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SELECTED EXAMPLES OF THE APPLICATION OF PAK SOFTWARE FOR MODELING THE BEHAVIOUR OF COMPLEX ENGINEERING STRUCTURES

Dejan Divac[1] [0000-0002-1578-1431], Slobodan Radovanović[1] [0000-0002-8437-9016], Maja Pavić[1] [0009-0008-6623-0331], Nikola Milivojević[1*] [0000-0001-5330-1219], Vladimir Milivojević[1] [0009-0005-9113-9896], Miroslav Živković[2] [0000-0002-0752-6289], Dragan Rakić[2] [0000-0001-5152-5788], Snežana Vulović[3] [0000-0001-5784-0906]

[1] Jaroslav Černi Water Institute, Belgrade, Serbia
e-mail: dejan.divac@jcerni.rs; slobodan.radovanovic@jcerni.rs; maja.pavic@jcerni.rs; nikola.milivojevic@jcerni.rs; vladimir.milivojevic@jcerni.rs
[2] University of Kragujevac, Faculty of Engineering, Kragujevac, Serbia
e-mail: zile@kg.ac.rs; drakic@kg.ac.rs
[3] Institute of Information Technologies, University of Kragujevac, Kragujevac, Serbia
e-mail: vsneza@kg.ac.rs
**corresponding author*

Abstract

The paper presents various examples of the application of the PAK software and its modules for thermal, filtration, and stress–strain analysis of different complex engineering structures such as gravity dams, arch dams, embankment dams, tunnels, and mines. Examples of research, studies, and projects in which PAK has been applied to assess the condition of various structures are provided. Special emphasis is placed on the application of PAK for modeling thermal, filtration, and stress–strain processes at the gravity dam, Iron Gate I Dam, and the arch dam, Grančarevo. PAK has been successfully used for a long time on these structures as part of the Dam Safety Management System, providing the basis for various analyses and reports on the assessment of dam condition and safety.

Keywords: dam, thermal processes, filtration processes, stress-strain processes, PAK

1. Introduction

Complex engineering structures are constructions composed of multiple interconnected components made of different materials, serving various functions and possessing distinct physical properties, whose design, analysis, and construction require the use of advanced engineering methods, tools, and software solutions. Due to their complexity, these structures typically involve an interdisciplinary approach and detailed modeling of mechanical, thermal, filtration, and other phenomena, including the interactions between different processes. Such structures include dams, as the most complex engineering structures, tunnels, bridges, various facilities, and similar constructions.

Dams are structures that have a significant impact on the economy and society. They create reservoirs that are used for electricity generation, water supply, irrigation, recreation, and other purposes. Dam failures and damages can have catastrophic consequences for society and the economy, primarily in terms of human casualties, and subsequently in other aspects. Around the world, there are numerous examples of dam failures and damage that occurred in the past, resulting in the loss of many lives, such as the failure of the Malpasset Dam in France in 1959 (Duffaut, 2013) or the damage to the Pacoima Dam in the United States (Ru & Jiang, 1995). Considering that most existing dams were constructed several decades ago, they have experienced complex operational conditions and, as such, have undergone various changes in their physical and mechanical properties, which affect their load-bearing capacity and safety. For this reason, it is almost always the responsibility of dam owners to monitor and assess the condition of the dam throughout its service life. If any issues are identified regarding the dam's functionality and safety, various remedial measures are undertaken to improve the safety and operational performance of the dam. In world practice, different countries apply different approaches to dam safety management (Radovanović, et al., 2022; Divac, et al., 2022; Jeon, et al., 2009; Wieland & Kirchen, 2012).

An indispensable part of dam safety management is the assessment of dam safety through the application of numerical models that account for the interaction between the dam structure, the surrounding rock mass, and water. These models are developed based on the actual geometry of the dam structure, while incorporating the structural and physico-mechanical characteristics of the rock mass, as well as various boundary conditions related to loads and temperature effects.

The resolution of real-world problems related to complex engineering structures typically relies on the use of sophisticated methods and appropriate software packages. In the early 1990s, the Jaroslav Černi Institute adopted PAK as a solution that could reliably address such problems, while simultaneously allowing further development of the software package to meet the evolving requirements posed by real engineering challenges. This paper presents representative examples of the application of PAK software (Živković, et al., 2019a; Živković, et al., 2019b; Živković, et al., 2019c; Živković, et al., 2019d) for modeling complex engineering structures such as gravity concrete dams, arch concrete dams, embankment dams, and tunnels. Special emphasis is placed on examples of PAK software applications related to the assessment of structural condition and safety evaluation, which are the most significant aspects of engineering practice concerning dam safety.

2. Application of PAK for Hydraulic Structures

In the early 1990s, at the Jaroslav Černi Institute, in cooperation with the Faculty of Mechanical Engineering, University of Kragujevac, the PAK software package was used for the first time to model subsidence processes and stress–strain changes caused by underground mining in the Bor mine (Jaroslav Černi Water Institute, 1996; Divac, et al., 1997). The task involved addressing the prevention of the negative impacts of mining activities on the town of Bor, located in the immediate vicinity of the mine. This required modeling the processes of rock mass deficit formation, subsidence, and the corresponding stress–strain changes in the surrounding rock mass due to the planned underground excavation, based on the actual physico-mechanical properties of the rock.

In the following period, PAK found application at the Jaroslav Černi Institute in dam design. Analyses were carried out for the design of the arch dam "Sv. Petka" on the Treska River in North Macedonia (Jaroslav Černi Water Institute, 2004), with a structural height of 64 m and a crest length of 115 m. Subsequently, it was applied in the design of the Prvonek Dam on the Banjska River (Jaroslav Černi Water Institute, 2012), an embankment dam with a structural height of 93 m and a crest length of 250 m. The Prvonek Dam was commissioned in 2005. PAK was also used in the design of the Bogovina gravity concrete dam (Jaroslav Černi Water Institute, 2006; Grujovic, et al., 2013), with a structural height of 54 m, where modeling of the interaction between the reservoir, the dam structure, and the rock mass was performed. As the most recent project, the Komarnica arch dam (Jaroslav Černi Water Institute & Hidroinženjering, 2023) on the Piva River can be mentioned, with a structural height of 176 m.

In addition to dams, PAK has also been used in the design of hydraulic tunnels, with notable examples including the auxiliary spillway at HPP Bočac (Jaroslav Černi Water Institute, 2014; Radovanović, et al., 2014) and the intake tunnel at HPP Dabar (Jaroslav Černi Water Institute & Stucky, 2014; Mirković, et al., 2022). The auxiliary high-water spillway was constructed at the existing Bočac HPP as an additional measure to ensure the required capacity of the dam's discharge facilities to safely pass a flood wave of a thousand-year return period. The Jaroslav Černi Institute conducted investigative and design works, as well as technical supervision during construction, in the period 2011–2012. Within these activities, numerous simulations were performed using the PAK-S module to obtain results for the assumed construction conditions and rock quality. Adjustments to technical solutions were carried out, both in terms of rationalization of structural components (arrangement of subgrade types along the tunnel) and in the execution of additional works to ensure the quality of the constructed elements.

Within the framework of the DyRes project, funded by the Science Fund of the Republic of Serbia, the stability of the Zavoj Dam was investigated (Rakić, et al., 2022). Zavoj Dam is an embankment dam located near Pirot in southeastern Serbia, with a height of 86 m and a crest length of 250 m. It was constructed at a site where, due to heavy rainfall, the bank of the Visočica River had previously collapsed, forming a natural dam. Therefore, assessing the stability of the riverbank and the embankment against external influences was of great importance. Based on models of the dam and the surrounding rock mass, the PAK-S module was used to examine the resistance of Zavoj Dam to various earthquake scenarios.

On the other hand, the need for a systematic approach to the assessment of the safety of dam structures and auxiliary facilities in the Republic of Serbia has been recognized. Consequently, the Jaroslav Černi Institute began, over a decade ago, the development of advanced software systems to support dam safety management. The concept of dam safety management includes a series of procedures whose goal is to create a physically based, software-supported system that will ensure the collection of all data that is important for dam safety, the analysis of that data, and their engineering interpretation through mathematical models of relevant processes, and control safety of the dam and making appropriate conclusions regarding the measures that need to be taken to achieve safety.

The concept of dam safety management is based on the following:

- Provision of data essential for dam safety. A data management subsystem is used which includes validation, acquisition, archiving, determination of technical data quality, and unified access.
- Use of physically-based FEM models for modeling associated processes.
- Dam safety monitoring using mathematical models for:
    - continuous monitoring of measurement results and determination of conformity of measured quantities and their expected values obtained by statistical models,
    - checking the dam's safety, i.e., determining the structure's condition and degree of protection using physically-based models.

These systems have previously been developed primarily for dams with hydroelectric purposes: Iron Gate I Dam (Divac, et al., 2022; Jaroslav Černi Water Institute, 2023; Jaroslav Černi Water Institute, 2017), Iron Gate II Dam (Jaroslav Černi Water Institute, 2020; Mirković, et al., 2021), Grančarevo (Radovanović, et al., 2022; Jaroslav Černi Water Institute, 2019; Rakić, et al., 2022), Vlasina (Jaroslav Černi Water Institute, 2017), and Lisina (Jaroslav Černi Water Institute, 2017).

The Serbian–Romanian hydroelectric and navigation systems Iron Gate I Dam and Iron Gate II Dam represent a unique and technologically inseparable unit, whose main function is to optimally utilize the hydroelectric potential of Europe's largest river, the Danube, through coordinated operation of the two systems. The system includes hydraulic structures of considerable scale, such as gravity concrete dams, hydroelectric power plants, and locks. The crest length of Iron Gate I Dam is 1,278 m, while Iron Gate II Dam has a crest length of 1,047 m.

The Grančarevo concrete arch dam is the first stage of the hydroelectric system on the Trebišnjica River, in the southeastern part of Bosnia and Herzegovina, with a structural height of 123 m and a crest length of approximately 440 m. It forms the Bileća Lake reservoir, with a volume of nearly 1.3 billion m^3.

The Vlasina and Lisina dams are key structures within the Vlasina hydroelectric system, located in southeastern Serbia. The system also includes channels and tunnels that convey water to the Vlasina Lake, as well as four hydroelectric power plants: HE Vrla 1, 2, 3, and 4. Both dams are embankment dams with crest lengths of approximately 240 m. The structural height of the Lisina Dam is 53 m, while that of the Vlasina Dam is 34 m.

For the aforementioned concrete dams, special modules for numerical modeling and simulation of thermal, filtration, and stress–strain processes have been developed within dedicated dam safety management systems. For the mentioned embankment dams, special modules have been created for numerical modeling and simulation of filtration and stress–strain processes. These processes are simulated using the numerical solvers PAK-PT (Živković, et al., 2019c; Živković, et al., 2019d) for thermal and filtration analysis, and PAK-S (Živković, et al., 2019a; Živković, et al., 2019b) for stress–strain analysis.

3. General Aspects of Modeling Relevant Processes in Dams

3.1 Application of Physically-Based Models

The prediction of dam behavior and the assessment of dam safety rely on the execution, analysis, and interpretation of computational simulations of relevant physical processes using appropriate physically based numerical models. For concrete dams, such models must be capable of simulating thermal, filtration, and stress–strain processes, while also accounting for their mutual interactions. For embankment dams, the physically based numerical models should simulate filtration and stress–strain processes, again taking into account their interactions.

Dams often consist of multiple segments, such as the spillway section, stilling basin, and embankment section. Consequently, particular attention must be paid when developing physically based models: they can be constructed either as integral representations of multiple dam segments or, in certain situations, as individual structures. In this sense, physically based models can become very complex depending on the processes being analyzed. They must also account for the rock mass foundation, with the complexity and size of the modeled domain adjusted to the processes under investigation. In general, modern numerical dam analysis represents a highly demanding engineering and computational task. The finite element method

(FEM) is currently regarded as the most reliable and widely used numerical technique for engineering analysis of complex structural systems in geotechnics.

It is essential that numerical models be representative of all processes simulated in the dam, so they can be considered trustworthy for safety control and usability assessment. To ensure representativeness, models must be developed on modern software platforms and grounded in real measurement data, replacing arbitrary assumptions with empirical facts wherever possible. Once developed, physically based numerical models must be continuously maintained to reflect changes in dam structure or behavior that may require model adjustments. For example, if remedial works such as grout curtain injections are performed, the zoning of the sealing system and permeability parameters must be updated. If rehabilitation works significantly alter the geometry or layout of the dam structures, the numerical models must also be corrected to match real conditions. The primary objective of forming and maintaining physically based numerical dam models in an up-to-date state is to enable their use for condition assessment and safety control.

Physically based numerical models must be calibrated for different types of processes (thermal, filtration, and stress–strain). This means that the optimal values of process parameters must be determined to achieve the best possible agreement between computed simulation results and observed in-situ measurements (temperature, piezometric levels, discharges, displacements, inclinations).

3.2 Modeling of Heat Transfer Processes

Thermal processes involve the analysis of the temperature field within the dam body, which is calculated based on external thermal boundary conditions (water temperature, air temperature) and thermal parameters. These processes are characterized by their time dependence; the effects of boundary conditions do not immediately manifest inside the dam body, but rather with a time delay. External thermal conditions may take several weeks or months to influence the internal thermal state of the dam. This time lag is more pronounced in gravity dams, due to their massive structure, than in arch dams, although it is significant in both cases. For this reason, dams must be analyzed under non-stationary thermal conditions to account for the duration of imposed boundary conditions. The air temperature boundary condition is defined based on technical monitoring data collected during dam operation. The water temperature boundary condition is defined from measurements (when available at certain points) and from the Bofang function describing water temperature variation with depth in the reservoir (Zhu, 1997).

In addition to boundary conditions, modeling thermal processes requires knowledge of parameters such as: the specific heat capacity of concrete (cp), thermal conductivity coefficient (k), heat transfer coefficient from air to concrete (ha), heat transfer coefficient from water to concrete (hwc), correction coefficient for measured radiation, temperature in the dam foundation (Tr), heat transfer coefficient from rock to concrete (hrc), and specific heat of the foundation rock mass (cr).

3.3 Modeling of Filtration Processes

Filtration processes involve analyzing filtration forces within the dam body and surrounding rock mass, calculated based on external filtration boundary conditions (groundwater levels, reservoir water levels) and filtration parameters. Filtration processes are analyzed under steady-state conditions, assuming that changes in boundary conditions do not produce significant temporal delays in filtration forces within the dam body and rock mass. Filtration boundary conditions are defined based on observations of reservoir water levels and piezometric levels in the ground. Filtration parameters are the permeability coefficients for different materials.

3.4 Modeling of Stress–Strain Processes

Stress–strain processes are the most complex dam processes, involving the analysis of stress and deformation states within the dam body and the surrounding rock mass. They are based on external boundary conditions, filtration forces obtained from filtration analysis, thermal deformations computed from thermal analysis, and relevant parameters describing the mechanical behavior of the rock mass and concrete.

The parameters of mechanical behavior for both rock mass and concrete depend on the selected constitutive material model. The simplest are elastic material models, followed by ideal elastoplastic models (Jaeger, et al., 2007; Hoek, et al., 2002), and finally, more advanced models that incorporate various types of stiffness and strength degradation (Lubliner, et al., 1989; Lee & Fenves, 1998; Lee & Fenves, 2001; Omidi & Lotfi, 2010).

4. Modeling and Estimation of the State of Relevant Processes at the Iron Gate I Dam

4.1 FEM Model of the Iron Gate I Dam

The HPP Iron Gate I Dam is a concrete gravity dam on the Danube River, forming a reservoir with a maximum volume of approximately 2.8 billion m³.

For the purpose of modeling thermal, filtration, and stress–strain processes at the Iron Gate I facilities, an integral three-dimensional model was developed using the finite element method (Jaroslav Černi Water Institute, 2017). The FEM model of the spillway dam includes the Serbian part of the spillway dam, the hydroelectric power plant, ship locks, the embankment dam, and part of the surrounding rock mass. The FEM model was constructed based on a previously developed three-dimensional geometric model of the dam, the engineering-geological characteristics of the terrain, and operational conditions on site. The finite element mesh of the integral model was created using ten-node tetrahedral elements, and the model comprises 4.37 million nodes and 2.97 million elements. Model boundaries extend 200 m upstream and 200 m downstream from the dam axis. The model width (perpendicular to the river flow) is 770 m, measured from the plane of the expansion joint between spillway dam blocks 14 and 15 towards the right bank.

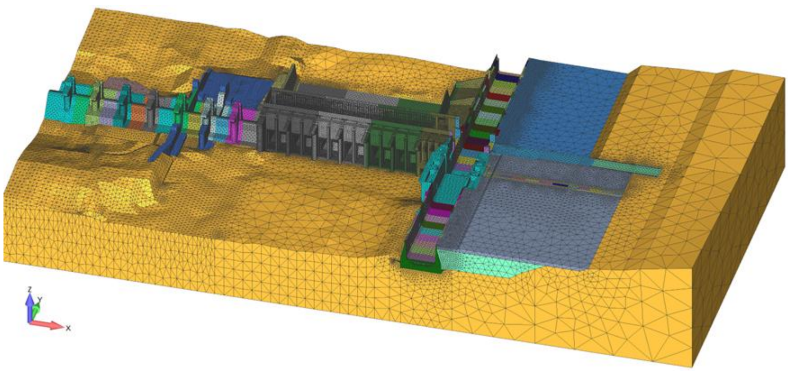

Fig. 1. Integral FEM model of the Serbian part of the Iron Gate I dam (spillway dam, power plant, ship lock, and embankment dam), view from the upstream side

In the integral model, the following groups of elements are distinguished:

– 3D elements: surrounding rock mass; concrete structure of the spillway dam (dam body and spillway, divided into 14 blocks); power plant (3 sections, assembled block, stilling basin, spillway of the low-level outlet); two-stage ship locks (25 upstream and downstream lock chambers); other concrete structures (partition wall between the power plant and spillway dam, structure on the embankment dam); grout curtain; drainage mats; drainage wells (36 units); clay core of the embankment dam; and the embankment.
– 2D elements: expansion joints; seals in the expansion joint zone.
– 1D elements: drainage boreholes and anchors in the spillway and power plant stilling basin zones.

4.2 Thermal Processes

Thermal processes at the Iron Gate I Dam were analyzed using transient boundary temperature conditions and the PAK-PT thermal solver (Živković, et al., 2019c; Živković, et al., 2019d). For temporally prescribed boundary conditions of measured external air temperature and the water temperature distribution with depth, obtained using the Bofang model (Zhu, 1997), calibration of the FEM model of thermal processes was performed. This calibration involved determining the values of thermal parameters that provide the best match between computed temperatures within the dam body and measured temperatures in the dam.

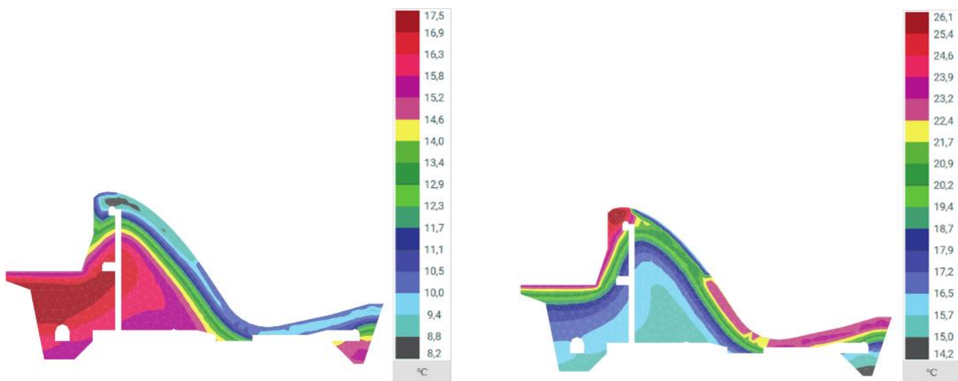

Fig. 2. Estimated thermal field of pier 7 of the spillway dam on March 14, 2025 (left), and September 13, 2024 (right)

4.3 Filtration Processes

Filtration processes at the Iron Gate I Dam were analyzed using steady-state boundary conditions based on potentials at the model boundaries and prescribed free surfaces in the unsaturated parts of the structure, employing the PAK-PT solver (Živković, et al., 2019c; Živković, et al., 2019d) on the integral model of the spillway dam, power plant, ship locks, and embankment dam. Calibration of the FEM model for filtration processes was performed, which involved determining optimal values of filtration parameters (permeability coefficients for different zones within the FEM model) to achieve the best match between computed and measured piezometric levels and flows at observation points. The filtration model can simulate various failure scenarios of the drainage and sealing system, as well as the grout curtain. Fig. 3 illustrates the functionality status of the drainage and sealing system and the grout curtain in the Serbian part of the Iron Gate I Dam. Fig. 4 shows the pore pressure field through the dam body and surrounding rock mass in the spillway section of the dam and the power plant structure.

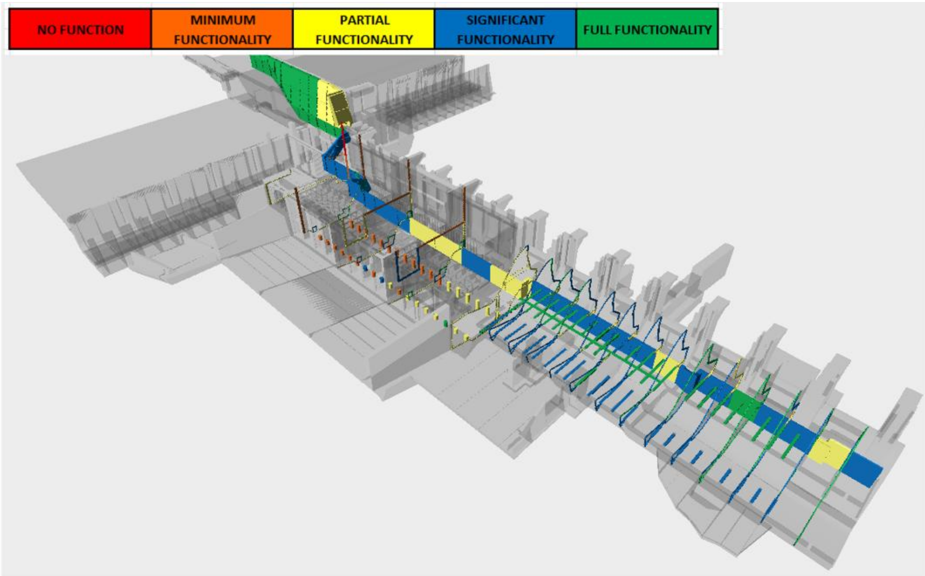

Fig. 3. An example of the results of the analysis of the condition of the seals on the joints, the drainage system, and the grout curtain on the Serbian part of the facility

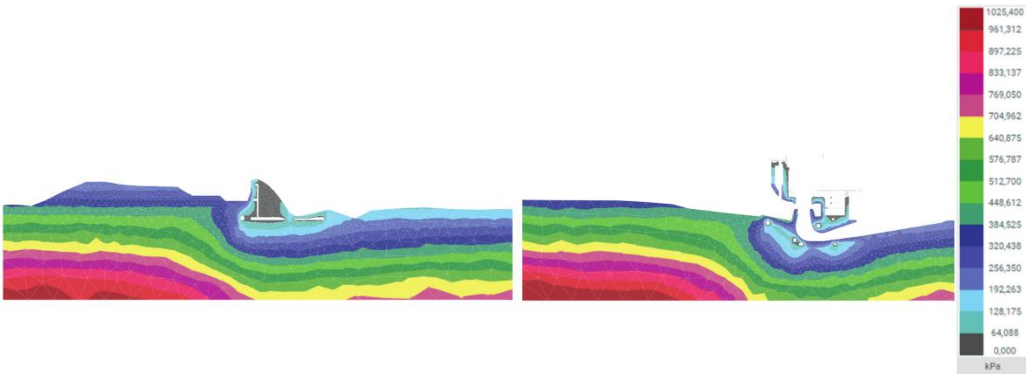

Fig. 4. Uplift pressure (kPa) on pier 7 of the spillway dam (left) and section I (right) of the power plant (in the axis of aggregate A3) with the estimated state of filtration, for WLR=69.30 masl and BWL=41.06 masl

4.4 Stress–Strain Processes

Stress–strain processes are used to determine the stress and deformation fields in dams and to assess dam safety. These processes are coupled with thermal and filtration processes. Simulation is performed using the PAK-S solver (Živković, et al., 2019a; Živković, et al., 2019b). For modeling the mechanical behavior of the rock mass and concrete, a material damage-plasticity model is applied (Lubliner, et al., 1989; Lee & Fenves, 1998; Lee & Fenves, 2001; Omidi & Lotfi, 2010), which is implemented in PAK-S. Dam safety assessment is carried out using the shear strength reduction method (Rakić, et al., 2023). Fig. 5 shows the displacement field in the dam body for the spillway section of the dam and the power plant structure.

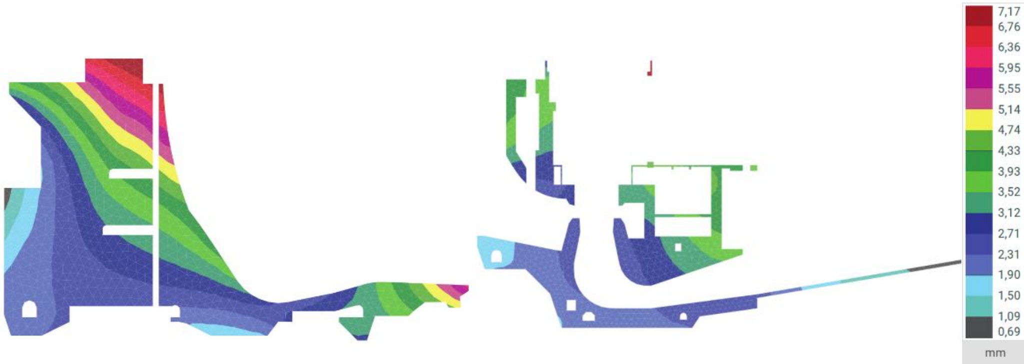

Fig. 5. Field of displacement in the direction of the Danube (mm) on pier 10 of the spillway dam and section III of the power plant

5. Modeling of Relevant Processes at the Grančarevo Dam

5.1 FEM Model of the Grančarevo Dam

For the simulation of relevant processes at the Grančarevo Dam, a three-dimensional FEM model of the dam and the surrounding rock mass was developed. The FEM model consists of 1,021,695 nodes and 718,920 elements (Radovanović, et al., 2022; Jaroslav Černi Water Institute, 2019; Rakić, et al., 2022). The FEM model (Fig. 6) is divided into an appropriate number of quasi-homogeneous zones: the concrete body of the dam is divided into a total of 91 quasi-homogeneous zones; the grout curtain is divided into 36 quasi-homogeneous zones; drainage boreholes are modeled as 1D finite elements, with each borehole modeled individually (a total of 41 drainage boreholes). The surrounding terrain is divided into three quasi-homogeneous rock mass zones.

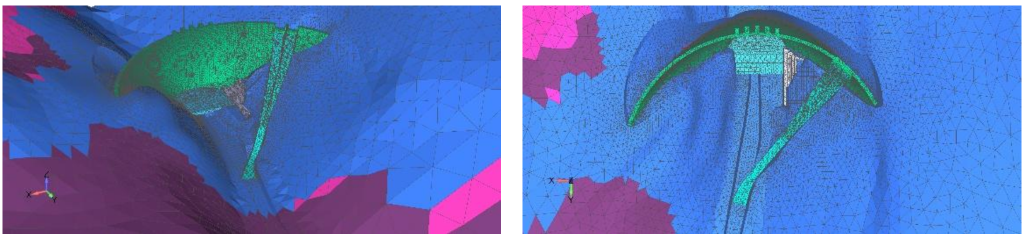

Fig. 6. FEM model of the dam and the surrounding rock mass

5.2 Thermal Processes

The thermal field in the dam was analyzed under steady-state boundary conditions using the PAK-PT solver (Živković, et al., 2019c; Živković, et al., 2019d). The model was previously calibrated, based on which the optimal values of thermal parameters were determined. Fig. 7 and 8 show comparisons between computed and measured temperature values at two observation points on the dam where concrete temperature monitoring is conducted.

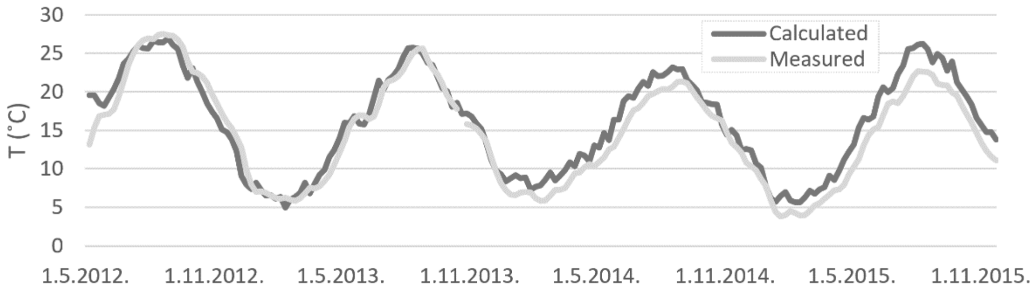

Fig. 7. Comparison of measured and computed values of concrete temperature on the embedded temperature sensor in concrete Tb-175 at the level of 401 masl (at the center of the dam body)

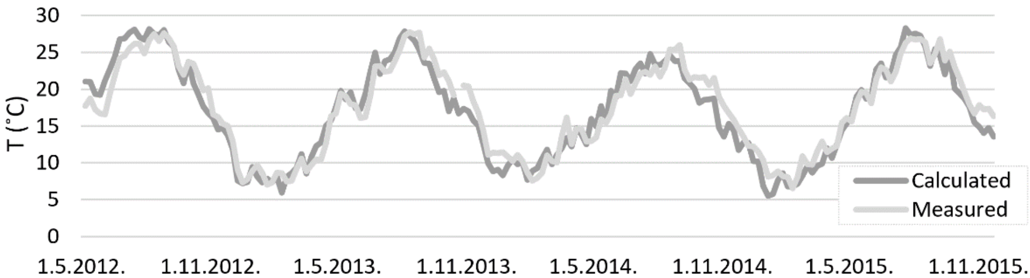

Fig. 8. Comparison of measured and computed values of concrete temperature on the embedded extensometer ER-22N at the level of 330 masl (downstream face of the dam)

For selected dates corresponding to the end of winter (winter thermal conditions), the thermal fields on the dam's downstream face and through the central cantilever are shown in Fig. 9.

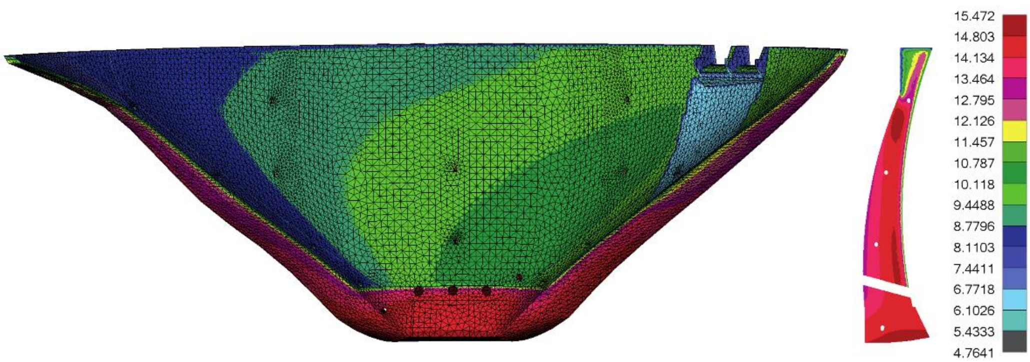

Fig. 9. Thermal field on the downstream face of the dam and in the section through the central cantilever on March 5, 2018

5.3 Filtration Processes

Based on filtration processes, the water level fields (Fig. 10) and filtration forces in the dam body and surrounding rock mass are obtained. Filtration processes are simulated using the PAK-PT solver (Živković, et al., 2019c; Živković, et al., 2019d). The filtration model was previously calibrated, and the values of filtration parameters were determined to achieve the best match between computed and measured piezometric levels and flows.

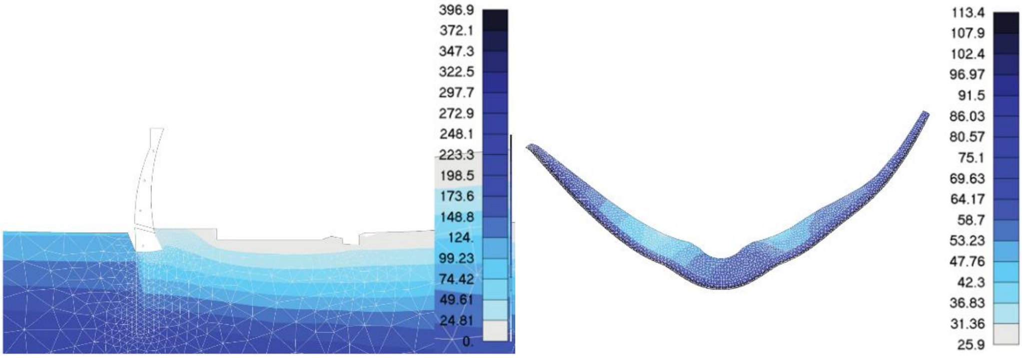

Fig.10. Water level height (m) - cross-section through the center cantilever (left) and the concrete-rock interface (right), for WLR=395.64 masl

5.4 Stress–Strain Processes

Based on stress–strain analyses, displacement, stress, strain, and damage fields in the dam body and surrounding rock mass are obtained. For modeling the mechanical behavior of the rock mass and concrete, a damage-plasticity model was applied (Rakić, et al., 2023). Fig. 11 shows a comparison between computed and measured tilt variations at six observation points on the dam. Fig. 12 illustrates the total displacement field in the dam body under summer boundary conditions.

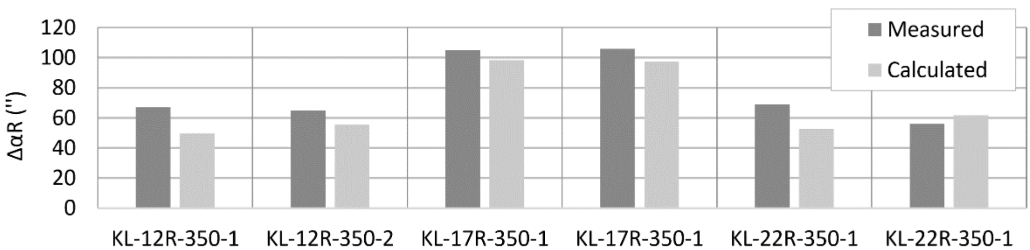

Fig.11. Change of inclination in the radial direction at elevation 350 masl (example of a comparison between measurement results and computation results)

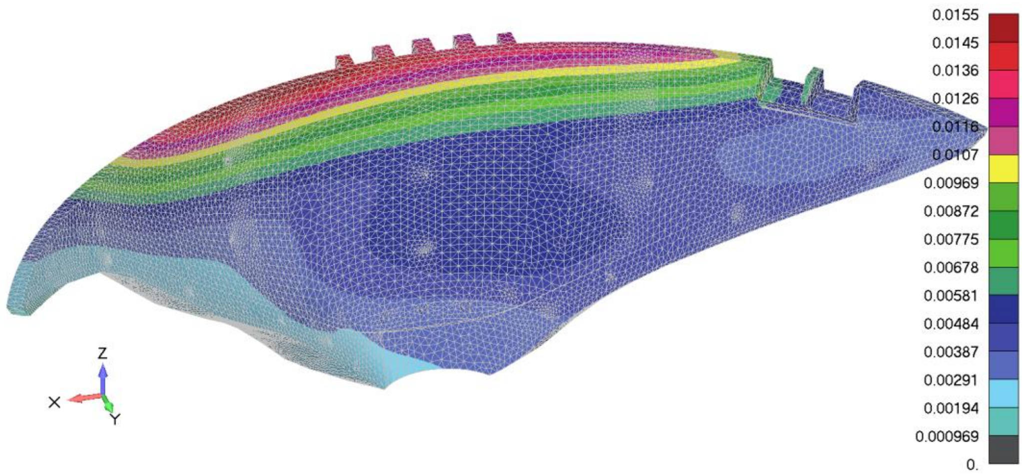

Fig.11. Field of total displacement on September 18, 2017, view from the downstream side

6. Conclusions

This paper presents various examples of the application of the PAK software for modeling and analyzing the state and processes in complex engineering structures, with detailed examples of its use on the Iron Gate I and Grančarevo dams. PAK has been successfully applied at these facilities for many years in the preparation of various studies, condition analyses, and dam safety assessments during operation. The advantage of using this software lies in its flexibility for modeling the most complex processes in dams and other engineering structures, through capabilities for extensions, external access to various applications, and control over the execution of calculations—features that standard commercial solvers rarely offer. Its adaptability to the requirements of the end user, realized through strong interaction between experts and researchers from different disciplines, represents an undeniable advantage compared to other similar commercial software used for modeling relevant processes in dams and other engineering structures.

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