Journal of the Serbian Society for Computational Mechanics / Vol. 19 / No. 1, 2025 / pp 139-145 / review paper
(10.24874/jsscm.2025.19.01.06)

OUR MULTISCALE FINITE ELEMENT MODEL OF GROUNDWATER FLOW TO RADIAL WELL

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Abstract

A water supply system with radial wells (RWs) extends usually over tens of kilometers horizontally and tens of meters deep within the soil. Water flows through the soil and then through several lateral screens to the vertical shaft. Lateral screens represent perforated pipes with lengths in meters and diameters measured in centimeters. A common approach in modeling the water flow is to use governing equations based on the Darcy law and transform them to the finite element form. The 3D finite element mesh follows the anisotropy of the space and the elements are dimensionally large. It would be impractical, inefficient, and complex to model lateral screens by 3D elements, and, additionally, to include colmated layers with a thickness of small size (measured in centimeters) around the screens. Therefore, this is dimensionally a multiscale modeling problem. We have resolved this task by modeling the screens by 1D finite elements aligned to the 3D mesh, with the flow according to the Hagen-Poiseuille law. The 1D and 3D element nodes are connected by fictitious (connectivity) 1D elements where a radial flow from the soil to the internal space of the screens is assumed.

We have implemented the multiscale model to our code PAK (Kojic et al., version in 2013) and applied it to the calibration of an RW of the Belgrade Groundwater Source

Keywords: groundwater flow, radial wells, multiscale finite element modeling

1. Introduction

Radial wells (RWs) installed by directional drilling significantly enhance the efficiency of water supply systems by facilitating groundwater abstraction in high-conductivity sand and gravel sediments beneath rivers. These wells allow water to flow from the river through the surrounding porous media, reducing the concentration of suspended solids and dissolved micropollutants through natural filtration and dilution (Ray, 2002). This filtration process minimizes the need for intensive post-treatment, ultimately lowering operational costs and improving water quality.

The uniqueness of RWs lies in their design, comprising a vertical caisson connected to multiple lateral screens that extend radially into the aquifer. These lateral screens, typically perforated pipes, introduce highly localized flow gradients that complicate numerical modeling. Over time, well ageing occurs due to the formation of low-conductivity deposits around the screens, reducing infiltration rates and necessitating periodic well maintenance or redevelopment. Various methods, including screen cleaning, sealing and replacing old screens, or constructing new RWs, have been employed to restore system performance.

The effectiveness of RWs, however, depends on accurate modeling and prediction of available flow rates under varying hydrogeological conditions. Groundwater flow modeling for RWs is inherently complex due to the three-dimensional nature of the flow field, the presence of multiple heterogeneous soil layers, and dynamic boundary conditions such as fluctuating water tables and river interactions (Dimkic, et al, 2013). Darcy's law is commonly employed as the fundamental principle governing flow in porous media, and when coupled with the continuity equation, it forms the basis for both steady-state and transient groundwater flow models. The challenge is further compounded by the anisotropy of the subsurface, where soil layers exhibit varying conductivities that can differ by several orders of magnitude.

The need for robust numerical and analytical models to simulate RW hydraulics has led to extensive research in this domain. Early studies by Huisman (1972) and Strack (1989) developed analytical solutions for two-dimensional (2D) flow to radial collector wells. Bischoff (1981) extended this work by employing boundary integral equation methods to model three-dimensional (3D) flow in confined aquifers with multiple lateral arms. Steward and Jin (2001) formulated a 3D analytical solution for horizontal wells, which are functionally equivalent to individual arms of a radial well. Further refinements were introduced by Zhan and Zlotnik (2002) and Zhan and Park (2003), who applied semi-analytical Laplace-transform-based solutions for flow into horizontal wells under unconfined and leaky aquifer conditions.

Ophori and Farvolden (1985) pioneered numerical modeling approaches by developing a finite element model for collector wells. Their initial implementation utilized a single-layer model before transitioning to a more advanced multi-layer representation incorporating point sinks. Subsequent improvements by Eberts and Bair (1990) involved applying the MODFLOW finite-difference model to simulate regional flow in a network of collector wells in Columbus, Ohio. These models were further refined by Chen et al. (2003), who introduced a polygonal finite-difference approach to simulate horizontal well performance, treating flow inside the screen as an equivalent porous medium with variable hydraulic conductivity. Later, Chen and Zhang (2009) expanded numerical well modeling by implementing various numerical schemes, including standard finite element method, control volume finite element method, and mixed finite element method. These approaches have improved the accuracy of groundwater flow simulations while maintaining computational efficiency. Recently, Perdikaki et al. (2022) analyzed the application of a modeling tool, for the simulation of the relative hydrologic processes between the open filter pipe of the injection well and the surrounding aquifer material with the finite difference method. The authors simulated groundwater flow in vicinity of horizontal direction injection well by using the Conduit Flow Process (CFP) of MODFLOW-2005 code.

Given the complex geometry of RW lateral screens and their interactions with the surrounding soil, finite element modeling (FEM) provides a viable framework for representing groundwater flow behavior. A highly refined 3D mesh can be used to explicitly model the lateral screens, but this approach is computationally expensive and challenging to implement due to mesh compatibility constraints. The generation of detailed 3D models requires substantial manual effort, as each screen must be precisely meshed, significantly increasing computational time and complexity.

To overcome these limitations, alternative modeling techniques have been proposed. Luther and Haitjema (2000) introduced a single-layer Dupuit-Forchheimer analytic element model to represent RW laterals without requiring a computational grid. This method directly simulates laterals as line-sinks while enforcing boundary conditions on the phreatic surface and seepage faces. Bakker et al. (2005) developed a multi-layer analytic element model to extend this concept to fully 3D systems, treating aquifers as horizontal layers with vertical flow resistance. Further refinement by Haitjema et al. (2010) incorporated a Cauchy boundary condition to simulate horizontal well screens in a regional Dupuit-Forchheimer model. By applying equivalent fictitious streams, their approach enables a more realistic representation of lateral screen flow interactions within the broader groundwater system.

A significant advantage would be achieved if the lateral screens could be represented using line finite elements with nodes aligned to the 3D mesh of the entire flow domain. This study presents a 3D-1D finite element coupling model, where 1D elements effectively replace the need for a fully detailed 3D representation of RW lateral screens, ensuring computational efficiency while maintaining accuracy (according to Dimkic et al., 2013).

The paper is structured as follows: Section 2 outlines the fundamental principles of the multiscale finite element model, detailing its formulation and methodology. Section 3 demonstrates the application of the proposed model to a real-world problem, specifically the calibration of a radial well in the Belgrade Groundwater Source. Finally, Section 4 presents the conclusion and summary, highlighting key findings and the broader implications of the study.

2. Multiscale finite element model

We here first present the theoretical basis of the finite element formulation, and then describe the connection between the 3D large domain with the screens as the lower-scale domain. The fundamental relations for water flow within the soil are represented by Darcy's law,

$$q_i = -k_{ij}\frac{\partial\phi}{\partial x_j} \tag{1}$$

where q_i is water velocity in the x_i direction, k_{ij} is the conductivity tensor, and ϕ is the potential defined as (Dimkic, et al., 2013)

$$\phi = \frac{p}{\gamma} + h \tag{2}$$

with p being the fluid pressure, γ is specific weight, and h is the height with respect to a reference plane. The tensor k_{ij} is defined by non-zero diagonal terms k_{xx}, k_{yy} and k_{zz}, so that the governing equation of the water flow, which relies on the mass balance, can be written as

$$k_{xx}\frac{\partial^2\phi}{\partial x^2} + k_{yy}\frac{\partial^2\phi}{\partial y^2} + k_{yy}\frac{\partial^2\phi}{\partial z^2} + q_V = S\frac{\partial\phi}{\partial t} \tag{3}$$

where q_V is the source term, and S is storage.

Using the standard procedure (Kojic et al. 2008), we obtain the incremental balance equation of a 3D finite element for a time step Δt, as

$$\left(\frac{1}{\Delta t}M_{IJ} + K_{IJ}\right)\Delta\phi_J = Q_I^{ext} + Q_I^V - \left(\frac{1}{\Delta t}M_{IJ} + K_{IJ}\right)\phi_J + \frac{1}{\Delta t}M_{IJ}\phi_J^t \tag{4}$$

where ϕ_J and ϕ_J^t are nodal values of the potential at the last equilibrium iteration and at the start of the time step, respectively; Q_I^{ext} and Q_I^V are external and volumetric fluxes, respectively; and the matrices are:

$$K_{IJ} = \int_V k_{ii} N_{I,i} N_{J,i} dV, \ sum\ on\ i: i = 1,2,3$$
$$M_{IJ} = \int_V S N_I N_J dV \tag{5}$$

We need the balance equation for a 1D finite element to model fluid flow within screens. Then, the Hagen-Poiseuille law can be applied (Kojic et al., 2022),

$$Q_x = -\frac{d^4\pi}{128\mu} dp/dx = -k_x dp/dx = -k_x \gamma d\phi/dx \tag{6}$$

where μ is the fluid viscosity coefficient, d is the screen internal diameter, and Q_x is the flux along the screen. The balance equation has the same form (4) , with the matrices:

$$K_{IJ} = \int_L k_x \gamma N_{I,x} N_{J,x} dx$$
$$M_{IJ} = A\rho \int_L N_I N_J dx \tag{7}$$

where A is the cross-sectional area and L is the length of the finite element; ρ is the fluid density.

Figure 1 shows a lateral screen aligned with the 3D mesh.

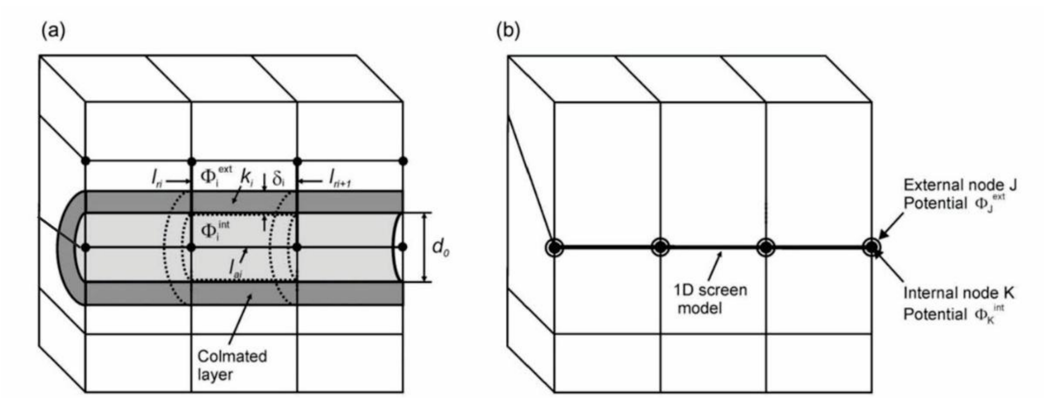

Fig. 1 Schematics of connection between large soil domain and the RW screens. a) Connectivity elements between 3D mesh for soil and 1D elements for screens. b) Doubled FE nodes with for the connectivity elements: J node belongs to 3D mesh, and K is the internal node at the same spatial position as node J. (according to Dimkic, et al, 2013)

As can be seen from Fig. 1b, we have two nodes at the same spatial position – one belonging to the soil and 3D mesh (external node of the connectivity element) and another representing the screen interior (internal node). The colmated layer thickness is δ as shown in Fig. 1a. It is assumed that the flow within the colmated layer is radial, hence we have the relation,

$$q_r = -k_{col} \frac{d\phi}{dr} \tag{8}$$

where k_{col} is the conductivity of the colmated layer, and r is the radial coordinate. This relation can further be expressed as

$$q_r = k_{col}\frac{\phi^{ext} - \phi^{int}}{\delta} \tag{9}$$

Then, we can write the incremental balance equation for the connectivity element, for iteration i, as:

$$K_{col}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \Delta\phi^{ext(i)} \\ \Delta\phi^{int(i)} \end{bmatrix} = -K_{col}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \phi^{ext(i-1)} \\ \phi^{int(i-1)} \end{bmatrix} \tag{10}$$

where

$$K_{col} = \frac{k_{col}}{\delta}A_{con} \tag{11}$$

Here A_{con} is the surface that belongs to the connectivity element, corresponding to a screen node I,

$$A_{con} = 0.5(d_0 + \delta)\pi(L_{I-1} + L_{I+1}) \tag{12}$$

where L_{I-1} and L_{I+1} are lengths of 1D elements with common node I.

We note that the flux to the screen at node I is

$$Q_I = K_{col}(\phi^{ext} - \phi^{int}) \tag{13}$$

Of course, the total flux to the radial well is the sum of fluxes through all screens.

It is further straightforward to include flow through the screens into the entire model of the underground flow. The potentials at internal connectivity nodes are coupled to the potentials at 3D mesh. Then the potential at the RW bottom represents the boundary condition.

The finite element balance equations for 1D elements are of the form (4), with the matrices (7).

3. Application: Model calibration for a radial well of the Belgrade Groundwater Source

We here apply our multiscale mode to the calibration of one of the RWs used in the Belgrade Groundwater Source, Fig. 2. The goal of the model is to numerically determine conductivities of the colmated layers which fit the groundwater potential measured by piezometers and the RW flow rate. This procedure is called model calibration (Dimkic et al., 2013).

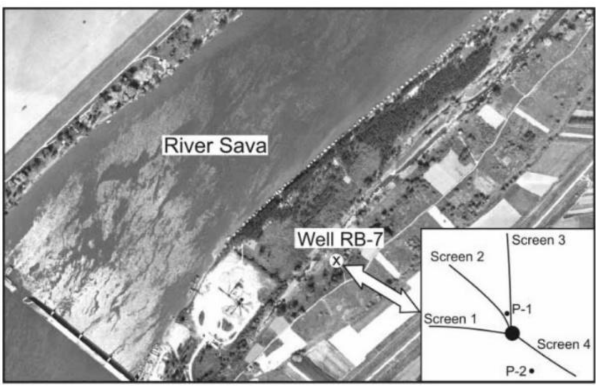

Fig. 2 One of the RWs within Belgrade Groundwater Source. Enlarged are shown horizontal screens and positions of piezometers P-1 and P-2. (according to Dimkic et al., 2013)

In Fig. 3 and Fig. 4 are shown the results of our model and the measurements, demonstrating the applicability of our finite element model and the software package PAK (Kojic et al., version in 2013).

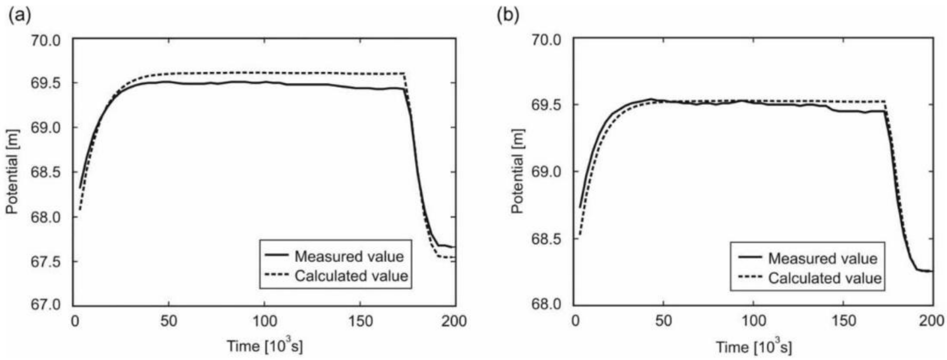

Fig. 3 Measured and calculated potentials at two piezometers shown in Fig. 2. a) Piezometer P-1; b) Piezometer P-2. (according to Dimkic et al., 2013).

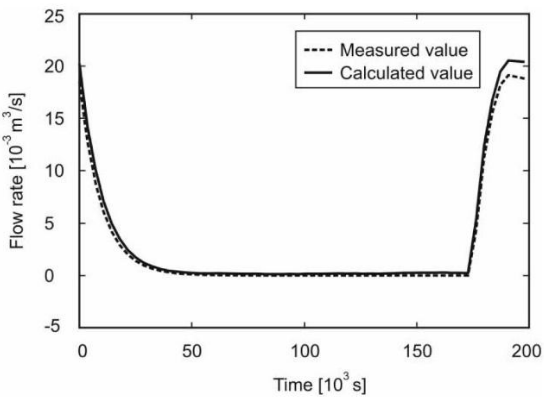

Fig. 4 Calculated and measured flow rate for the RW shown in Fig. 2. (according to Dimkic et al., 2013).

4. Summary and concluding remarks

We have presented in this short review our formulation of the dimensionally multiscale finite element model by directly coupling the underground water flow within two domains - different in size by orders of magnitude. Water flows through the soil with various conductivity layers and kilometer sizes and then is collected through RW screens of centimeters in diameter and meters in length. The FE model is composed of dimensionally large 3D finite elements for the soil and 1D elements for the screens, aligned to the 3D FE mesh. The 1D and 3D models are coupled by the connectivity (fictitious) 1D elements. A 2-node connectivity element contains one node (external) of the 3D mesh and another, at the same spatial position, representing the screen interior. The conductivity of a connectivity element is specified by the characteristic of the colmated layer, while the cross-section is determined by the external surface of the colmated layer associated with the external connectivity node. Our model is accurate, efficient and simple for generation. It can be used in engineering practice, as is shown in our example.

This paper is devoted to the memory of our great friend Professor Dr. Milan Dimkic, who was leading this exciting research as the director of the Institute “Jaroslav Cerni”.

Acknowledgements: This research was funded by the Institute “Jaroslav Cerni” during the period 2006-2013. We acknowledge the Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac (grant number F-134 of the SASA). The authors are thankful to the City of Kragujevac, Serbia.

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