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FATIGUE TO FRACTURE INTEGRITY ASSESSMENT IN ENGINEERING AND BIOMEDICAL ENGINEERING

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Abstract

Degradation of structural integrity and fatigue life estimation remain critical challenges in engineering, including biomedical applications. Fracture mechanics and crack propagation prediction are highly sensitive to material parameters, with the Stress Intensity Factor being the most significant physical parameter for the estimation of crack stress fields. This paper applies a fatigue crack growth model and structural integrity assessment using advanced numerical methods. The model calculates Stress Intensity Factor via the J-Equivalent Domain method, implemented in the in-house PAK software. Crack growth is simulated using the Extended Finite Element Method, incorporating discontinuous functions and asymptotic crack-tip displacement fields through Partition of Unity and Fast Marching-Level Set methods, which eliminates explicit crack meshing. The approach is validated through case studies from both classic engineering and biomedical structures.

Keywords: Fatigue crack growth, Integrity assessment, Stress Intensity Factor, J-Equivalent Domain Integral, Extended Finite Element Method

1. Introduction

The Finite Element Method (FEM) has become the gold standard for structural analysis of engineering structures, including applications in the field of biomedical engineering. Today, there is no serious analysis of engineering structures that does not involve the application of FEM. Academician prof. Miloš Kojić and prof. Radovan Slavković, at the Faculty of Mechanical Engineering, University of Kragujevac, started the pioneering venture of developing in-house FEM based software PAK (Kojić et al., 1998, 2010), 50 years ago. During this time, a number of PhD and master's students have implemented specific modules within the in-house software PAK, including PAK – Multiphysics. Also, it should be emphasized that

PAK is the only FEM software in Serbia and more widely in the South East Europe that is open-source and allows young researchers to develop an original approach and produce a top quality research in applied mechanics. PAK-Multiphysics includes a wide range of solvers: PAK-S (for static analysis), PAK-F (for fluid dynamics), PAK-FM&F (for fracture mechanics and fatigue), etc. Special emphasis should be placed on the application of PAK in Biomedical Engineering, where academician Kojić and his research team produced pioneering work. The FEM has found a large area of application in the biomedical engineering, including both solid mechanics such as bone tissues, stents, etc., and in fluid mechanics for the simulation of blood flow.

The advantage of PAK software is reflected in the implementation of the novel numerical methods that were not available at the time in other commercial software. Some examples include the implementation of Fast Marching-Level Set (FM&LS), Extended Finite Element Method (XFEM) and J-Equivalent Domain Integral (J-EDI) methods in PAK, as advanced numerical methods for simulating crack growth in the structure. Fatigue crack growth simulation as well as integrity assessment could be performed both by classical FEM and advanced computational techniques such as XFEM. The numerical methods mentioned above have proven to be the only reliable computational resource for simulating fatigue crack growth because analytical solutions for complex structures have not been available. At that time XFEM, was not available in any other commercial software.

The essence of applying advanced FEM methods in the simulation of fatigue crack growth is to correctly predict the discontinuous physical fields around the crack. The first attempts to simulate discontinuities in the finite element method, the so-called "weak" discontinuities were published by researchers Ortiz et al. (1987) and Belytschko et al. (1988). They defined the discontinuity in the deformation field using the multi-field variational principle. Dworkin et al. (1990) developed "strong" discontinuity in the displacement field by modifying the principle of virtual work. In numerical modeling of strong discontinuity, the physical displacement field consists of normal and added components, where the added components in the displacement field arise from enrichment functions. Belytschko and Black (1999) introduced Near Tip (NT) functions into finite element (FE) approximation. These functions allowed for a jump of physical quantity near crack tip is achieved. The procedure for introducing additional functions into the finite element approximation is based on the Partition of Unity (PU) principle. NT functions were initially introduced by Belytschko et al. (1993) and Fleming et al. (1997) in the development of the Element Free Galerkin (EFG) method. An additional improvement of the "strong" discontinuity in the displacement field around the crack face was achieved by Moes et al. (1999) by introducing the Heaviside function in the FE interpolation while respecting the PU principle. So, by inserting additional enrichment functions (NT and Heaviside) into FEM, XFEM was created.

However, this idea of inserting additional functions into XFEM is not new but is taken from the EFG method. A NT functions and the Heaviside function are used to enrich the finite element approximation in the XFEM. Combination of NT and Heaviside functions resulted in an extremely robust numerical method for simulating discontinuities in the physical fields on the crack faces as well as the "singular" stress field around the crack tip. The robustness of XFEM compared to other methods is reflected in the following: there is no scaling factor inherent in EFG, and it must be adopted empirically; or there is no remeshing of the mesh as with the classic FEM when simulating crack propagation. This XFEM methodology is implemented in the software PAK based on the standard finite element approximation.

Structural integrity assessment of critiral engineering or biomedical devices is important to ensure the safety, durability and reliability. In that case, the assessment includes tasks in many areas, such as structural and failure analysis, non-destructive testing, structural monitoring and instrumentation, fatigue analysis, fatigue life assessment, safe-operation assessment, etc. From the mechanical aspect, the failure may occur due to a static load exceeding the material strength or due to damage accumulation caused by cyclic loading. Cyclic load causes stress changes which, over time, may cause degradation of mechanical properties (i.e. damage accumulation), initiation and growing of micro-cracks and, consequently fracture – fatigue failure. Material fatigue can cause the following processes in a structure: initial damage accumulation, damage growth caused crack initiation, crack propagation and total failure i.e., fracture.

Structural damage is identified as the degradation of material stiffness and is very important as an inverse optimization problem for fatigue assessment and residual lifetime prediction. The first step in failure is localization of damage. The principal aim is to define and apply numerical preventing procedures to assess durability of engineering or biomedical structure based on the estimation of safety from fatigue to fracture. This procedure cosnsists of three phases: **safe-**operation i.e., no-fatigue failure or fracture from failure; initial fatigue analysis i.e., stress-based fatigue estimation; fatigue lifetime assessment based on fatigue crack growth simulation using Paris-power law.

It is well known that fracture mechanics alone may not predict the full scenario of the: degradation of mechanical properties, damage localization, crack initiation, stable crack growth and unstable crack propagation i.e., total failure. Crack initiation is based on accumulation of damage and requires damage mechanics to model the gradual loss of stiffness in a small area Moes et al. (2011). Damage localization was achieved by using a Level Set (LS) method, while planar damage propagation was simulated using Fast Marching Level Set (FM&LS) method. The LS method is a numerical scheme tailored, among others, to model arbitrary cracks, holes and material interfaces (inclusions), without meshing the internal boundaries. Osher and Sethian (1988) introduced the Level Set Method (LS method) to represent the interface as the zero level surface of a function of one dimension or higher. The LS method is established on Initial value formulation. This technique is based on the finite difference method for hyperbolic conservation laws enabling the accurate and stable evolution of sharp corners and cusps in interface. FM&LS methods are used to track the motion of an arbitrary monotonically advancing interface.

This paper presents applications of these advanced numerical methods within PAK for fatigue crack growth modeling and integrity assessment, with case studies including assessment of stable crack growth in engineering and Fatigue to Fracture (FtF) assessment in biomedical engineering examples.

2. Methods for fatigue to fracture crack growth and integrity assessment

In the assessment of durability and reliability of the structures, there are two approaches that depend on the size of the structure and whether or not damage occurred in the form of an initial crack. In order to achieve appropriate and acceptable reliability in the framework of critical structures and to prevent FtF. Therefore, FtF is a process that can develop in structures subjected to cyclic loading without indications of an initial crack. The theory of small cracks is necessarily used in the FtF approach, and as such it should certainly be implemented if the structure is of small dimensions. In the case when a crack with dimensions larger than the initial one is observed in the structure, which is a frequent case in engineering, a stable crack growth simulation procedure and assessment of the residual life of such structure is carried out.

2.1 Residual lifetime assessment in case of stable crack growth

Determination of physical variables (displacements, deformations and stresses) using the XFEM or FEM method in a mechanical structure containing a crack is the starting point for assessing the integrity of a structure. The assessment of the integrity of a structure that has a macro-crack is reflected in the definition of stress intensity factors (SIFs) as a basic parameter of fracture mechanics. In commercial software, there are different methods for defining SIFs, one being the Equivalent Domain Integral (EDI) method for evaluation of the J-integral. The J-EDI numerical method is very useful for defining the SIFs parameters. This method could also be applied for post-processing in the FE framework as well as in the XFEM approach in the PAK software. Integrity assessment of existing structures with an observed crack is based on defining SIFs, which is in turn calculated based on the stress field around the crack. This is followed by the simulation of crack growth and the calculation of the remaining life. In order to carry out the simulation of crack growth in an effective way, the application of FM&LS techniques for automatic identification of damage is required.

2.1.1 Identification of the damage zone using FM&LS techniques

FM&LS techniques were first introduced by Sethian (1998). The FM method is established on the Boundary value formulation, employs no time step, and is not subject to time step restriction, unlike LS methods. These techniques have been used in a large variety of applications, including fluid interface motion, two phase flow simulation, combustion, dendritic solidification, etching and deposition semi-conductor manufacturing, robotic navigation and the path planning, computation of seismic travel times, image segmentation in medical imaging scans, see (Sethian and Chopp, 1995; Sethian, 1999):.

The model derivation is based on the assumption that the initial position of the damage front is the zero level set of a higher dimension LS function, ψ. Evolution of this function ψ is then associated with propagation of the damage front itself through a time dependent initial value problem. At any time instance, the front is given by the zero level set of the ψ in all points in the computational domain (Sethian and Chopp, 1995; Sukumur et al., 2002; Jovičić, 2005). In order to derive an equation of motion for the LS-function ψ, the zero level set always matches the propagating hyper-surface. It means that:

$$\psi(\mathbf{x}(t),t)=0 \tag{1}$$

In order to derive partial differential equation for the time evaluation of ψ one can use the chain rule:

$$\psi_t+\nabla\psi(\mathbf{x}(t),t)\,\mathbf{x}'(t)=0 \tag{2}$$

where ψ_t denotes a time derivative of the function and $\mathbf{x}'(t)$ is partial derivative coordinates by time. It then follows that :

$$\mathbf{x}'(t)\cdot\mathbf{n}=F \tag{3}$$

In Eq. (3), F is a speed of interface propagation in the outward normal direction on damage interface, where the outward normal direction is obtained from the level set function ψ, as:

$$\mathbf{n}=\frac{\nabla\psi}{\|\psi\|} \tag{4}$$

Equation (2) then becomes

$$\begin{aligned}&\psi_t+F\,\|\nabla\psi\|=0\\&\psi(\mathbf{x},t=0)=\pm d(\mathbf{x})\end{aligned} \tag{5}$$

where the right hand side in Equation 5b) is given. The term $\pm d(\mathbf{x})$ is the signed distance from $\mathbf{x}$ to the initial front. Eqn. (5b) is the level set equation introduced by Osher and Sethian (1988). This formulation can be applied for the arbitrary speed function F . As analyzed by Sethian (1998), the efficient solution of these front propagation problems requires the use of upwind difference schemes taken from the solution of hyperbolic conservation laws.

Two LS functions are used for crack identification in the XFEM method, which fully define discontinuity across the sides of the crack as the position of the crack type and the area in front of the crack tip. Linear segments representation of the crack is shown in Fig. 1, by using Level Set functions:

$$\psi(\mathbf{x}) = sign\left[\mathbf{n}\cdot(\mathbf{x}-\mathbf{X_{CT}})\right]\min_{\mathbf{x}\in\Gamma_c}\left|\mathbf{x}-\mathbf{X_{CT}}\right|, \quad \phi(\mathbf{x}) = (\mathbf{x}-\mathbf{X}_{CT})\cdot\mathbf{t} \tag{6}$$

where are: $\mathbf{n}$ is the unit normal to crack at crack tip; $\mathbf{t}$ is the unit tangent to crack at the crack tip; $\mathbf{X}_{CT}$ are coordinates of the crack tip, $\mathbf{x}$ coordinate of the point around the crack.

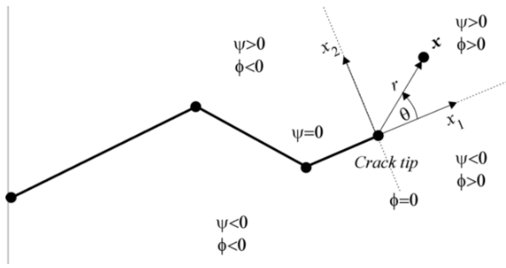


Fig. 1. Linear segments domain representation of the crack using Level Set functions.

One way to characterize the position of this expanding front is to compute the arrival time $T(x,y)$ of the front as the crosses each point (x,y). The equation that describes arrival time surface $T(x,y)$ is derived using: *distance=rate*time*, ($\Delta x = F\Delta t$):

$$1 = F\frac{dT}{dx} \tag{7}$$

In multi dimension, the spatial derivative of the solution surface $T(x,y)$ becomes the gradient, and hence we have:

$$\left\|\nabla T\right\| = \frac{1}{F} \tag{8}$$

Eqn. (8) represents the boundary value partial differential equation describing the interface motion (Sethian, 1998). If the speed F depends only on position, then the equation reduces to the familiar Eikonal equation. If ψ is a signed distance function so that

$$\left\|\nabla\psi\right\| = 1 \tag{9}$$

is one of Equation (8) solutions. The function ψ remains the signed distance function for all time in all regions where ψ and F are smooth if:

$$\nabla F\cdot\nabla\psi = 0 \tag{10}$$

Eqn. (10) assures that function ψ remains the signed distance function that satisfies Eqn (8) for all time. Solution of Eqn (10) gives the extension velocity *F*. Also, with (10) it is assured that *F=const.* along the normal direction of the damage front. The boundary value perspective is restricted to the front that always moves in the same direction, i.e. outward, because it requires crossing time (T=ψ) at the each grid point, and hence a point cannot be revisited. In the study (Sethian, 1998), the assumption was made that the front velocity *F* depends only on the front position. Solving equations (8) and (9) can be done by using the FM method which is the optimal technique for solving Eikonal equation, coupled with a bi-cubic interpolation scheme.

2.1.2 Displacement approximation near a crack using XFEM

The displacement approximation $\mathbf{u}(\mathbf{x})$ in the X-FEM consists of a continuous and an enrichment part:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}_{con}(\mathbf{x}) + \mathbf{u}_{enrh}(\mathbf{x}) \tag{11}$$

where: the continuous displacement approximation $\mathbf{u}^h_{con}(\mathbf{x}) = \sum N_I(\mathbf{x})\mathbf{u}_I$ is standard approximation in the FEM, and $\mathbf{u}_{enrh}(\mathbf{x})$ is enrichment part of displacement approximation near the crack (Fig. 2). Therefore, in the XFEM, the enrichment functions are added to the finite element approximation to represent the intra-element discontinuous field. In particular instance of 2D crack modeling, the enriched displacement approximation is written as (Sukumur et al., 2002; Jovičić, 2005):

$$\mathbf{u}^h_{enrh}(\mathbf{x}) = \sum_{I \in \mathcal{N}_a} N_I(\mathbf{x}) H(\mathbf{x}) \mathbf{a}_I + \sum_{I \in \mathcal{N}_b} N_I(\mathbf{x}) F_\alpha(\mathbf{x})\, \mathbf{b}^\alpha_I, \quad \alpha = 1,4 \tag{12}$$

where N_I, $I = (1, N)$ are the finite element shape functions; $\mathbf{a}_I$ are additional degrees of freedom associated with the Heaviside (discontinuous) function; $\mathbf{b}^\alpha_I$ are additional degrees of freedom associated with the Westergaard asymptotic crack-tip functions; $H(\mathbf{x})$ is the Heaviside function:

$$H(\mathbf{X}) = \begin{cases} 1 & \text{if} \quad (\mathbf{X} - \mathbf{X}^*) \cdot \mathbf{n} \geq 0 \\ -1 & \text{if} \quad (\mathbf{X} - \mathbf{X}^*) \cdot \mathbf{n} < 0 \end{cases} \tag{13}$$

where $\mathbf{X}$ is the sample (Gauss) point, $\mathbf{X}^*$ (lies on the crack) is the closest point to $\mathbf{X}$, and $\mathbf{n}$ is unit outward normal to crack at $\mathbf{X}^*$. The $F_\alpha(\mathbf{x})$, $\alpha = (1, 4)$ are Westergaard asymptotic Near-Tip (NT) functions:

$$F(\mathbf{x}) = \{F_1, F_2, F_3, F_4\} = \left[\sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right] \tag{14}$$

where are: $r(\mathbf{x})$ and $\theta(\mathbf{x})$ polar coordinates of the point $\mathbf{x}$. The polar coordinate system is attached to the crack tip, see Fig. 2.

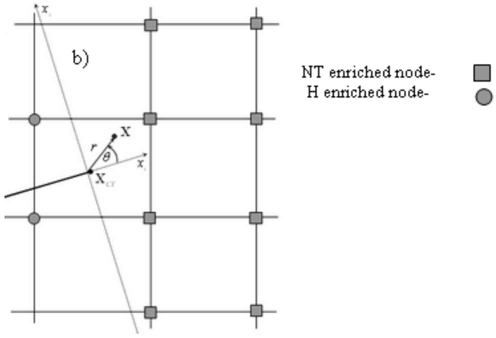


Fig. 2. Enrichment nodes near the crack tip.

In Fig. 2 the circle nodes are enriched by Heavisade function, the square nodes are enriched by NT functions, while the other nodes are not enriched. Enrichment by the H function is applied only behind the crack, hence discontinuity occurred.

2.1.3 Determination of the SIFs using J-EDI method.

The J-EDI numerical method is very useful for defining the SIFs parameters and can be applied for post-processing in both the FE and the XFEM framework. The EDI approach (Moes et al. (1999); Jovičić, (2005)) has the advantage that the effect of body forces can be easily included. Discretized form of the J-EDI integral is:

$$J_k = \int_A \left(\sigma_{ij} u_{i,k} - W\delta_{kj}\right) q_{,j} dA + \int_A \left(\sigma_{ij} u_{i,k} - W\delta_{kj}\right)_{,j} q dA \quad (15)$$

where: W is the strain energy density, σ_{ij} is stress tensor, u_i are components of the displacement vector, $u_{i,k}$ is derivative of the displacement with respect to x_k, $q_{,j}$ is derivate of the weight function with respect to coordinates x_j. The present formulation is for a structure of homogeneous material in which no body forces are present. Once the numerical calculation of the J_k, the Stress Intensity Factors K_I and K_{II} for modes I and II, respectively, can be obtained from the following equations:

$$J_1 = \frac{K_I^2 + K_{II}^2}{E^*}, \quad J_2 = \frac{-2K_I K_{II}}{E^*} \quad (16)$$

where: $E^* = E$ for plane strain, $E^* = E/\left(1-\nu^2\right)$ for plane stress, E is Young's module, ν is Poison's ratio.

2.1.4 Residual Lifetime Prediction for Fatigue Crack Growth

Paris power law of fatigue crack growth is used for the crack growth prediction in this model. It is noteworthy that Paris power law cannot predict fracture, but gives good results in the region of stable crack growth. Since the pre-existing flaws are typically present in the structures, this approach is damage-tolerant for assessing fatigue life. Therefore, residual fatigue lifetime prediction is based on the simulation of fatigue crack growth and the Paris law in a conservative formulation:

$$da_i = dN \cdot C\left(dK_i\right)^m \quad i = 1, n \tag{17}$$

where; C , m are material fatigue parameters.

In the case of a variable operating mode of the structure, it is necessary to take into account the influence of stress fluctuation on the remaining life of the structure by using the stress alternation factor. Consequently, the crack growth simulation is performed by applying the corrected Paris law:

$$da_i = C\left(1-R\right)^{0.5m}\left(dK_i\right)^m dN \tag{18}$$

When $R \to 0$ the corrected Paris law (18) reduces to the Paris law in Eqn. (17). In the previous relations, the length of the crack in the i+1 iteration is defined based on the crack growth increment in the i-th iteration, i.e.: $a_{i+1} = a_i + da_i$. For the Theory of small cracks, the stress intensity range (in eq. (17)) is defined as $\Delta K = Y\Delta\sigma\,(\pi a)^{1/2}$, where; Y is a constant dependent upon the geometry, flaw size and shape; and $\Delta\sigma$ is far-field tensile stress range ahead the crack tip. For the small flaw, it is reasonable to assume that Y does not change during the crack growth.

For the macro-crack theory, which is most common in engineering practice, SIFs is defined numerically based on Eqn. (16), and the range of the SIF for eq. (17) is determined based on the equivalent stress intensity factor:

$$K_{Ieq} = K_I \cos^3\frac{\theta_c}{2} - 3K_{II}\cos^2\frac{\theta_c}{2}\sin\frac{\theta_c}{2} \tag{19}$$

where θ_c denotes the direction in which the crack is likely to propagate relative to the coordinate system attached to the crack tip ,see Fig. 3.

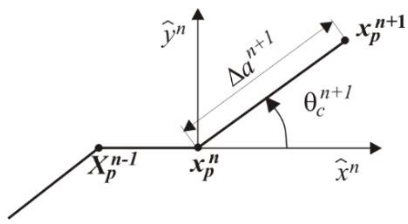


Fig. 3. The crack growth angle in (n+1) step of the simulation.

The crack growth angle in which the crack propagates is (Jovičić, 2005):

$$\theta_c = \pm\cos^{-1}\left(\frac{3K_{II}^2 + K_I^2\sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2}\right) \tag{20}$$

The above equation was developed in the numerical simulations of the crack growth in PAK-XFEM. The crack growth simulation in (n+1) step is calculated based on the SIFs value from the nth step. Also, the crack propagation angle in n+1 step, eq. (16), is performed based on the value of SIFs in the nth step.

Based on numerically calculated values of SIFs and simulation of crack growth, an R-curve can be formed, see Fig 4. This curve is useful in engineering practice because it defines the critical zone for the operation of the structure.

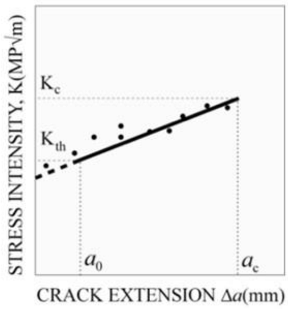


Fig. 4. R-curve with specific values of crack size and stress intensity factor.

The R-curve is the fracture resistance curve and defines the relationship between the stress intensity factor (or J-integral for nonlinear case) and the crack length. In the theory of macro cracks, it is considered that crack growth is stable in the zone between the threshold stress intensity factor and the fracture toughness.

2.2 FtF approach of the integrity assessment

The FtF approach for integrity assessment can be defined through several post-processing codes: safe operation assessment, stress-based fatigue estimation, and residual lifetime prediction for fatigue crack growth. Which of the approaches will be applied largely depends on: the size of the structure, the presence of damage in the form of a crack, crack size (micro or macro), significance of the component the integrity assessment is performed for. The safe-operation assessment is based on the prediction of critical fatigue stress range in operation for structure with pre-existing flaws. This form of integrity assessment is necessary with structures of small dimensions, such as a cardiovascular stent. Safe-operation estimation is also necessary for high-responsibility engineering structures such as nuclear reactors. Stress-based fatigue estimation is used for structures without visible cracks that are exposed to monotonous or complex cyclic loading. Residual lifetime prediction is used in the numerical simulation of the growth of an existing crack.

2.2.1 Safe-Operation Assessment in Theory of small crack

Fatigue crack growth typically occurs in stages: crack initiation (small cracks) followed by long crack propagation until unstable propagation and failure. The theory of small cracks is crucial for the crack initiation stage, where conventional long-crack fracture mechanics does not fully apply. Small cracks, typically of microstructural size, can grow at stress levels that would not propagate larger cracks. The Kitagawa–Takahashi (K–T) diagram provides a framework to combine fatigue limit (endurance strength) with fracture mechanics, mapping regions of safe vs. unsafe crack growth behavior (Ritchie, 1999; Jovičić et al., 2014; Vukićević et al., 2015). Fig. 5 schematically illustrates the K–T diagram, which defines a safe-fatigue zone by the line A–B–C.

Below the A–B line in the K–T diagram, the crack is smaller than a certain transient crack size a_0, meaning classical continuum fracture mechanics cannot be directly applied. This sub-A–B region is essentially a no-fatigue-failure zone where very small cracks do not yet propagate in the conventional sense. Point B on the diagram corresponds to the crack reaching the transitional size a_0, beyond which the material's full long-crack threshold behavior is reached. The portion B–C represents the threshold stress intensity range (ΔK) required for a long crack to grow at the fatigue limit. Stresses or flaw sizes above line A–B–C will cause crack growth or even immediate fracture, whereas operating below that line ensures the crack remains dormant (non-propagating).

The transient crack size a_0 is a material-specific length that marks the boundary between small-crack behavior and long-crack behavior. If a flaw is smaller than a_0, the effective fatigue threshold is lower and varies with crack length. In this regime, the threshold stress intensity factor range ΔK_{th} for crack growth increases as the crack grows. The transitional length a_0 itself can be calculated from the intrinsic threshold ΔK_{th0} (the fatigue threshold for long cracks) and the material's fatigue limit (Δσ) as:

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}^0}{Y\sigma_e}\right)^2 \tag{21}$$

where Y is a geometry factor for the crack. Above a_0, the material's threshold ΔK_{th0} is essentially constant, intrinsic to the material, independent of crack size.

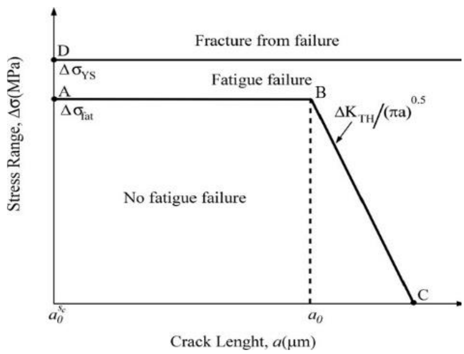


Fig. 5. KT diagram for safe-operation assessment.

The previous procedure is extremely important for critical structures of small dimensions, which requires use the Theory of Small Cracks.

3. Example of assessment of stable crack growth in engineering using FM&LS and XFEM

In most engineering structures, fracture mechanics theory is generally used when the existence of an initial crack has already been observed. In that case, the stable growth of the crack and the remaining life span are assessed. The FM&LS method is used in the case of planar damage for tracking its propagation, .

3.1 Simulation of damage propagation using the FM&LS method

To illustrate influence of the velocity function along the front on the propagation of damage in Eqn. (7), two cases of the planar damage growth are shown in Fig. (6): a) the propagation velocity along the front is constant and b) the velocity along the front is not constant.

Figure 6a shows the results of a numerical simulation of the propagation of planar polygonal damage during 100 time steps. For the constant speed along the front, $F = const$ shown in Fig. 6a, the crack retains its original shape during propagation. That is, the new positions of the crack are defined by lines of constant values of the transition time and those lines are equidistant. Figure 6b shows the propagation of a planar polygonal crack in the case where the velocity along the front depends on the location, which implies that the crack gradually changes its shape, i.e. it elongates in the direction of the highest SIFs value.

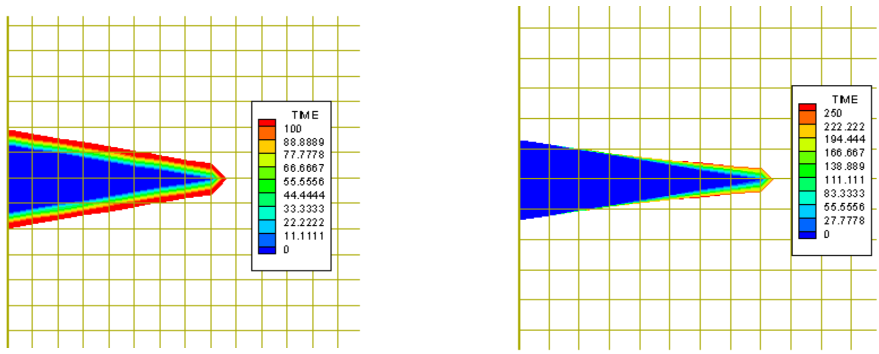


Fig. 6. Propagation of planar damage. a) Constant speed along the front; b) Speed at the front that depends on the position of the front point.

3.2 Simulation of crack growth in the steam turbine housing for cogeneration electricity production

An example of the application of the XFEM method in engineering practice is illustrated for the case of stable crack growth in the steam turbine housing for cogeneration electricity production. As previously indicated, in order to evaluate stable macro-crack growth, it was necessary to experimentally determine fracture toughness. In the XFEM, four nodes linear elements are used and 6x6 Gauss quadrature only in the part of the domain enriched by NT and Heaviside functions.

Effective stress for 2D turbine model without insulation, for a number of crack lengths is determined by using PAK-XFEM. The crack path is independent of the mesh structure, i.e. the crack overlaps the elements edges, and there is no physical separation of the joint sides of elements, see Fig. 7. One of the critical parts of the turbine is a hole for the steam supply, with the sharp edges which cause stress concentration and where the cracks appear.

Figure 8 shows crack growth in case of load fluctuation in the steam turbine casing. The load fluctuation in a steam turbine for cogeneration electricity production is defined through the stress alternation factor R by applying the corrected Paris law. Changing the level of load fluctuation in the turbine housing significantly affects the residual life.

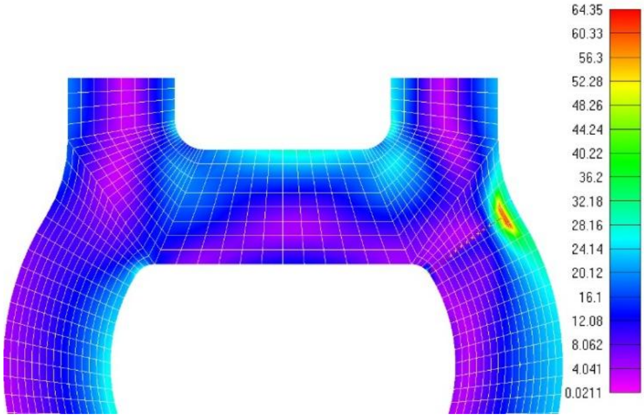


Fig. 7. Effective stress field due to crack growth in the lower housing part without insulation (using PAK-XFEM).

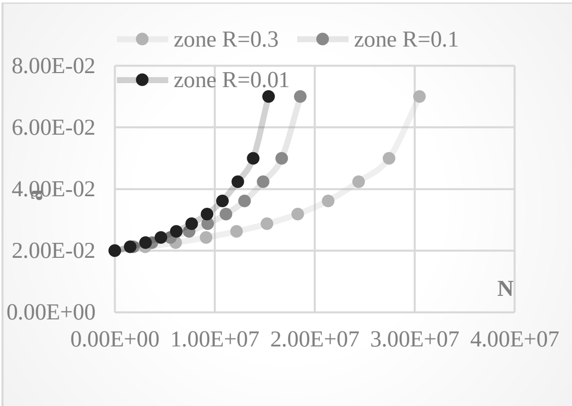


Fig. 8. Dependence of the crack length on the number of cycles obtained by applying the corrected Paris law of growth.

Figure 9 shows the numerically obtained R-curve, i.e. the fracture resistance curve, for the casing of the cogeneration turbine housing, for a range of crack lengths between 20mm and 70mm, by using classic FEM and XFEM implemented in PAK. The same mesh density was used in both models, where FEM result was obtained with eight node quadratic elements, whilst the XFEM solution was based on the linear four node elements. In both numerical results shown in Figure 9, a sudden change in the growth rate occurs beyond the crack length of 60 mm. The crack lengths below 20mm were not considered, as this is a typical crack length observed during standard turbine housing inspection.

A slightly lower slope of the resistance curve up to the crack length of 60 mm was obtained by using PAK-XFEM, which is a consequence of the use of linear four-node finite elements.

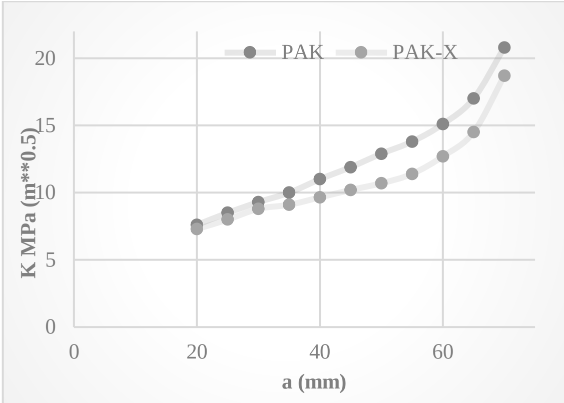
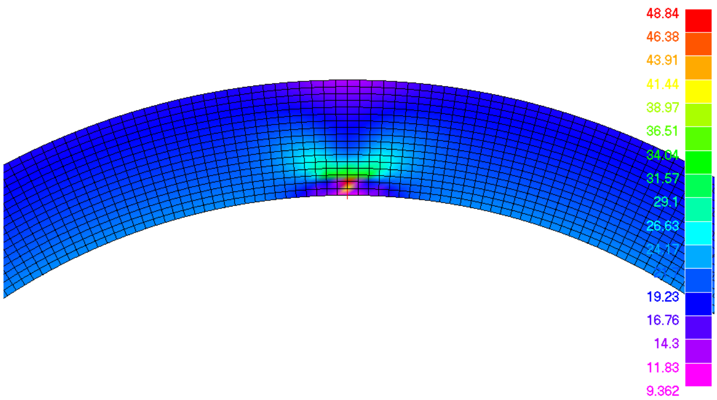


Fig. 9. R-curves for the turbine housing obtained by PAK-FEM and PAK-XFEM.

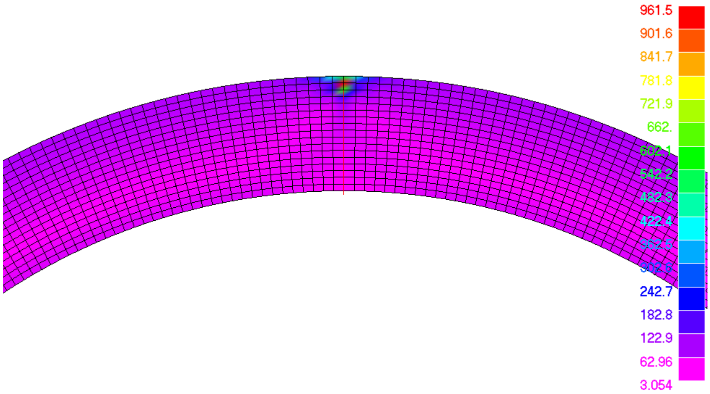
Essentially, both methods gave the same assessment of the change in crack growth rate, which in this case occurred at a crack length of 62 mm. The advantage of XFEM over FEM is reflected in the simulation of crack growth obtained on a fixed finite element mesh.

3.3 FtF Assessment application on a high-pressure steam pipe in a thermal power plant

In the case of high-responsibility engineering structures where one dimension is significantly smaller than the other two, the use of FtF tips is extremely justified. In order to simulate a stable operation of such structure using the FtF method, a previous simulation of crack growth is necessary. In this case, considered a is a pipe of a high-pressure steam pipeline, which is modeled using the XFEM method and the space is discretized with four-node finite elements, see Fig. 10.



a=1mm



a=15mm

Fig. 10. Crack growth on high pressure steam pipe.

In the case of crack growth simulation, the crack growth increment corresponding to the initial crack length is adopted. The crack growth simulation was performed in 16 steps. Stress intensity factors were numerically calculated using the J-EDI method. The simulation of crack growth was carried out on a fixed finite elements mesh in the PAK-XFEM, while remeshing was performed at each calculated step of the crack growth in the PAK-FEM.

During the simulation of crack growth on a high-pressure pipe, due to the existence of axial symmetry, half of the steam pipe was modeled. Inside the tube, there is a pressure of 36 bar. Fig 11 shows a magnified view of the area on the high-pressure pipe, where the crack is propagating, during the 1st and 15th increments of crack growth.

Numerical results of KI SIF obtained by PAK-XFEM were compared with the analytical solution and numerical results obtained by PAK-FEM in Fig. 11.

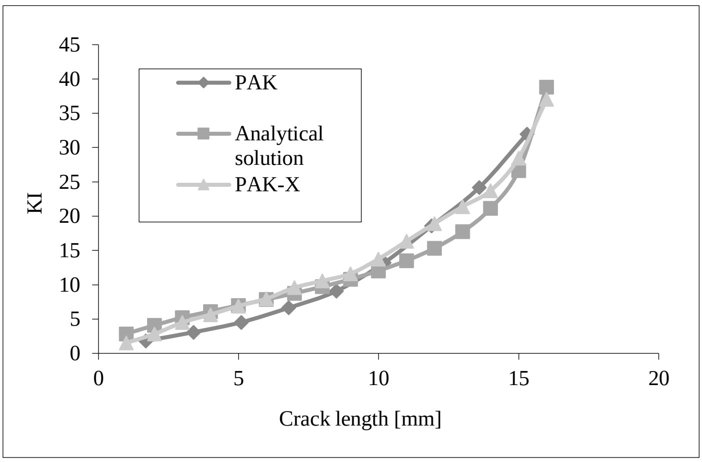


Fig. 11. Comparative view of K_I obtained with PAK-XFEM, PAK-FEM and analytical.

PAK-XFEM solution for the crack growth agrees better with the analytical solution than PAK-FEM. Using PAK-FEM, further evaluation of the KI value beyond the crack length of 15mm was not possible.

4. Example FtF Assessment in Biomedical Engineering

The structural evaluation of critical biomedical devices is essential to ensure their safety, durability, and reliability. To achieve the required reliability in biomedical devices and prevent fatigue to fracture, a thorough assessment must be carried out. A cardiovascular stent, as a representative biomedical implant, must be designed with a focus on failure prevention and structural integrity. The primary goal is to ensure that the stent remains fatigue-resistant for a lifespan of 10–15 years without failure. According to the ASTM F2477-06 standard (ASTM, 2019), a cardiovascular implant must withstand at least 4×10^8 loading cycles.

The structural assessment of stents is performed using FEM (Jovičić et al., 2014; Jovičić et al, 2024). A 3D FEM simulation was carried out to analyze the durability of a coronary stent. The geometry of the 3D model is illustrated in Figure 12 (Jovičić et al., 2014). The stent is made of L-605 Co-Cr alloy, with a length of 7 mm, an initial internal diameter of 1.5 mm, and an outer diameter of approximately 1.65 mm before implantation. Mechanical properties of the high-quality L-605 Co-Cr alloy are detailed in Table 1.

Young's modulus $E = 243GPa$ *Yield strength* $\sigma_Y = 547MPa$	*Endurance strength* $\sigma_e = 207MPa$ *Ultimate tensile strength* $\sigma_u = 1449MPa$	*Threshold range* $\Delta K_{th}^0 = 2.58MPa\sqrt{m}$ *Fracture toughness* $\Delta K_C = 60MPa\sqrt{m}$	*Fatigue parameters* $C = 4.74 \times 10^{-13}$ $m = 10.39$

Table 1. Material properties for L-605 Co-Cr

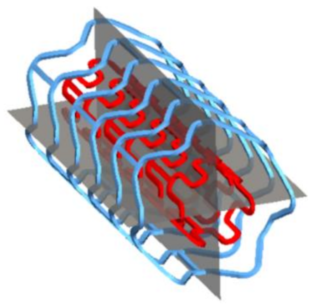


Fig. 12. 3D model of coronary stent before and after expanding.

The concepts of small-crack theory and fracture toughness were used in assessing a cardiovascular stent. Stents experience cyclic loads in service (pulsatile blood pressure) and may contain micro-scale manufacturing flaws. The analysis here considered a small surface flaw of about $83\mu m$ depth in a stent structure and evaluated its behavior under physiological loading. The Kitagawa–Takahashi diagram was used to plot stress range versus crack length in relation to the material's fatigue limit and threshold curves. Fig. 13 shows the K–T diagram with the stent's operating stress range and the flaw size, along with numerical results of crack propagation analysis in the stent.

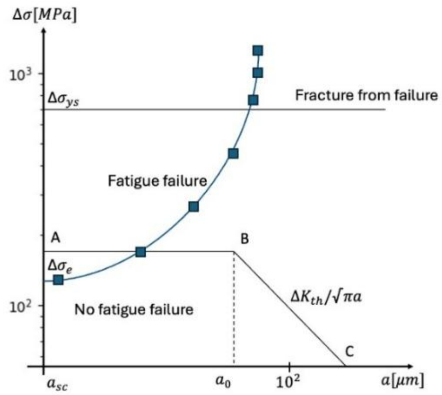


Fig. 13. KT-diagram and numerical values of flaw propagation in the stent.

Using fracture mechanics, the stress intensity factor K for the flaw was calculated throughout the load cycle. The maximum K experienced at the flaw (Mode I) was about 18.08 MPa•√m, which is approximately 3.3 times lower than the alloy's fracture toughness. In other words, Kmax ≈ $18.08MPa\sqrt{m}$ vs. KIC ≈ 60 MPa•√m, based on the material properties, see Table 1 for the exact fracture toughness value. This comparison indicates that an immediate fracture event is not likely for the given flaw size and loading – the crack simply cannot attain a critical stress intensity in service. Indeed, the analysis concluded that the stent will safely achieve its projected life without fracturing.

Furthermore, the position of the flaw on the K–T diagram fell below the threshold line A–B–C, but only slightly. This means the flaw is just at the margin of propagating. In the simulation, the crack did grow, but in a stable, sub-critical manner. The flaw did not remain entirely in the "no-fatigue-failure" zone, since some crack growth was observed, but it never entered the catastrophic fatigue failure zone. Instead, the crack growth rate corresponded to a steady, slow propagation consistent with life span. Both the very early stage (small crack regime) and later stage (larger crack) growth remained stable – the crack did not suddenly accelerate at any point, indicating that neither the endurance limit nor the fracture toughness was exceeded under the given loading.

The stent case study illustrates the practical application of the fatigue-to-fracture approach. By comparing the flaw's stress intensity to the fatigue threshold and to KIC, engineers can determine whether a flaw will grow and whether it poses a risk of fracture. In this example, the flaw's K was below the critical toughness and near or below the threshold required for sustained growth, placing it in a safe regime for the device's lifetime. Thus, neither immediate fracture nor runaway fatigue crack growth is expected.

5. Conclusions

This paper presents an extensive overview of the developed numerical code for simulating crack growth using the XFEM and FM&LS methods. The numerical code integrated in PAK is very complex and consists of pre-processing (based on FM&LS), processing (based on XFEM/or FEM) and post-processing. The post-processing phase enables structural integrity assessment through numerical computation of the J-integral and the resistance curve, with a broader evaluation based on the Fatigue-to-Fracture (FtF) approach. Integrity assessment in post-processing code of the FEM was based on Fatigue to Fracture approach: Safe-Operation Assessment, Stress-Based Fatigue Estimation and Residual lifetime Prediction for Fatigue Crack Growth.

The aim of this paper was to present developed numerical procedure for assessment of engineering or biomedical structures durability based on continuum mechanics. The methodology used depends on whether a crack is present, crack size (micro/macro), and the criticality of the structure. For structures where a macro-crack has been detected, fatigue lifetime assessment based on the crack growth simulation is applied. Safe-operation assessment is used to define three critical zones: (1) safe operation without fatigue damage, (2) fatigue failure onset, and (3) fracture due to fatigue damage. This approach was demonstrated in the case of a biomedical device—a cardiovascular stent.

Further, the FtF-based residual lifetime prediction was applied to engineering structures, including a turbine for cogenerative electricity production and a high-pressure steam pipeline. In both cases, macro-cracks were present, and integrity assessment was performed using resistance curves derived from numerical (XFEM and FEM) and analytical methods. The robustness of XFEM was particularly evident in its ability to simulate crack growth on a fixed finite element mesh without remeshing, making it a reliable tool for structural integrity assessment. Furthermore, due to operational fluctuations in the turbine, a remaining life assessment was conducted via numerical crack growth simulation. The results confirmed that the proposed FtF post-processing algorithm provides a conservative and reliable estimation of structural durability, ensuring safety in critical applications.

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