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OVERVIEW OF EXPLICIT AND IMPLICIT DYNAMIC FEM ANALYSIS IN PAK SOFTWARE

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Abstract

Dynamic analysis of engineering structures is essential in the design process, providing insights into their behavior under various loading conditions. This paper focuses on the comparative study of implicit and explicit dynamic analysis methods using the PAK software and FEMAP with the Nastran software. Both methods have distinct advantages and disadvantages, which are examined in detail through the simulations of simple examples. The influence of the solution methods on the results is critically analyzed, offering valuable insights into selecting the appropriate dynamic analysis technique. The results from PAK software are validated through comparison with FEMAP with Nastran software, highlighting the accuracy of both software in dynamic analysis.

Keywords: PAK Software, Dynamic Analysis, Explicit Method, Implicit Method

1. Introduction

Dynamic Finite Element Method (FEM) analysis of engineering structures behavior is an indispensable aspect of modern engineering design, crucial for assessing the safety and functionality. In civil and mechanical engineering, dynamic analysis is essential as structures are often subjected to dynamic forces such as earthquakes, winds, explosions, and traffic loads, which can have substantial implications on their integrity and durability. Appropriate and accurate dynamic analysis allows prediction of response to such impacts, ensuring stability and safety, but also optimizing material use and design costs (Harris and Sabnis, 1999).

In engineering design, the dynamic simulations can be realized by implementation of the implicit and the explicit methods into the FEM code. These methods enable accurate and efficient dynamic simulation, allowing a better understanding of potential issues and improvements. The appropriate choice of the analysis method can significantly influence the accuracy and efficiency (Chopra, 2012; Clough and Penzien, 1993). The selection between the implicit and the explicit methods depends heavily on specific factors like the computational resources, the complexity of the structure, and the nature of the dynamic loading. The implicit methods are more stable and less sensitive to the time step size but can be computationally more expensive for large problems (Newmark, 1982). The explicit methods are more suitable for non-linear problems or impulse loading, such as impacts or explosions (Wilson, 2003).

This paper focuses on the application of the FEM software PAK (Kojić et al., 1999) for structural dynamic analysis, exploring and comparing the advantages and limitations of both methods. A comprehensive analysis of both the implicit and the explicit dynamic analysis methods using the PAK software is conducted. The study aims to systematically compare these two approaches and to identify their optimal application scenarios based on computational efficiency, accuracy, and the capacity to handle various types of dynamic loads. The functionality and limitations of each method are demonstrated through detailed simulations. Furthermore, all results obtained from the PAK software are compared with those from the FEMAP with Nastran software (Femap v2021.2) to benchmark and validate the findings in this paper. This study will also discuss the influence of factors such as time step size, integration schemes, and the nature of dynamic loading.

2. Dynamic analysis theory in FEM

In the field of structural analysis, the choice of appropriate method can greatly impact the accuracy, efficiency, and applicability of the simulation results (Zienkiewicz and Taylor, 2005; Cook et al., 2002). In this section the theoretical background of both the explicit and the implicit methods are explained as it is implemented in the FEM software PAK. Based on the equilibrium of internal and external virtual work in a deformable body $\delta A_{\text{int}} = \delta A_{\text{ext}}$, it is possible to derive the equation of motion as (Bathe, 2006):

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{B}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}(t) \ , \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{B}$ is the damping matrix, and $\mathbf{K}$ is the stiffness matrix. The vector $\mathbf{U}$ is the vector of nodal displacements, while $\dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$ are the derivatives that represent the nodal velocity and the acceleration vectors. The equation of motion (1) can be solved by the implicit or the explicit methods.

2.1 Theoretical Basis of the Explicit Dynamics

The Central Difference Method (CDM) is widely used in the dynamics analysis. It approximates the acceleration of a system at a given time step by using finite difference approximations for the velocities and the displacements. The explicit dynamic analysis is highly effective for simulations involving high strain rates, severe impact loads, and large deformations. This method employs the CDM integration scheme within the FEM to solve the discretized equations of motion. The vector of nodal accelerations can be defined as follows (Bathe, 2006):

$$\ddot{\mathbf{U}}_{t+\Delta t} = \frac{\mathbf{U}_{t+\Delta t} - 2\mathbf{U}_t + \mathbf{U}_{t-\Delta t}}{\Delta t^2} \ , \tag{2}$$

while the vector of nodal velocities can be expressed as (Bathe, 2006):

$$\dot{\mathbf{U}}_{t+\Delta t} = \frac{\mathbf{U}_{t+\Delta t} - \mathbf{U}_{t-\Delta t}}{2\Delta t} \ , \tag{3}$$

where Δt represents the time step increment. The explicit method tends to bypass the global system stiffness matrix inversion, which speeds up calculations significantly, which is particularly useful in scenarios where the computational cost is a concern (Bathe, 2006;

Belytschko et al., 2000). The equations (2) and (3) are replaced in the equation of motion (1), which gives the solution for $\mathbf{U}_{t+\Delta t}$:

$$\left(\frac{1}{\Delta t^2}\mathbf{M}+\frac{1}{2\Delta t}\mathbf{B}\right)\mathbf{U}_{t+\Delta t}=\mathbf{R}_t-\left(\mathbf{K}-\frac{1}{2\Delta t^2}\mathbf{M}\right)\mathbf{U}_t-\left(\frac{1}{\Delta t^2}\mathbf{M}+\frac{1}{2\Delta t}\mathbf{B}\right)\mathbf{U}_{t-\Delta t}\ . \tag{4}$$

This approach can effectively handle the resulting non-linearities with relatively lower computational effort compared to the implicit methods. However, its stability is conditional on maintaining a sufficiently small time step, which is directly tied to the system's highest natural frequency (Belytschko et al., 2000). The explicit methods require very small time steps, typically on the order of microseconds, to maintain numerical stability. This requirement is dictated by the Courant-Friedrichs-Lewy (CFL) condition (Courant et al., 1927), which is generally expressed as:

$$\Delta t \le \frac{C\cdot\Delta x}{|u|}\ , \tag{5}$$

where Δx is the spatial element size (the distance between adjacent element nodes), u is the maximum speed at which any signal (or wave) can travel through the medium, often referred to as the wave propagation speed in the context of the simulation, and C is the Courant number, which typically must be less than or equal to 1 for stability (the optimal value of C depends on the specifics of the differential equation and the numerical scheme used).

2.2 Theoretical Basis of Implicit Dynamics

The most popular implicit dynamic method is the Newmark algorithm, which integrates the motion equations stably and accurately. It is particularly useful for systems where the load is applied slowly, allowing for the larger time steps without losing accuracy. The Newmark method can adaptively balance between higher accuracy and numerical damping, making it suitable for a wide range of dynamic problems (Chopra, 2012). The general form of the Newmark integration method for implicit dynamics can be expressed as (Bathe, 2006):

$$\mathbf{U}_{t+\Delta t}=\mathbf{U}_t+\dot{\mathbf{U}}_t\Delta t+\frac{\Delta t^2}{2}\left((1-\gamma)\ddot{\mathbf{U}}_t+\gamma\ddot{\mathbf{U}}_{t+\Delta t}\right)\ , \tag{6}$$

and for the velocity:

$$\dot{\mathbf{U}}_{t+\Delta t}=\dot{\mathbf{U}}_t+\Delta t\left((1-\gamma)\ddot{\mathbf{U}}_t+\gamma\ddot{\mathbf{U}}_{t+\Delta t}\right)\ , \tag{7}$$

where $0\le\gamma\le1$ is another parameter influencing numerical damping. After the appropriate transformation of equations (6) and (7) and introducing them into equation (1), the system of algebraic equations is derived in the following form:

$$\hat{\mathbf{K}}\mathbf{U}_{t+\Delta t}=\hat{\mathbf{R}}_{t+\Delta t} \tag{8}$$

where the matrices $\hat{\mathbf{K}}$ and $\hat{\mathbf{R}}$ are:

$$\begin{aligned}&\hat{\mathbf{K}}=\mathbf{K}+a_0\mathbf{M}+a_1\mathbf{B}\\&\hat{\mathbf{R}}_{t+\Delta t}=\mathbf{R}_{t+\Delta t}+\mathbf{M}\left(a_0\mathbf{U}_t+a_2\dot{\mathbf{U}}_t+a_3\ddot{\mathbf{U}}_t\right)+\mathbf{B}\left(a_1\mathbf{U}_t+a_4\dot{\mathbf{U}}_t+a_5\ddot{\mathbf{U}}_t\right)\end{aligned}\ . \tag{9}$$

In equations (9), $a_{0,}a_1,...,a_5$ are the coefficients (Bathe, 2006). This method is well-suited for analyzing the quasi-static problems or the dynamics involving small deformations over longer time periods (Hughes, 1987; Reddy, 2004). The implicit methods are especially beneficial for analyses where the load is applied slowly relative to the structure's natural frequencies or in cases requiring high accuracy over long periods. These methods are inherently more stable and can handle larger time steps without sacrificing accuracy.

Both methods, the explicit and the implicit, have their distinct applications, and choosing the correct approach depends critically on the specific requirements of the problem. The subsequent sections will describe how these theories are applied using the PAK software, comparing it with the results from the FEMAP with Nastran software to validate findings and to understand the practical implications of selecting one method over the other.

3. FE examples of dynamic structural analysis

3.1 Steel plate cantilever beam loaded by pressure and concentrated forces

The first FE example (Djordjević, 2004) utilizes 3D hexahedral eight-node finite elements (FE). The cantilever dimensions are 130 mm x 48 mm x 2.5 mm, and the finite elements dimensions are 6.5 mm x 6 mm x 1.25 mm. The FE example consists of 320 elements and 567 nodes. The plate is modelled with the steel material characteristics (2.07 x 105 MPa as the Young Modulus, 7.801 x 10-9 t/mm3 as the density, and 0.3 as the Poisson ratio). The pressure is applied to the bottom side of the plate in the positive direction of the vertical axis, with 1.105 Pa applied per element, resulting in a total pressure of 6895 Pa. A force is applied on the free edge of the plate, also in the positive direction of the vertical axis, with a force value of 24.722 N on each node, resulting in a total force of 222.5 N. Both the pressure and the force are modulated by a function over time, allowing for the dynamic simulation of varying load intensities. The pressure and the force functions over time are shown in Fig.1.

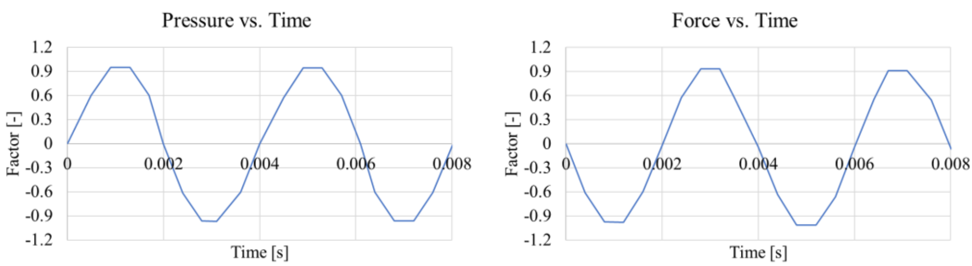

Fig. 1. Pressure vs. Time function – left, Force vs. Time function – right.

The boundary conditions are set on the side opposite to the concentrated forces, with nodes being clamped. The FE model of the first example is shown in Fig. 2.

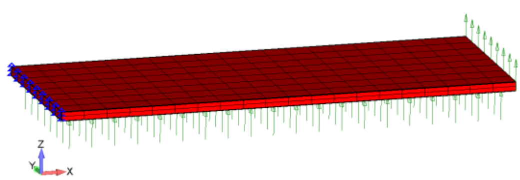

Fig. 2. Cantilever plate with loading and boundary conditions.

3.2 A 2D cantilever beam loaded by concentrated force

The second FE example (Đorđević, 2004) utilizes the 2D isoparametric four-node elements. The cantilever beam dimensions are 254 mm x 25.4 mm x 2.54 mm, and the finite element dimensions are 25.4 mm x 25.4 mm x 2.54 mm. The FE example comprises 10 elements and 22 nodes. The cantilever is modelled with 2.07 x 105 MPa as the Young Modulus, 3.20616 x 10-6 t/mm3 as the density, and 0.3 as the Poisson ratio. A concentrated force is applied on the upper node of the free edge of the cantilever beam, in the positive direction of the vertical axis, with the force value of 1 N. The force is modulated by a function over time, allowing for the dynamic simulation of varying load intensities. The force function over time is shown in Fig. 3.

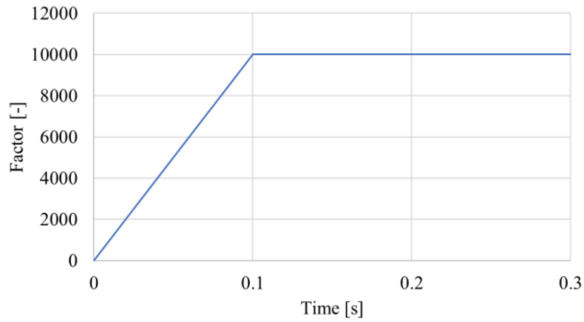

Fig. 3. Force vs. Time function for 2D cantilever.

The boundary conditions are set on the side opposite to the force, with the nodes being clamped. The second FE example is shown in Fig. 4.

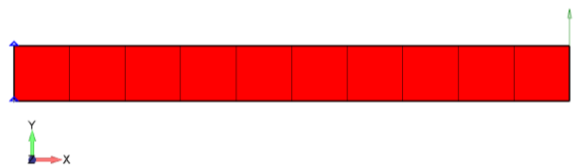

Fig. 4. Cantilever 2D example with the loading and the boundary conditions.

4. Results and Discussion

The primary objective of this paper is to analyze the vertical displacement of the node over time for both the FE examples and the stress field distribution for the first FE example. The study employs two types of finite elements: the 3D hexahedral eight-node elements for the first FE example and the 2D isoparametric four-node elements for the second FE example. Additionally, the results obtained by using both the implicit and the explicit methods are compared. The analysis also includes the variations in vertical displacement results using the full and the diagonal mass matrices. All these results are then compared with findings obtained using the FEMAP with Nastran software to validate the models and the results.

The linear dynamic analysis was conducted for both FE examples. The total time for the first FE example was 0.008s over 100 time-steps for the implicit solution for both full and diagonal mass matrices, 0.008 s over 80000 time-steps for the explicit solution with full mass matrix, and 0.008 s over 40000 time-steps for the explicit solution with diagonal mass matrix. For the second FE example, the total time was 0.3 s over 300 time-steps for the implicit solution for both full and diagonal mass matrices and 0.3 s over 30000 time-steps for the explicit solution with full and diagonal mass matrices. Different numbers of time steps between the implicit and the explicit methods across two FE examples were highlighted. The implicit methods, known for their stability, allowed for larger time steps (100 for the first FE example and 300 for the second FE example), efficiently covering the simulation duration with fewer computational demands per step despite the overall complexity. In contrast, the explicit methods required significantly more steps (80000 time-steps with full mass matrix and 40000 with diagonal mass matrix for the first FE example, and 30000 time-steps with both, full and diagonal mass matrix for the second FE example) to maintain numerical stability due to the Courant-Friedrichs-Lewy condition (explained in Section 2.1), which mandates smaller time steps to accurately capture high-frequency dynamic responses. This distinction underscores the computational efficiency of implicit methods for longer durations and the precision of explicit methods in capturing rapid dynamics, illustrating the trade-offs between the computational speed and the accuracy in dynamic simulations.

The results for the vertical displacement from the first FE example presented in Fig. 5 illustrate the simulation results of the PAK software using both the implicit and explicit methods for solving differential equations, applied with both full and diagonal mass matrices. These results are labeled as PAK IS – FMM (Implicit Solution with Full Mass Matrix), PAK IS – DMM (Implicit Solution with Diagonal Mass Matrix), PAK ES – FMM (Explicit Solution with Full Mass Matrix), and PAK ES - DMM (Explicit Solution with Diagonal Mass Matrix), respectively. These results are compared with the results obtained using the implicit solution approach in FEMAP with Nastran software, denoted as NASTRAN IS - FMM (Implicit Solution with Full Mass Matrix) and NASTRAN IS - DMM (Implicit Solution with Diagonal Mass Matrix). The results for the first FE example are shown through 100 steps.

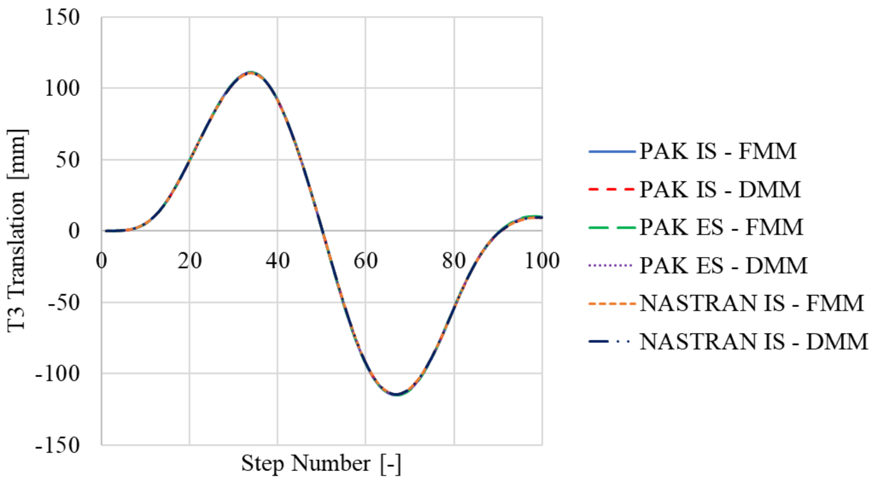

Fig. 5. The first FE example – Comparative analysis of results obtained from PAK software and FEMAP with Nastran software.

Both PAK software and the FEMAP with Nastran software display consistent trends in node displacement, which suggests that despite the inherent differences in computational techniques and settings, there is a reliable cross-validation of dynamic behavior across software platforms.

Fig. 6 displays the stress results obtained from the first FE example, presenting the outcomes at the same simulation time points for both PAK and FEMAP with Nastran software using different methods. Fig. 6 presents (a) the result from linear explicit dynamics in PAK software, (b) the result from linear implicit dynamics in PAK software, and (c) the result from linear implicit dynamics in FEMAP with Nastran software.

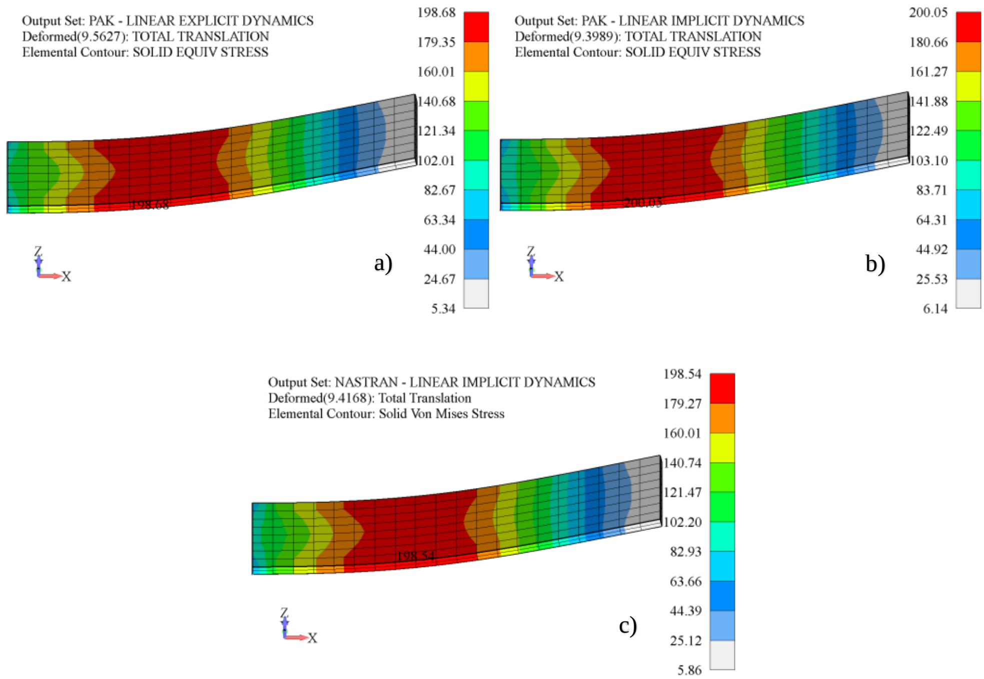

Fig. 6. Comparative stress field distribution results obtained from PAK and FEMAP with Nastran software.

Both the PAK and FEMAP with Nastran software show a high degree of consistency in the stress filed distribution along the length of the cantilever plate. The peak stress values are very close across all simulations, with the PAK software results from the implicit and the explicit simulations obtaining slightly higher peak stresses (200.05 MPa and 198.68 MPa, respectively) compared to FEMAP with Nastran software implicit result (198.54 MPa). The minor differences can be attributed to the different numerical methods and software used.

The comparative stress analysis between the PAK and the FEMAP with Nastran software validates the use of PAK software for numerical calculations in dynamic analysis, confirming its reliability and effectiveness in this field.

Fig. 7 presents a comparative analysis of the vertical displacement results from linear dynamic analysis performed using both software packages for the second FE example. The labeling of the second FE example remains consistent with that used for the previous FE example (Fig. 5). Results for the second FE example are shown through 300 steps.

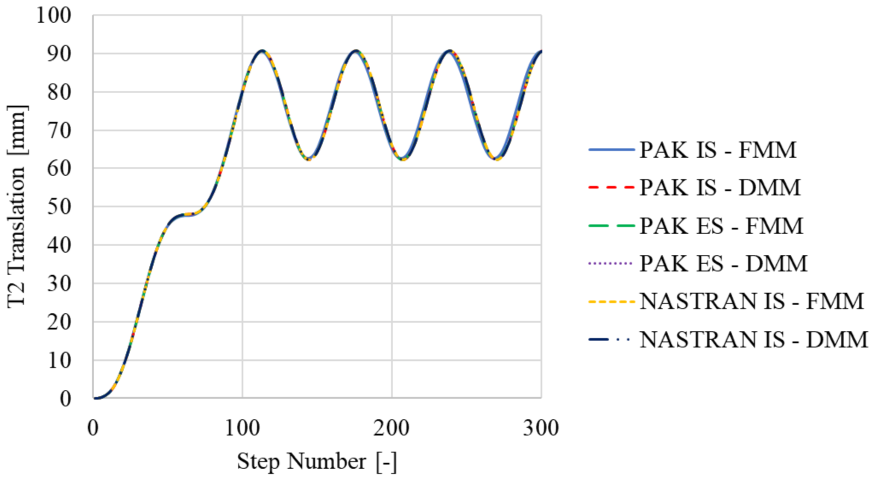

Fig. 7. The second FE example – Comparative analysis of results obtained from PAK software and FEMAP with Nastran software.

All diagrams follow a similar overall trajectory, indicating a good result matching in the dynamic behavior captured by the different configurations. This consistency is crucial for validating the computational models and their applicability in the dynamic analysis. The results demonstrate that the PAK software is capable of accurately simulating the dynamic behavior of the FE example, with minor differences influenced by the choice of the solver method and the mass matrix formulation.

The results from the linear dynamic analyses conducted using both the PAK software and the FEMAP with Nastran software across two illustrative examples have demonstrated their capability to accurately simulate dynamic behaviors. Results are presented in Fig. 5 and 7, where node displacement was monitored over time under various configurations, including both implicit and explicit methods with the full and the diagonal mass matrices, and in Fig. 6 where stress field distribution results from the first FE example across both software are compared. Despite the variations in solver methods and mass matrix formulations, both software platforms showed a high degree of consistency in the trends of node displacement and stress field distribution. The close similarity between the results from the PAK software and those from the FEMAP with Nastran software effectively verifies the PAK software used in resolving different dynamic problems. This confirmation of obtained results from the PAK software through direct comparison with the FEMAP with Nastran software results confirmed the PAK software as a reliable tool for the dynamic analysis of various engineering problems.

4. Conclusions

This study has successfully validated dynamic analysis implementation in the PAK software through a comprehensive comparison to the FEMAP with Nastran software. By monitoring node displacement over time and stress field distribution across two benchmark FE examples, this study demonstrated that the PAK software can accurately simulate dynamic behaviors. This cross-validation confirms the robustness and reliability of PAK software for dynamic simulations. The alignment of results from different software platforms, involving various configurations and solver methods, further supports the precision and adaptability of the PAK software in diverse simulation environments.

The use of implicit solutions demonstrated superior stability and required fewer time steps to achieve accurate results, making them preferable for scenarios where computational efficiency and larger time steps are necessary. In contrast, explicit solutions, while requiring a finer temporal resolution due to their dependency on the CFL condition, have excelled in capturing the high-frequency dynamic responses that are critical in scenarios involving sudden loads or impacts.

The comparison between full and diagonal mass matrices has highlighted how these choices affect the accuracy and computational demands of the simulations. The full mass matrices, while computationally more intensive, provided a more accurate representation of dynamic behavior, particularly in complex simulations where mass distribution plays a critical role. In contrast, diagonal mass matrices, though less accurate in certain dynamic conditions, offered computational speed advantages and sufficed in simpler scenarios where mass representation was less critical.

Each method and matrix configuration has its advantages and disadvantages, which must be carefully considered based on the specific requirements of the simulation task. The choice between the implicit and the explicit methods, as well as between full and diagonal mass matrices, should, therefore, be guided by the specific dynamics of the problem, the desired accuracy, and the computational resources available.

For future research, the development of PAK software should focus on expanding algorithms for nonlinear dynamics. By implementing the algorithms for non-linear dynamic problems, PAK software would be able to address a broader spectrum of dynamic problems, making it even more versatile and powerful in practical applications. Additionally, integrating the capability to utilize various types of finite elements would offer greater flexibility and precision in modeling.

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