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**Special Issue Devoted to 50 Years of
the Finite Element Program PAK**

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**I PAK – FINITE ELEMENT PROGRAM FOR ENGINEERING AND
BIOENGINEERING**

PREFACE

It is already 50 years from the decision that we are going to try to develop our own finite element program. Nobody could imagine, even in their best dreams, that we could reach the stage where we are today. This special issue of our journal and the written text as a monograph serve to witness our journey, how, from scratch and technologically a “stone age” (using punched cards, one computer in town with 1Mb memory), it is now possible to have the PAK software as a huge system. We document this development and application in various technical and scientific fields by a large number of participants and institutions.

In order to keep the form of the journal, but also provide the basic information about our package, we have divided the entire content into two parts: I PAK – Finite Element Program for Engineering and Bioengineering, and II Selected Review Papers. The first part includes the main information of the PAK software: Authors, History of the PAK Development, PAK Versions-Moduli, PAK Elements, Material Models, and Bibliography. A large number of researchers and engineers participated in the PAK development and applications, and we list them in the Section Authors. The next two sections provide the main characteristics of the PAK system. The overall impact of the PAK project is documented in the last section, where our books and papers related to the PAK are listed. We mention here that the history section gives the entire journey from 1975 to 2025, with notifications of the main, decisive moments, institutions, and persons who shaped the entire journey along the road of success. The PAK software represents a large number of versions, which are specific to each of the fields since the PAK has been developed primarily for research, education, and application, rather than a commercial package. We believe that this part of the Special Issue of the JSSCM gives a global picture of what the PAK is.

The second part contains a selection of review papers that cover most of the areas of PAK application. Also, most of the PAK developers and participants in the PAK use took part in PART II so that the complete picture could be viewed.

It is a great opportunity and a pleasure to thank many domestic and foreign institutions and the enormous number of people all over the world who helped us keep this project expanding over the decades. We have mentioned some of them in our History section, but the list is far from being complete. Besides the Publishers who helped this Special Issue, we would particularly emphasize the support of the industry of the Kragujevac region and Serbia, the City of Kragujevac, the Ministry of Science of the Republic of Serbia, and the Serbian Academy of Sciences and Arts.

Last, but most important, is the support of our families to all of us involved in the PAK project over many years. We owe our families gratitude and admiration.

CO-PUBLISHERS

Faculty of Engineering University of Kragujevac



**Faculty of Engineering
(earlier Faculty of Mechanical Engineering)**



**Dr. Slobodan Savić,
Head of the Faculty
University of Kragujevac**

The development of the software PAK started 50 years ago under the leadership of a young professor of Mechanics Miloš Kojić. The new Finite Element Method was attractive to talented students as well as to the Ministry of Sciences, so the Faculty started to be supported by numerous grants that continued over decades. Our Department for Mechanics became the leader in collaboration with scientific organizations and industry in Serbia and all over the world. The curriculum of our undergraduate and graduate studies was becoming richer by new courses, such as Numerical Methods for Structural Analysis, Finite Element Method, Modeling and Simulation, Constitutive Modeling of Engineering Materials, Dynamics of Structures, Nonlinear analysis of structures. The software PAK became the main tool for the research, and numerous Diploma Work, MS and Ph. D. Theses relied on the PAK. The PAK software has been continuously extended to new fields, such as fluid mechanics, heat transfer, flow through porous media, nonlinear problems in structural analysis and physical fields, biomechanics, and coupled problems. New laboratories emerged as the result of the achievements in computational methods and the PAK use and development. A number of young talents joined our laboratories with a significant impact on education and research. Many scientific conferences related to computational methods have been organized by our Faculty. The Serbian Society for Computational Mechanics was established during an EU conference at our Faculty in 2006.

Our Faculty became the center for education and research in computational mechanics, and we can assess the “Serbian School for Computational Mechanics“ has been firmly established. Our Faculty and the University of Kragujevac become a reference institution for computational methods and their applications in Serbia and abroad.

Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac



Serbian Academy of Sciences and Arts

**University of Kragujevac
Office of the Rector**

**Academician Miloš Đuran
Director of the Center**

The Scientific Program Bioengineering at the Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac (further referred to as the Center) was introduced by Miloš Kojić in 1997. The research grant was supported by the Ministry of Science of Serbia (Minister academician Dušan Kanazir) in order to expand the team for bioengineering, composed of young talents and medical doctors. After a few years, in 2002, the City of Kragujevac, led by Mayor Veroljub Stevanović, decided to support 5 young talents to be full-time employed at the Center. The Center expanded its research, and new grants and contracts were realized with various national and international institutions: the Ministry of Sciences of Serbia, the Institute “Jaroslav Černi”, the University of Hong Kong, the Harvard School of Public Health (HSPH), and the University of Houston. It is important to emphasize that the contract with the HSPH brought the research to an internationally recognized level. The new computational methods in Bioengineering were built into the general-purpose finite element program PAK. The achievement of the Center also had a strong impact on the overall research and educational programs at the University of Kragujevac, such as the graduate Multidisciplinary and Interdisciplinary Studies. Several Ph.D. theses were completed within this graduate program, among which are 3 theses of the collaborators of the Center, who became professors of the University of Kragujevac; they significantly strengthened the courses related to modern informatics and its application.

The Scientific Program Bioengineering had strong and continuous support from the Serbian Academy of Sciences and Arts – academicians Dušan Kanazir and Nikola Hajdin (the President of SASA), and Directors of the Center, academicians Dragoslav Srejović and Nikola Tasić. The research at the Center also had support from the Rectors of the University, Radoslav Senić, Miloš Kojić, Mirko Rosić, and Miloš Đuran. As stated above, the support for the City of Kragujevac had a very important, decisive role.

The Scientific Program for Bioengineering was terminated by an unjustified rector’s decision in 2008, after the Rector Đuran’s tenure. The Program continued within the Bioengineering Research and Development Center BioIRC, founded by the Serbian Society for Computational Mechanics and the City of Kragujevac.

Bioengineering Research and Development Center BioIrc, Kragujevac
Founders of BioIRC



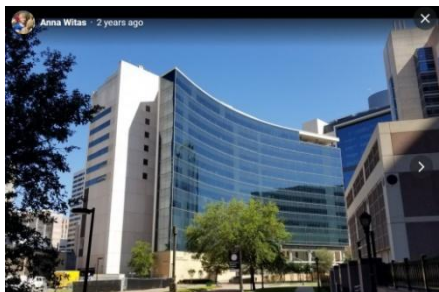
Academician Miloš Kojić, Director

Dr. Nenad Filipović, Vice-director

The president of the University of Kragujevac decided in 2008., without any rationale reason, to terminate the Scientific Program Bioengineering at the Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac. Milos Kojic, president of the Serbian Society for Computational Mechanics, and Veroljub Stevanovic, mayor of the City of Kragujevac, decided to open the Bioengineering R&D center BioIRC. The City provided the space and financial support for the researchers who moved from the University. Due to the enthusiasm ever-present among young researchers, and the leaders Milos Kojic and Nenad Filipovic, Bioirc started to be recognized in Serbia and abroad. Scientific grants realized in BioIRC were supported by the Ministry of Science of Serbia, and from foreign sources. Nenad Filipovic was particularly successful in applications to the EU calls and brought numerous research grants. The topics included heart models, laboratory investigations and computational modeling of stents, and others. The PAK development was strengthened by the 15-year collaboration of BioIRC and the Houston Methodist Research Institute (HMRI) led by Professors Mauro Ferrari and Milos Kojic – laboratory investigations were conducted at the HMRI, while the computational models were built to the PAK and executed at BioIRC. This research was primarily devoted to nanomedicine and drug delivery within tissue and tumors.

The overall research in Bioengineering has been so intensive and versatile that the number of collaborators within BioIRC reached sometimes around 40.

Houston Methodist Research Institute Department of Nanomedicine



Houston Methodist Research Institute

**Dr. Alesandro Grattoni,
the Head of the Department of Nanomedicine**

Within the Institute, our Department is oriented to interdisciplinary research coupling the fundamental and technological sciences with the medical practice. The main component of our research occurs within our laboratories, but it is accompanied by computational modeling. During his visit to the University of Kragujevac in 2006, Professor Mauro Ferrari got to know the achievements in computational mechanics and software development of the team led by Professor Milos Kojic, and, as the director of the Institute, decided to engage Professor Kojic at the Institute. As Professor of Nanomedicine, during his tenure in our Department in the period 2010-2024, he was leading the efforts in the development of computational methods and their applications in nanomedicine. Research within the scientific grants of the Department where supported by computational models developed within the Department and the Professor Kojic research group in Serbia. The continuous collaboration between our Department, the Research and Development Center for Bioengineering BIOIRC, and the University of Kragujevac, Serbia, supported by the software PAK, resulted in significant scientific achievements and the success of our grants over the years. We specifically emphasize the results in the development of the original methodologies for elucidating complex processes of drug transport within nanodevices, from nanoparticles, within capillary systems, tissue, tumors, and cells. Numerous papers were published in the most prestigious journals, including the book published by Elsevier in 2022 where the results of the research are summarized. Among other achievements, our Department has become recognized in the bioengineering community for its original contributions to computational methodologies and applications. This important work remains a legacy, a basis, and an inspiration for our future endeavors.

Jaroslav Černi Water Institute



Jaroslav Černi Water Institute (JCWI)

Dr. Dejan Divac, Director

Founded in 1947, the Jaroslav Černi Water Institute (JCWI) has grown into a distinguished center of expertise, committed to addressing the broad spectrum of issues in water resources and management. From the very beginning, JCWI has been a regional leading organization in the water sector, successfully implementing large and complex projects in the Republic of Serbia and surrounding countries, as well as in dozens of countries worldwide. Building on this legacy, the Institute today covers a wide range of expertise, including dams and reservoirs, hydroelectric power plants, water supply systems, sewage and wastewater treatment, irrigation and drainage, flood protection, river engineering, erosion control, infrastructure development (including roads, metro lines, and gas pipelines), rehabilitation of mines and waste disposal sites, as well as comprehensive water management and environmental protection.

JCWI provides a strong scientific foundation and expert solutions for improved water management. Grounded in scientific and developmental research, its activities extend to design, engineering, consulting, and the preparation of studies, ensuring that knowledge is effectively translated into practice. By combining scientific expertise with engineering experience and relying on advanced measurements, testing, and laboratory research, the Institute delivers innovative and effective solutions to the most complex challenges in the field of water. Building on its scientific and professional foundation, JCWI continuously develops knowledge and innovative approaches through close cooperation with both industry and academic institutions.

In the late 1990s, the Institute was first introduced to the PAK finite-element program during a presentation by academician Milos Kojic at the Serbian Institute for Materials in Belgrade. Recognizing the software's potential for modeling a wide range of engineering problems relevant to the Institute's work, collaborative teams were soon established with Professor Kojic's group at the Faculty of Mechanical Engineering and with the Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac. The most challenging problems were addressed in the modeling of underground water flow, the design of dams, embankments, and in mine operations planning, among others. In the last decade, this intensive cooperation has resulted in the implementation of several dam safety management systems at major hydroelectric facilities in the region, the most complex of which is the Dam Safety Management System for Iron Gate Dam. These systems are based on advanced software solutions that integrate complex information systems with FEM tools to automate numerous calculations and provide intuitive user interaction with data, models, and calculation results. New material models have been developed for concrete behavior, rock mass properties, and similar applications, along with new methodological approaches for calculations and their processing for engineering use. This fruitful collaboration, which has enabled JCWI to develop the knowledge and expertise required to solve such complex problems while providing the University of Kragujevac with the opportunity to apply FEM tools to real and complex cases, has also resulted in numerous scientific publications, including articles in leading journals, doctoral dissertations, monographs, and more.

Institute for Information Technologies Kragujevac, University of Kragujevac



Dr. Zoran Markovic
First Director 2019-2023



Dr. Igor Saveljic
Current Director 2023-

The Institute for Information Technologies Kragujevac, established on June 6, 2019, by the decision of the Government of Serbia as a part of the University of Kragujevac with help of the Rector Prof. dr Nenad Filipovic, serves as a modern, multidisciplinary scientific research center that fosters collaboration across natural-mathematical, technical-technological, medical, biotechnical, and social sciences. Its primary mission is to advance research, development, and education in information technologies, with a focus on fundamental and applied research, technological development, and the transfer of knowledge and technologies. By promoting multidisciplinary approaches, the Institute aims to enhance the quality and scope of scientific work, particularly in areas such as biomedicine, bioengineering, and bioinformatics, while establishing partnerships with industry, universities, and international programs like Erasmus+ and Horizon 2020 to facilitate internationalization and innovation. Moreover, it actively engages in national and international projects, supports the development of laboratory and semi-industrial technologies, provides consulting services across various scientific fields, and strives to attract returning scientists from the diaspora through various grant programs, all with the overarching goal of fostering scientific progress and technological advancement aligned with global trends.

Center of numerical modelling and biomechanics of Institute uses software PAK for multidisciplinary approach to the study of biomechanical systems through the use of advanced numerical methods and simulations. Focused on the integration of knowledge from the fields of engineering, micro and molecular biology, medical sciences and computer science, the center aims to improve the understanding and analysis of biomechanical processes in order to improve medical treatments and technologies.

Software PAK is mostly used for numerical research activities and the center deals with the following thematic units: biomedical engineering – through an interdisciplinary approach, the center combines knowledge from biomedicine and engineering to develop innovative solutions for the advancement of health technology.

HISTORY OF PAK DEVELOPMENT

Miloš Kojić
With Contributions by PAK Co-Authors
Radovan Slavković, Miroslav Živković, Nenad Grujović, Nenad Filipović
HISTORY OF PAK DEVELOPMENT

I will present the history of our PAK software as a personal story since this has been my life in last more than fifty years. The co-authors will add parts where they were involved as leaders of the specific areas of research. I will select the events that had the most important, sometimes decisive, effects on me and numerous contributors in our fifty-year journey. For me, this is a very emotional story, and I might go into certain details to illustrate the dramatic character of these events.

In the spring of 1972, I returned from my Ph. D. 1.5 years of studies at Rice University in Houston, which was the most pleasant and successful time in my life, to my Mechanical Engineering Faculty (today Faculty of Engineering). I started to think about what I should do next in my professional life. Somehow, I got to read a paper about the finite element method (FEM). I immediately realized that this scientific field was something for me: understandable from the view of the fundamentals of physical laws, and with excessive demands for software development. All that came later was a realization of that, I would say, an intuitive and lucky decision.

We had the great luck of obtaining SAP IV FEM software, written in FORTRAN IV language, from my close friend during Rice University studies, later a fantastic professor of Civil Engineering in Sarajevo, Branko Verbic. We punched the cards from the software listing at the “Zastava” car factory Computer Center in Kragujevac (Fig.1), and that was the starting point for learning the concept of the FEM software. My main collaborator at that time was Radovan Slavkovic, an undergraduate student at the Faculty of Mechanical Engineering. We were lucky to have with us the fantastic Milutin Marinkovic, MS in electrical engineering, a specialist in computer programming (in Fig.1).



Fig. 1 Left: Group from the “Zastava” factory visiting an IBM center in Germany in 1978 to get familiar with our further computer use for engineering applications in the factory. In front, from left to right: second is director Milan Perovic, Milos Kojic, director Bozidar-Zida Blagojevic, Milutin Marinkovic, and Dimitrije Obradovic. Right: Computer Center of the factory where the development of PAK software started in 1975. The building of the Center after the NATO bombing in 1999.

In analogy with the SAP IV, we started our code under the name PAK (abbreviation for the program for structural analysis – in Serbian: **P**rogram za **A**nalizu **K**onstrukcija). This was an FE program for linear analysis. From today's standards, that was the “Stone Age”. We had to punch hundreds of cards, make a listing, and try to find the errors. The machine was an IBM 360 with 1MB of memory! We had to wait in the Computer Center for a window in the process of the salary data handling and printing for about 15,000

employees at the factory. Very often, our working time at the Center started at 10 pm and finished early in the morning. Our work was something very new at the factory and was supported by all directors, from Zdravko Menjak, Dusan Slavkovic, and Dimitrije Obradovic at the “Zastava Institute”, director of the factory for special structures Bozidar-Zida Blagojevic, to the director of the Computer Center Milan Perovic, to the General Director Radoljub Micic, and Vice-Director Slobodan Smiljanic. It did not take too long (about 1 year), and we had the first diploma thesis of Radovan Slavkovic and Djuro Pavic (April 1977) with our beam element. Our work started to attract many young engineers, Aleksandra Jankovic, Vera Nikolic, Ivo Vlastelica, who later became professors at our University; engineers Gradimir Zivkovic, Dragoljub Grujovic (later director in the Factory for Special Structures within the “Zastava” system). The number of built-in finite elements increased, and other fields, besides linear mechanics, were included, such as heat transfer. The first master's thesis was completed by Dragoljub Grujovic in 1980, followed by the mentioned engineers and others.

I started in 1976 to apply for scientific grants to the Ministry of Science of Serbia, and my proposals were, during subsequent years, till 2012, always at the top of the list of accepted grants. Our work started to be the most attractive, and the group was becoming larger and larger, sometimes with more than 10 members. The best students were joining us each year, Radovan Slavkovic became an assistant in our Faculty, and was joined a bit later by Miroslav Zivkovic and Nenad Grujovic. The picture of us as the first main PAK developers at that time is shown in Fig. 2.

Fig. 2 The main authors of the PAK software, around 1990. Standing, from right to left: Miroslav Zivkovic, Nenad Grujovic, Radovan Slavkovic – all became professors at the Mechanical Engineering Faculty (MEF); seated – Milos Kojic



They and others from the industry were completing their MS theses, and we started to go into nonlinear problems in the FEM. I have also introduced new courses, such as Structural Analysis, and others at the undergraduate and graduate levels at our Faculty. I established the Laboratory for Engineering Software within the Faculty as the unit for engineering software development. Our PAK becomes a tool in education. Courses at the undergraduate and graduate levels were designed according to our book (Kojic, Slavkovic, Zivkovic, Grujovic, 1998). We had support not only from industry but also from other institutions and companies as “Jugobanka” (director Predrag Ggalovic), or “Prizma” Kragujevac (the owner Tanasije Katanic).

The next, maybe the most important and decisive event, happened in 1983 when I got information from the Yugoslavian-American Commission for Scientific Collaboration that my grant proposal (a month after my application) was accepted. I was offered to select a university in the USA to spend half of a year on research - there was no dilemma: I immediately answered that I would like to go to Massachusetts Institute of Technology (MIT) in Boston in the lab of Professor Klaus-Jurgen Bathe (Fig.3). At that time, we were developing the “Bathe school” in Serbia since he had a profound influence on us through his papers. Also, he was a Ph. D. student of Professor Wilson at Berkeley, California, the author of the SAP IV software.

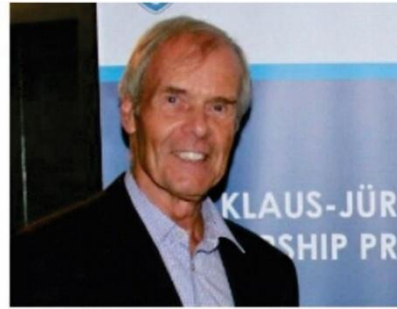
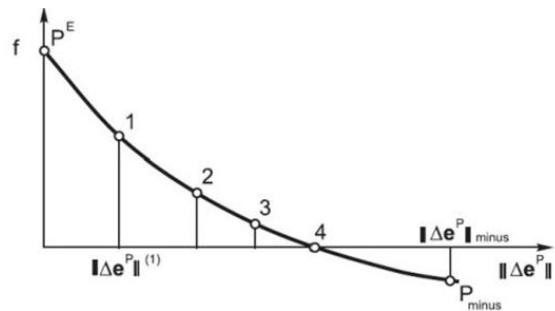


Fig. 3 Massachusetts Institute of Technology (MIT). Klaus Juren-Bathe, professor at MIT, had an enormous influence on our development of PAK in the early years (1975-1990), and later during my work at ADINA R&D, 1985-1990.

Professor Bathe agreed that I could join him, and I came to Boston in the Spring of 1983. The next day after my arrival, I asked Professor Bathe about the topic on which I should work with him. He proposed that I should test the current methodology related to thermoplasticity and creep of metals, built into his FE software, ADINA. After a month, I got an idea, a completely new computational algorithm, that resolved obvious deficiencies of the existing algorithm. I introduced an “effective stress function (ESF)” whose zero provides the solution without any uncertainty (Fig. 4).

Fig. 4 Effective stress function $f(\Delta e^P)$,

where Δe^P is the increment of the effective plastic strain, as the basis for implicit stress integration built in the PAK software [Kojic and Bathe, 1987a,b]



Over the decades after that, I used to joke that without the curve representing the EFS, many things might have happened later at our university, or would never have happened. After my return home, my assistant Slavkovic spent 1.5 years at MIT implementing my ESF to large strains and completed his Ph. D. thesis. I spent 4 years, with some short breaks, at ADINA R&D, the company of Bathe, to implement the ESF idea into metal thermoplasticity and geological materials into the code ADINA. Later, I generalized the ESF into the governing parameter method (GPM), showing that in plasticity models, we can find the solution by establishing a governing parameter. After the ESF concept was published [Kojic and Bathe, 1987a,b], it was implemented, with my participation, into our PAK code by Radovan Slavkovic, Miroslav Zivkovic, Nenad Grujovic, Dusan Begovic, and others. Fig. 5 illustrates the accuracy and applicability of the ESF to real engineering problems.

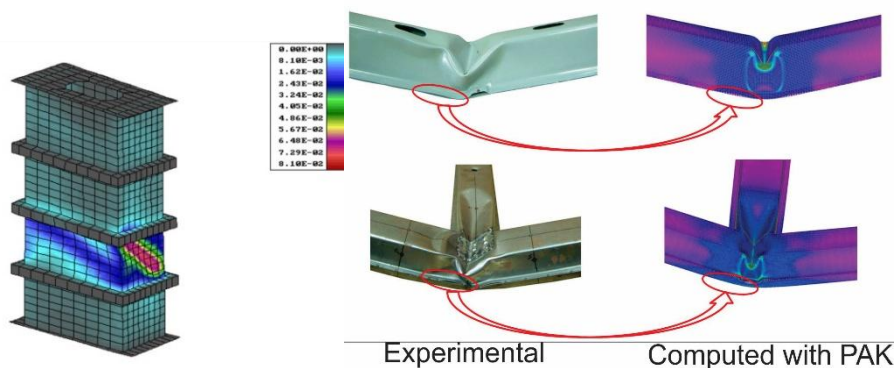
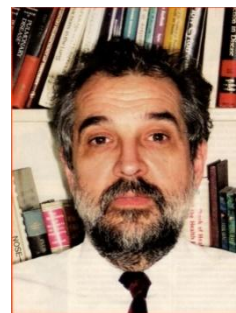


Fig. 5 Application of the ESF in the PAK to modeling structural collapse. Left: Thin-walled beam used in the bridges, subjected to torsion (field of the effective plastic deformation in red color; Ph. D thesis of B. Stipanac from Civil Engineering Faculty in Belgrade, mentor academician Nikola Hajdin); Right: Collapse of the typical car supporting elements (model created by M. Živković).

In the early nineties, the PAK code was reorganized to be more efficient, where the main role belonging to Miroslav Živković. I have established our Laboratory for Engineering Software, where around 10 young collaborators have been constantly involved. New fields were included in the code, such as enhanced elements, rigid body elements (Zoran Bogdanović), flow through porous media, fluid mechanics (Nenad Filipović), and others. It was the time of the start of the application to large-scale industrial problems, such as dams, structures of the Electro-distribution company of Serbia, collaboration with the Institute “Jaroslav Černi”, and others.

The next important and decisive year for the overall development of the PAK was 1995. I was privately in Boston when I got basic information about computational modeling in biomedical engineering. My close friend Srboľjub–Srba Mijailović (in Fig. 6) suggested that I should switch from my engineering to the biomedical field. Several fortunate events occurred regarding Srba. I was employed at ADINA R&D when one day Professor Bathe asked me: Do you know Srba? Yes, I know him; he is the most talented engineer I know. Srba became a graduate student of Bathe and showed his talent and knowledge. Since Bathe asked Srba to first go to MS (which Srba already possessed), Srba changed his mentor and switched to bioengineering, completed Ph. D., and started to work at Harvard School of Public Health with his new mentor.

Fig. 6 Srboľjub–Srba Mijailović initiated 1995 our involvement in the development of methods and software for bioengineering. Since that time, he has participated in research and significantly contributed to the overall advances in this field at the University of Kragujevac (UK). He possesses an honorary doctorate from the UK.



Since 1995, Srba has been involved in the development of our bioengineering methodology, with many publications as the principal author, scientific grants, and in Ph. D. theses at the University of Kragujevac. Research in the engineering field and upgrades and modernization of the PAK system of programs continued at the Laboratory for Engineering Software under the guidance of professors Radovan Slavković, Miroslav Živković, and Nenad Grujović. In the last 24 years, the Lab, under the leadership of Miroslav Živković, reached national and international recognition through the FE modeling of the most complex and large structures, with the continuous engagement of more than 10 researchers. In this period, with the support of the Ministry of Science of Republic of Serbia, several specific versions of the PAK software were developed: Software for fracture mechanics PAK-FM, Software for fatigue PAK-FAT,

Software for geomechanics PAK-GEO, Software for numerical solving of electrostatic problems PAK-E, Software for fast generation of blood vessels FEM models - STL2FEM, Software for turbulent fluid flow PAKF-Turbulent, Software for explicit solving of dynamic problems PAK-EXP, Software for coupling of PAK with SPH method SPH-LifeCycle, Software for automatic testing of PAK moduli PAK-Testing, Software for pre- and post-processing PAK-G. All these software coupled for solving complex problems are named Software for Solving Coupled Problems, PAK-Multiphysics. At the moment, the most recent development is related to the Serbian Science Fund project, which supports the development of software for the prediction of damage evolution – PAK-DAM. The realized projects coordinated by Miroslav Živković are:

- Technological project of the Ministry of Science, Technology and Development of the Republic of Serbia: Development of methods and software for the analysis, simulation, and optimization of large deformation processes in the mechanical industry, TR258, 2002–2004.
- Technological project of the Ministry of Science and Environmental Protection of the Republic of Serbia: Development of software for strength analysis and assessment of structural service life, TR6204, 2005–2007.
- Project of the Ministry of Science and Environmental Protection of the Republic of Serbia on a designated topic: Revitalization of critical components of thermal power plants based on their integrity, TD7066, 2005–2007.
- RRSCD INNCODE 043820, Reinforcement of Research Capacity in Software Development and Innovative Collaborative Design and Engineering in Serbia and Montenegro, 09.2006–09.2008.
- Technological project of the Ministry of Science and Technological Development of the Republic of Serbia: Development of software for explicit nonlinear dynamic analysis, TR12005, 2008–2010.
- Technological project of the Ministry of Education, Science and Technological Development of the Republic of Serbia: Development of software for solving coupled multiphysics problems, TR32036, 2011–2019.
- IPA ADRIA HUB - Bridge technical differences and social suspicions contributing to transforming the Adriatic area into a stable hub for sustainable technological development, 2013-2015.
- H2020 – Cloudifactory – OSICS - Optimization of the Production Process of Metal Structures Using OSICE. Grant agreement by the EC no. 768892, 2019-2020.
- The Science Fund of the Republic of Serbia, #GRANT No 7475, Prediction of damage evolution in engineering structures – PROMINENT, 2024-2026.

In the framework of the realized projects, new collaborators joined the Lab and finished their Ph. D. theses. Gordana Jovicic developed the PAK code in the field of XFEM (fracture mechanics). Dragan Rakic and Vladimir Dunic contributed as co-authors of the PAK for structural analysis. Dragan Rakic worked in the field of the new constitutive models implementation to soil mechanics, while Vladimir Dunic was active in the development and implementation of shape memory alloys and concrete damage plasticity constitutive models, thermo-mechanical coupling, phase-field damage modeling, and explicit dynamics. Snezana Vulovic developed a code for solving penalty contact problems, fatigue, seepage, and heat transfer problems. Marko Topalovic developed a methodology for modeling granular materials and fluid flow by a smoothed-particle hydrodynamics (SPH) solver, which is coupled with PAK. Aleksandar Nikolic implemented the turbulent fluid flow model into PAK. Vladimir Milovanovic worked in the field of experimental mechanics and contributed to the validation of computational models in the field of fatigue. Vukasin Slavkovic worked on the development of crystal plasticity and shape memory polymers' constitutive models. Jelena Zivkovic contributed to the field of phase-field damage modeling in metallic materials.

In parallel with the above-cited research supported by the Serbian governmental institutions, the PAK research group established in the late 1980s of the last century a continuous collaboration with important institutions in Serbia, such as the Military-technical Institute and Aerotechnical Institute (with Mladen Berkovic and Stevan Maksimovic), leading the research to a higher level. The collaboration with the Institute “Jaroslav Černi” was very important and has been advancing over the decades. The PAK research group started with the modeling of dams and underground water flow, under the guidance of directors Milan Dimkic and Dejan Divac (Fig. 7).

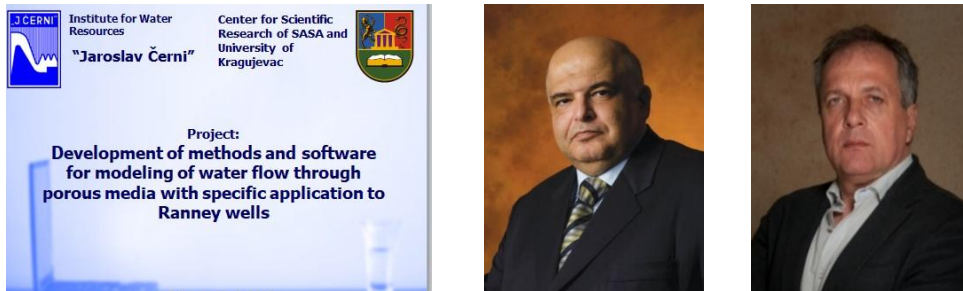


Fig. 7 Left: One of the projects with the Institute “Jaroslav Černi”; Right: Directors of the Institute “Jaroslav Černi, Milan Dimkic (left) and Dejan Divac (right), who significantly contributed to the development of PAK in the fields of the underground water flow and dam structures.

I emphasize here not only the highest professional and scientific level of both directors but also their friendship, support, and inspiration for our PAK team over the decades. We also published together with them several very nice papers. I give here a small detail. We introduced a fictitious dimensionless 1D element to connect a 3D space to the lateral screens of radial wells. In analogy with that, I was able to generate my composite smeared finite element for the gradient-driven physical fields in composite media as tissue. Here are listed a few of the typical structures modeled by our PAK in collaboration with the Institute “Jaroslav Černi” (Figs. 8, 9, 10):

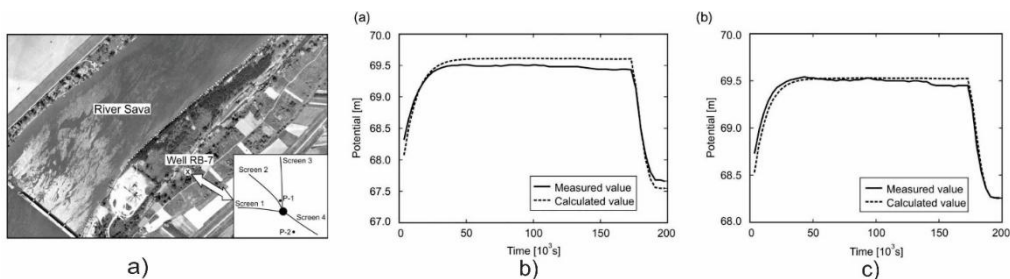


Fig. 8 Model of Belgrade Groundwater Source. a) Position of the radial well near the Sava River; b) and c) Measured and calculated potentials at piezometers P-1 and P-2.

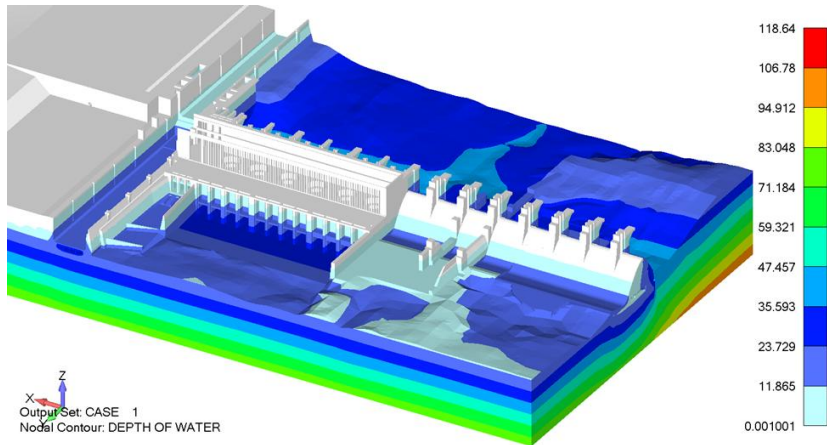


Fig. 9 Numerical model of Djerdap 1 dam. Depth of water field (m)

The numerical model of the Đerdap 1 dam (concrete gravity dam) consists of 4.3 million nodes and 3 million finite elements. It includes an overflow dam section, a power plant, an assembly block, a lock and an embankment dam on the Serbian side of the Danube River.

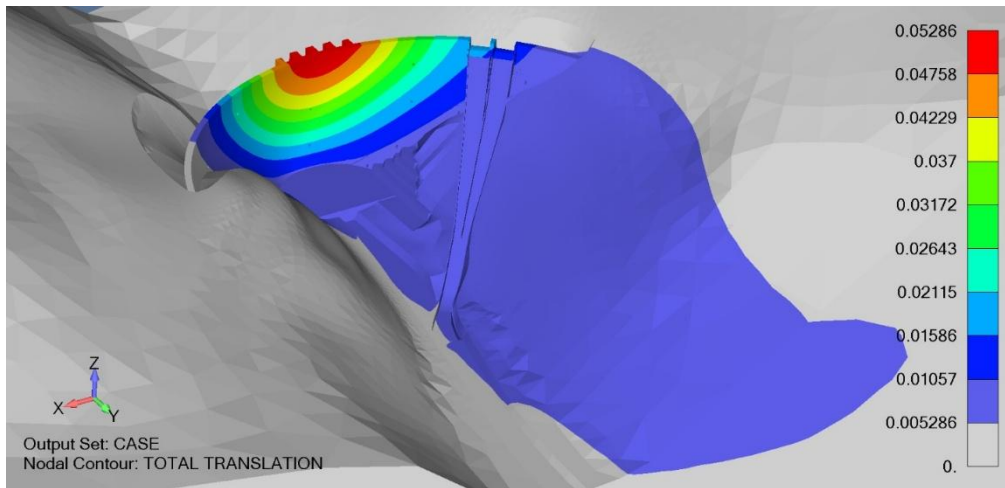


Fig. 10 Numerical model of Grančarevo dam. Total translation field (m)

The numerical model of the Grančarevo dam (concrete arch dam) consists of 1 million nodes and 700 thousand finite elements. A very detailed model has been developed that contains all the necessary structural elements for the analysis of thermal, filtration, and stress-strain processes on the dam and the surrounding rock mass.

Returning to 1995, I presented new ideas about bioengineering to my collaborators, and my decision to further focus on this, for us, unknown field. My offer to join me was accepted by young collaborators Nenad Filipovic and Nebojsa Zdravkovic. Together with Srba, we developed our first biological models, as muscle, tissue, and cartilage in PAK, with published papers in journals, and that was everyday excitement. Then, in 1997, academician Dusan Kanazir, at that time Minister of Science of Serbia, visited our Faculty (Fig. 11), and I had the opportunity to show him some of our results in bioengineering. He

was surprised and so impressed that he concluded: “Serbia has to support this research immediately”. I formulated a research program, under the title “Scientific Program in Bioengineering”, and got financial support from the Ministry to form a larger group of researchers.

Fig. 11 Academician Dusan Kanazir visiting the Mechanical Engineering Faculty in 1997. At left is academician Kanazir, on the right, head of the Faculty Ratko Mitrovic, and in the center, Milos Kojic.



The meeting was a decisive moment for the introduction of the Scientific program Bioengineering at the Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac.

The Program was allocated to the recently opened Center for Supercomputing at the University of Kragujevac. The Program was enthusiastically supported by the Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac, led by academician Dragoslav Srejavic and later by academician Nikola Tasic (Fig. 12).

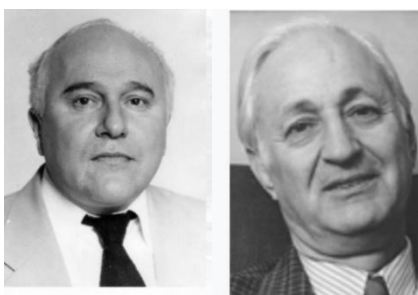


Fig. 12 Academicians Dragoslav Srejavic (left) and Nikola Tasic (right), directors of the Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac, supported the Scientific Program Bioengineering.

I further presented a unique idea to my friends Sasa Milenic – the vice mayor of the City of Kragujevac, and Slobodan Pavicevic, in charge of cultural affairs of our City, that the City should employ 5 young talented people (according to myself and Nenad Filipovic's selection) to full-time employment at the Center. It was accepted by the City Mayor Veroljub-Verko Stevanovic, and our new team joined our research at the Center. Later, this specific scientific program was signed by the president of the Serbian Academy of Sciences, Nikola Hajdin, Mayer Veroljub Stevanovic, and the President (Rector) of the University of Kragujevac, on 10 October 2005, Fig. 13.



Fig. 13 (From left to right) Academician Nikola Hajdin, Mayor of the city of Kragujevac Veroljub-Verko Stevanovic, and Milos Djuran, president (rector) of the University of Kragujevac, signed the contract of support of the Scientific Program Bioengineering, 10.10.2005.

In 2000. I got, according to the recommendation of Srba Mijailovic, an offer from Akira Tsuda (Fig. 14), the PI of an NSF grant at Harvard School of Public Health (HSPH), to join his team. Working at HSPH in the Akira team and Srba, our team at the Center for Supercomputing in Serbia was involved in research, with such advances that resulted in a contract between HSPH and the University of Kragujevac. Hence, the University of Kragujevac got financial support from the NIH fund.

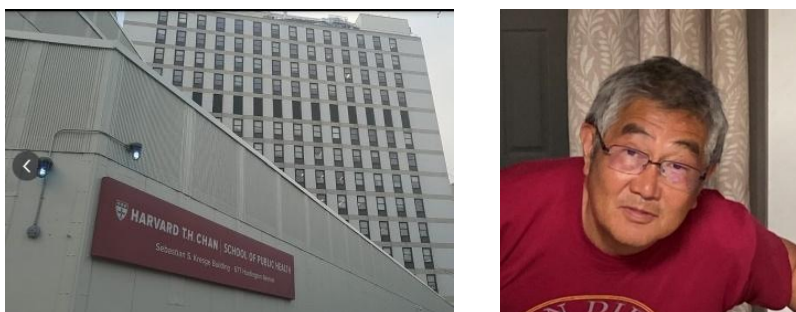


Fig. 14 Harvard School of Public Health (HSPH). Dr Akira Tsuda from the HSPH, as PI of an NIH grant, directed part of the grant to the University of Kragujevac. The Center for Supercomputing was involved in the research of the Dr. Tsuda grant for almost 10 years.

Later, in 2006, Professor Mauro Ferrari from the University of Texas at Houston came to a conference in Kragujevac and visited our research center (Fig.15). He was impressed by our work, so he decided to collaborate with us. For his support of our research in the next years, he got the highest recognition, the Plaque of St. George of the City of Kragujevac, 2009 (Fig. 16).

It should be noted that three of the five members of the Center completed Ph.D. theses in bioengineering, participating in the PAK development, and they are now professors at the University of Kragujevac: Boban Stojanovic, Vladimir Rankovic, and Milos Ivanovic. I should mention that, besides collaborators from the Center, Nenad Filipovic and Ivo Vlastelica significantly contributed to our methodology and PAK development. Our research during this period included several big topics, primarily related to lung problems, but also to muscle and cartilage modeling; transport within lung microstructure and spider channels (Nikola Kojic), and other related fields. We published papers in various world journals and summarized our work in the period 1995-2008 in the book [Kojic,M., Filipovic,N., Stojanovic, B., Kojic,N.].

Fig. 15 Professor Mauro Ferrari from the University of Texas at Houston with the team from the Center for Supercomputing of the University of Kragujevac, 2006. From left to right: Boban Stojanovic, Mileta Nedeljkovic, Lazar Otasevic, Nikola Kojic (Ph. D. student at MIT), Mauro Ferrari, Milos Kojic, Vladimir Rankovic, Milos Ivanovic, Nenad Filipovic



By the unjustified decision of the president of the University, the Center for Supercomputing was closed, and the Scientific Program Bioengineering moved to the Bioengineering Research and Development Center BioIRC. I was lucky to have such respect from the City of Kragujevac, so that Mayor Verko Stevanovic offered me support. We opened BioIRC as a joint research center by the City of Kragujevac and the Serbian Society for Computational Mechanics (established in 2006; I am the president). The Center was financially supported by the City, and through grants (Ministry of Science of Serbia, Institut “Jaroslav Cerni”, European and USA grants). A new era of our research started, and new young collaborators started to come to BioIRC: Velibor Isailovic, Miljan Milosevic (later university professors), Dejan Petrovic, Vladimir Simic, Bogdan Milicevic, Dalibor Nikolic, and others (Fig. 17), who significantly contributed to further advances in computational methodology and PAK development.

Fig. 16 Professor Mauro Ferrari at the ceremony of awarding him the Plaque of St. George of the City of Kragujevac, 2009. From left to right: academician Nikola Hajdin, Mauro Ferrari, Mayor Veroljub Stevanovic, President of the City Parliament Dobrica Milovanovic, Vice-Mayor Sasa Milenic.





Fig. 17 BioIRC 2009. From the left: Dalibor Nikolic, Milos Ivanovic (behind), Dejan Petrovic, Nenad Filipovic (behind), Tijana Djukic, Velibor Isailovic, Milos Kojic, Danko Milasinovic, Dejan Veljkovic, Zarko Milosevic, Radun Vulovic, Miljan Milosevic, Milos Radovic, Dejan Milenkovic.

One of the very significant developments was our methodology for modeling the motion of deformable bodies within a fluid with a remeshing procedure, and contact between bodies using 1D elements (Velibor Isailovic, Miljan Milosevic; and application to motion of cancer cells within capillaries – Nikola Kojic).

BioIRC was becoming a stronger and larger research center, recognized not only in Serbia but also in Europe and the USA, particularly due to the engagement and leadership of Nenad Filipovic. A large number of EU grants, bilateral grants between Serbia and other countries, were realized in BioIRC and University of Kragujevac with Nenad Filipovic as PI, and the number of full- and part-time employees was sometimes over 40. New biomedical problems were studied and the corresponding models developed within the PAK: atherosclerosis, biodegradable and metal stents with applications, heart model, cardiomyopathy, and heart failure.

Some of the EU projects with PI Nenad Filipovic are included in PAK, as one of the main software and open source for numerical calculation in various problems, and are presented here.

- FP7-NMP-2007-LARGE-1, (EU, 1/6/2008-31/12/2013), MUST, Multi-Level Protection of Materials for Vehicles by "SMART" Nanocontainers, European Commission. Part of the research in this project was built in the Ph.D. thesis of Dejan Petrovic.
- ICT-FP7 – 224297, (EU, 01/09/2008– 31/08/2011) with title: ARTreat, Multi-level patient-specific artery and atherogenesis model for outcome prediction, decision support treatment, and virtual hand-on training, University of Kragujevac. Part of the research is built in the Ph.D. thesis of Aleksandar Milovanovic.
- European Union's Horizon 2020 (EU, 1/7/2017-31/12/2022) with title: PanBiora, Personalised and generalized integrated biomaterial risk assessment. This project has provided a set of tools, composed of protocols and instruments, standardising the evaluation of new biomaterials. Part of the research is built into the Ph.D. thesis of Milica Nikolic.
- SCOPES Joint Research Project (Switzerland, 09/01/14-08/31/18) with title: Role of blood flow and sdf-1/cxcr4-induced recruitment of mononuclear cells in intussusceptive angiogenesis. In this project specific model of shear stress for PAK-F was used for the calculation of the recruitment of mononuclear cells in intussusceptive angiogenesis. Part of the Ph.D. thesis of Tijane Djukic was devoted to this topic.
- H2020-PHC-2015-single-stage (EU, 01/01/16-06/30/19) with title: SMARTool: Simulation Modeling of coronary ARTery disease: a tool for clinical decision support. In this project model

of atherosclerosis was used, and a specific model for Fractional Flow Reserve. Part of the project for a model of atherosclerosis was built in the Ph.D. thesis of Milos Radovic.

- H2020-SC1-PM-15–2017 (EU, 01/10/17-30/09/20) with title: InSilc: In Silico trials for drug-eluting BVS development and evaluation. In this project model of biodegradable, metal, and nitinol stents was used for in silico clinical trials. Some implementation of the project was built in the Ph.D. thesis of Dalibor Nikolic. Also, a model of the aorta with fluid-structure interaction was developed and used in the Ph.D. theses of Igor Saveljic and Dejan Krsmanovic.
- H2020-SC1-PM-15–2017 (EU, 01/12/17-05/31/21) with title: HOLOBALANCE: HOLOgrams for personalised virtual coaching and motivation in an ageing population with BALANCE disorders. In this project balance model for rehabilitation was developed. Dr Aleksandra Vulovic has incorporated some part in her Ph.D. thesis.
- H2020-MSCA-ITN-2020 (EU, 01/01/21-31/05/25) with title: DECODE: Drug-coated balloon simulation and optimization system for the improved treatment of peripheral artery disease. Some of the numerical models from PAK-FS were implemented in the Ph.D. theses: Safi Ur Rehman Qamar, Lemana Spahic, and Leo Beolic.
- H2020-WIDESPREAD-2018-2020 (EU, 01/10/20-30/09/24) with title SGABU: Increasing scientific, technological and innovation capacity of Serbia as a Widening country in the domain of multiscale modelling and medical informatics in biomedical engineering. In this project web platform for M.S. and Ph.D. students is developed. Parts of the project are incorporated in Ph.D. theses of Bogdan Milicevic, Vladimir Simic, Miljan Milosevic, Tijana Geroski, and Aleksandra Vulovic.
- H2020-SC1-2016-2017 (EU, 01/06/18-31/03/22) with title: SILICOFCM: In Silico trials for drug tracing the effects of sarcomeric protein mutations leading to familial cardiomyopathy. In this project it is developed a computational platform for *in silico* clinical trials of Familial cardiomyopathies (FCMs) that takes into consideration a comprehensive list of patient-specific features (genetic, biological, pharmacologic, clinical, imaging, and patient-specific cellular aspects) capable of optimizing and testing medical treatment strategies with the purpose of maximizing positive therapeutic outcomes. Parts of this project are incorporated in Ph.D. theses of Smiljane Tomasevic and Bogdan Milicevic.
- HORIZON-HLTH-2022-STAYHLTH-01-04 (EU, 01/6/23-31/05/27) with title: STRATIFYHF: Artificial intelligence-based decision support system for risk stratification and early detection of heart failure in primary and secondary care. In this project, accurate risk stratification and early diagnosis of heart failure allow implementation of evidence-based prevention and treatment strategies, which reduce heart failure morbidity and mortality and its burden on healthcare. It uses computational mechanics combined with artificial intelligence methods for risk stratification, diagnosis, and prognosis of patients with heart failure. Part of the research is implemented in the Ph.D. thesis of Jelena Pavic.

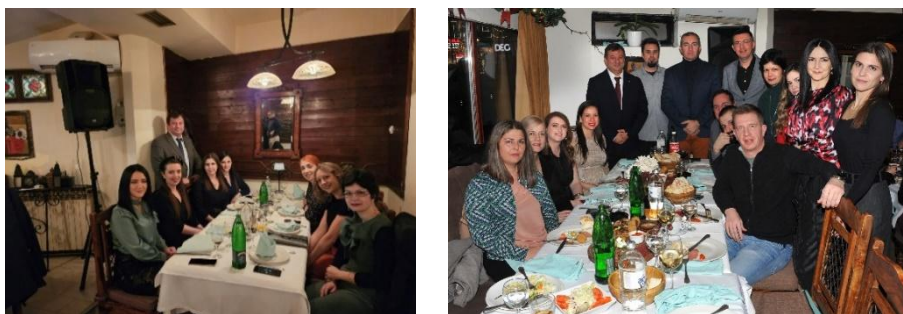


Fig. 18 Typical celebration parties of the Professor Filipovic Ph.D. students.

A part of the recent history of the PAK development is related to our collaboration with Houston Methodist Research Institute (HMRI, Fig. 19). This collaboration started in 2010 when I was invited by Professor Ferrari, the director of HMRI, to join his research team.

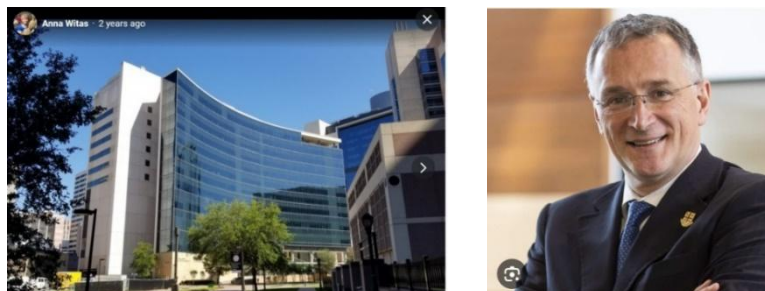


Fig. 19 Houston Methodist Research Institute (HMRI). Professor Mauro Ferrari, director of the HMRI, Foreign Member of the Serbian Academy of Sciences and Arts, initiated collaboration between the HMRI and BIOIRC

The laboratory investigations at HMRI were coupled with our computational teams at BioIRC and the University of Kragujevac. We had contracts to realize joint research at HMRI and to further develop our PAK program. The grant was under the title:

Modeling of Blood Microcirculation, Margination, and Endocytosis of Particles

between the HMRI and BIOIRC, during the period 2011-2013, and served to establish the basis for the joint research over the years, until the end of 2024. The main topic of the research was nanomedicine with drug delivery, mainly within the Ferrari grants. Our key collaborators at HMRI, besides Ferrari, were Arturas Ziemus (biochemical physicist, whose contribution was fundamental in connecting lab research and our modeling procedures), Kenji Yokoi, Alessandro Grattoni, Elvin Blanco, Xuewu Liu, Carly Filgueira, Antonio Martino, Rossana Terracciano; also, Eugene Jon Koay from MD Anderson Cancer Center, Professor Annapragada from the Texas Childrens Hospital, and Professor Bernhard Shefler from Italy.

We have developed specific computational procedures, such as numerical homogenization, and later the smeared concept for physical fields within composite media, as tissue (Kojic Transport Model - KTM). Implementation, testing, and corrections of the KTM were realized in BioIRC under the leadership of Miljan Milosevic; he significantly contributed to the PAK solver and CAD pre-and post-processor CAD. The Ph. D. theses of Miljan Milosevic and Vladimir Simic came directly from this research. Bogdan Milicevic and Vladimir Simic from BioIRC also made important contributions. Our achievements are summarized in the book [Kojic, Ziemys, Milosevic, 2022]. Also, one of the important achievements is related to models of heart mechanics and electrophysiology, and lung models.

In Fig. 20 is shown the composite smeared element and its application to determine concentration within the pancreas of a mouse, which includes several scales: large blood vessels, capillaries, extracellular space, two types of cells, and organelles. I consider that the formulation of this element is one of the main representatives of my scientific ID.

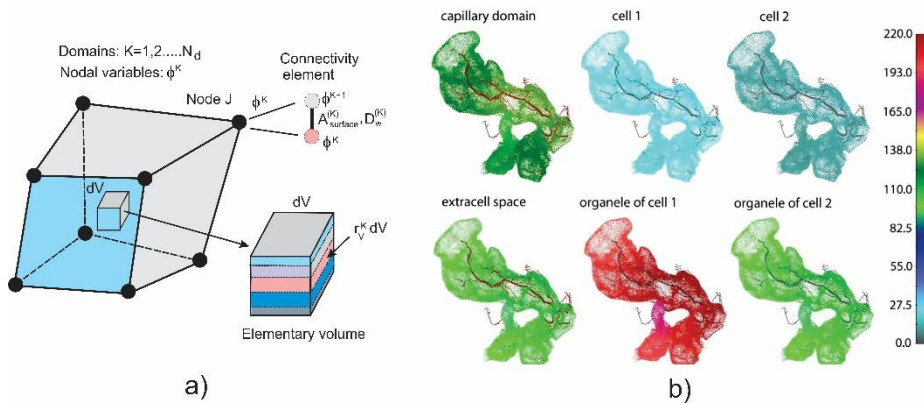


Fig. 20 a) Composite smeared finite element for gradient-driven physical fields as the basis for the KTM; b) Concentration field within several domains of different length scales of the pancreas.

Finally, I would like to show the photo of my retirement ceremony at the HMRI (Fig. 21), as a document of my official end of employment, which fortunately lasted around 60 years.

Fig. 21 Retirement ceremony at the Houston Methodist Research Institute on 09 December 2024. From the left: Junhua Mai, Carly Filgueira, Xiewu Liu, Elvin Blanco, Milos Kojic, Alessandro Grattoni (Head of the Department for Nanomedicine), Biana Godin.



In the end, I want to point out that this brief summary of our 50-year journey brings the most important events and contributions of the people who shaped our work. It can be said that we have formed a PAK family that has worked in almost ideal harmony over decades with many challenges, but always claiming to reach newer horizons. Of course, our big PAK community owes to many institutions, friends, and supporters; some of them are mentioned in this text, but there are many of them in Serbia and all over the world, not included here – and I would like, on behalf of the PAK authors and contributors, to apologize for that to those people and institutions. Also, the decisive role in the success of the PAK community belongs to our families, and we express our admiration and thanks for their patience and support.

I cannot avoid mentioning a few personal notes. I may say, as a testimony, that I consider that I was born under the “Lucky Star” and that I had the luck and privilege to work with so many talented and wonderful people, particularly with my close collaborators; and others at my universities, in the city of Kragujevac, in our country and abroad; I am deeply grateful to all of them. The PAK project turns out to be my everyday life. I can assure all the people with whom I worked that I was not able to do more – I feel that I owe to all, and particularly to my family, from the early age in my village Zakuta, to today.

Main References

- Milos Kojic, Radovan Slavkovic, Miroslav Zivkovic, and Nenad Grujovic, Finite Element Method I - Linear Analysis, in Serbian, Faculty of Mechanical Engineering, Kragujevac, 1998.
- M. Kojic, K. J. Bathe, The "effective stress-function" algorithm for thermo-elasto-plasticity and creep, Int. J. Num. Meth., Engng., Vol. 24, pp. 1509-1532, 1987.
- M. Kojic, K. J. Bathe, Thermo-elastic-plastic and creep analysis of shell structures, Computers and Structures, Vol. 26, No. 1/2, pp. 135-143, 1987.
- M. Kojic, N. Filipovic, B. Stojanovic, N. Kojic, Computer Modeling in Bioengineering – Theoretical Background and Software, J. Wiley and Sons, 2008.
- M. Kojic, A. Ziemys, M. Milosevic, Computational Methods in Biomedical Engineering – Finite Element Models Based on Smeared Physical Fields: Theory, Solutions, and Software, Elsevier, 2022.

AUTHORS AND CONTRIBUTORS

During 50 years of the development and application of the PAK finite element package, tens of researchers contributed to the success of this software. The development of the theoretical background and its implementation into the PAK framework involves a significant effort. The initial FORTRAN code was devoted to structural analysis, representing the basis for the general concept and software organization. The basic software, which served further for the development of numerous versions for specific problems, including a large field of Bioengineering from 1995, was developed by Miloš Kojić, Radovan Slavković, Miroslav Živković, and Nenad Grujović, followed by extensions by Nenad Filipović from 1993. Snezana Vulovic, Boban Stojanovic, Vadinir Rankovic, Milos Ivanovic, and others; then by Miljan Milošević, Vladimir Dunić, Dragan Rakić, Vladimir Simic, Bogdan Milicevic, and others from the beginning of the decade 2010-2020. Over the decades, different fields of science, such as engineering, physics, bioengineering, biology, and medicine, were included and led to a quite large number of PAK versions. This concept of the PAK development and diversification follows the basic idea that the PAK should be a research, rather than commercial software, enabling the freedom of researchers to pursue their advances in academic careers and practical applications.

Here are presented short biographies of the (A) – the Main Authors, and (B) – Contributing Authors, followed by (C) a Complete List of Participants in the PAK Development and Application. The main authors played the crucial role in the formulation of the theoretical background and code development, leading their groups of M.S. and Ph. D. students, and engineers. The group (B) represents researchers who contributed to the theory, as well as to the code development. Finally, the complete list of participants provides information about the field of participation, the module of the PAK, and the mentor in their research.

Altogether, this section aims to give an overview of the manpower of the PAK community that brought a profound positive impact on Serbian society and became respected in the world.

A. THE MAIN AUTHORS OF PAK

MILOŠ KOJIĆ

He completed his B.S. in engineering at the Faculty of Mechanical Engineering, Belgrade – Department at Kragujevac in 1964 as the best student in his generation and also the first graduate at the future University of Kragujevac. He completed his M.S. in mechanics at the Faculty of Natural Sciences and Mathematics, University of Belgrade, in 1969, in the field of aerodynamics. The Ph.D. studies he completed in engineering at Rice University, Houston, TX, in 1972, in a record short time in the university's history, of one year and four months. His Ph.D. thesis was related to the stability of soils in the presence of water flow. From 1964 to 2007, he served as an assistant and professor of Mechanics at the Mechanical Engineering Faculty of the University of Kragujevac. He was the head of the Mechanical Engineering Faculty from 1979-1980, and the rector of the University of Kragujevac from 1980-1981. He became a member of the Serbian Academy of Sciences and Arts in 2009.



Miloš Kojić, as a young professor, initiated the use of computers for structural analysis at the car factory “Crvena Zastava” in 1972, and then the development of the FE software named PAK (abbreviation in Serbian for a program for structural analysis). Since that time, he has been the PI of the PAK development in engineering and, since 1995, in bioengineering. His group of young, talented collaborators was large over the decades, sometimes around 20 researchers. He was the PI of numerous national and international scientific and applied grants over decades, all based on the finite element method with software development and application of the PAK package.

He was a visiting scholar at MIT in 1983, where the collaboration with Professor Klaus-Jurgen Bathe had a significant impact on the PAK development in structural analysis. Further, he was engaged at ADINA R&D 1985-1990 in the implementation of his “effective stress function” (ESF) to the ADINA code. This methodology was published in 2005 by Springer in a book co-authored with Professor Bathe.

Since 1995, he has been leading the development of the methodology and implementation into the PAK in the field of bioengineering. This research was summarized in the book published by J. Wiley and Sons in 2008, where Professor Kojic was the first author among three others and 20 contributors. He was the founder of the Bioengineering R&D Center BioIRC, in 2008, and has been the director since that year. From 2010 to 2025, he was engaged at Houston Methodist Research Institute (HMRI) as a Professor of Nanomedicine, leading the development of computational methods devoted to nanomedicine and mass transport within tissue. This research is summarized in the book published in 2022 by Elsevier, where the central topic is the multiscale smeared concept for the gradient-driven physical fields within tissue, termed the Kojic Transport Model (KTM).

He introduced to the PAK numerous original concepts, such as the ESF, the general beam element, shell element for heart tissue, a model for the motion of deformable bodies within fluid, numerical homogenization, membrane tissue model, and his KTM framework. He also developed a significant amount of the code (thousands of lines in FORTRAN), starting from PAK-S to PAK-FIS and PAK-KTM. His fundamental goal has been to foster scientific connections between the University of Kragujevac and world-leading universities and institutions.

Academician Kojic is the author or co-author of around 250 papers; author or first author of 14 textbooks or monographs, 11 in Serbian, and 3 published by Springer, J. Wiley and Sons, and Elsevier, devoted to fundamental mechanics and the finite element method. Kojic is the first President of the Serbian Society for Computational Mechanics and the first Editor of the Journal of the Serbian Society for Computational Mechanics. For his contribution to science, impact on the University of Kragujevac advances, industry, and society, academician Kojic earned numerous domestic and international awards and recognitions.

RADOVAN SLAVKOVIĆ

Born on 27.12.1952. Čumić, Kragujevac. Graduated from the Mechanical Engineering Faculty (today Faculty of Engineering), University of Kragujevac, in 1977, as the best student in his generation. At the same Faculty, he completed his MS studies in 1982 and Ph. D. in 1987. His diploma work, as well as both theses, were in the field of Applied Mechanics (Finite Element Method, FEM). He was employed at the faculty as a research associate in 1977, and a professor assistant 1978-1987; elected as docent 1987, associate professor 1993, and full professor from 1998. He was teaching Mechanics and several courses related to the FEM and its application. He retired in 2018.



He spent 17 months, starting in 1984, at MIT working with Professor K. J. Bathe within the grant of Professor M. Kojic (one year). He was implementing the “Effective Stress Function” concept, formulated in 1983. by M. Kojic, to large strains of the ADINA code, and further using this implementation as a basis for his Ph. D. thesis research.

Professor Slavković has been one of the principal developers of the PAK finite element code from its very beginning in 1976, when he joined the project as a fourth-year undergraduate student, and throughout the subsequent decades of its continuous development. His contributions have been both methodological, through the formulation of finite element procedures, and practical, through direct coding and software implementation within PAK. Over the years, his work has encompassed a wide spectrum of fundamental areas: the development and implementation concept of advanced material models, including plasticity of metals and geological materials; the large strain problems with various strain measures such as logarithmic strains; the formulation and improvement of finite elements with incompatible displacement modes; the development of beam elements with arbitrary cross-sections and shell elements tailored for complex structural analyses; as well as the extension of PAK to field and coupled problems, including specific constitutive models for shape memory alloys and polymers. In addition to his own research and technical contributions, Professor Slavković played a central role in coordinating various research topics within the PAK group, mentoring younger collaborators, and guiding the application of the software to complex engineering problems. Alongside the software work, he has co-authored a large number of publications in leading international journals, books, monographs, and technical solutions, and has contributed to the establishment of a strong research group within the Department of Mechanics. Professor Slavković, in collaboration with his colleague Professor Miroslav Živković, developed a module for the **PAK** software aimed at thermal analysis – **PAK-T**. The module was designed to perform advanced simulations of heat conduction problems, based on the finite element method (FEM). PAK-T enabled precise modeling of transient and steady-state thermal processes in complex engineering structures.

From 2000 to 2006, he served as Dean of the Faculty of Mechanical Engineering at the University of Kragujevac (today the Faculty of Engineering). During his tenure, a comprehensive reform of curricula was carried out through several international projects, resulting in their modernization and alignment with the European Higher Education Area. He acted as coordinator and principal investigator of numerous international projects, including TEMPUS, FP6, and FP7, with a strong focus on improving teaching quality, curriculum development, and strengthening research capacity. As professor and head of the institution, he established and developed long-term academic cooperation with prestigious European universities, including the National Technical University of Athens (NTUA), Universitat Politècnica de Catalunya (UPC), and Technische Universität Braunschweig (TUB). During his tenure as Dean, he spearheaded the reform of the BS/MS (3+2) curriculum through international projects and close collaboration with institutional partners, a framework that remains in use to this day.

He served as Head of the Department of Applied Mechanics and Automatic Control from 2008 to 2018.

MIROSLAV ŽIVKOVIĆ

Born on March 23, 1962, in Kragujevac, Serbia. Živković completed his studies at the Faculty of Mechanical Engineering, University of Kragujevac, in 1985, graduating as the best student of his generation and at the University level. At the same Faculty, he obtained his master's degree in 1989 and defended his doctoral dissertation in 1996 under the supervision of Prof. Miloš Kojić. Živković began his academic career as a teaching assistant at the same Faculty in 1986. He was promoted to Assistant Professor in 1997, to Associate Professor in 2001, and to Full Professor in 2007.

In 1984, he joined Prof. Kojić's team, working on the extension of the PAKT program (transient nonlinear heat conduction), with the significant help of Prof. Slavković. In the late 1980s, he developed the core PAKS program (structural analysis), and in the early 1990s, the basic PAKF program (laminar fluid flow). In 1999, during the NATO bombing, he merged the then-current versions of PAKS and PAKF into PAKFS (fluid–structure interaction). Since 2001, Živković has led a research team at the Laboratory for Engineering Software, continuing the development of the PAK software package for structural analysis. With continuous support from the Ministry of Science of the Republic of Serbia (2002–2023), the Laboratory has developed specialized PAK modules, including PAK-FM (fracture mechanics), PAK-FAT (fatigue), PAK-GEO (geomechanics), PAK-PT (seepage flow and heat transfer), PAK-E (electrostatics), STL2FEM (vascular modeling), PAK-Turbulent (fluid flow), PAK-EXP (explicit dynamics), SPH-LifeCycle (coupling with smoothed particle hydrodynamics), PAK-Testing (automatic module testing), and PAK-G (pre/post-processing). Together, these have evolved into PAK-Multiphysics. Through these projects, many young researchers completed their PhD studies. The most recent development, supported by the Serbian Science Fund through the PROMINENT project (2024–2026), is PAK-DAM, designed for predicting damage evolution in engineering structures. Živković has also played a significant role in more than ten international projects. In 2012, he founded the Center for Engineering Software and Dynamic Testing, which is equipped with a Hydrodynamic Pulsator, a Strain-Controlled Press, and a Hopkinson Tension Bar for testing material properties.

Over the course of his career, Živković has authored or co-authored more than 400 scientific papers in international and domestic journals and conference proceedings, including over 50 papers in SCI-indexed journals. He is the author of one monograph, co-author of three monographs and three textbooks, and has contributed to the development of fifteen technical solutions. He has supervised eight doctoral dissertations and mentored many master's theses. In addition, he has coordinated and managed numerous scientific projects and more than 300 applied projects for industrial purposes. He has collaborated with a large number of universities worldwide, where he presented the results of the Laboratory for Engineering Software and fostered international cooperation in computational mechanics.

Beyond research, Živković has held important academic and administrative positions. He served as Dean of the Faculty of Engineering, University of Kragujevac (2013–2016), and since 2018 has been Head of the Department of Applied Mechanics.



NENAD GRUJOVIĆ

He was born on June 5, 1966. In 1982, at age 16 and after completing his first year at Gymnasium in Kragujevac, he became the youngest university student in the country. He enrolled at the Faculty of Mechanical Engineering, University of Kragujevac, after passing the entrance exams with distinction. During his second year (1983/1984), he met Prof. Kojić and began a mentorship focused on the Finite Element Method (FEM) and computer programming. His dedication led to an early graduation, six months ahead of schedule, supported by a scholarship from the Serbian Academy of Sciences. In September 1986, he completed his diploma thesis, Application of FEM for Modeling Anisotropic Beams. Through intensive teamwork, he contributed to the design and development of the PAK software, a universal and unified programming platform, with emphasis on robust solution technologies for generally nonlinear problems. His Magisterium thesis (1989), Incremental and Iterative Methods in FEM, made a significant contribution to PAK with both methodology and efficient code, forming a foundation for future developers.



Further education abroad included: Harvard School of Public Health, USA (1990); National Technical University of Athens (2001–2006); Technical University of Braunschweig (2001–2012); and Technical University of Crete (2007–2012). His academic career began in 1987 as a teaching assistant in Mechanics. After receiving his PhD in 1996 related to Contact Problems by FEM, he became an assistant professor at the Faculty of Mechanical Engineering, University of Kragujevac, then associate professor in 2001, and full professor in 2006. He focused on Applied Mechanics, Applied Informatics, and Computer Engineering within the same department. From 2001 to 2004, he served as Vice Dean for Scientific Research at the Faculty of Mechanical Engineering.

He authored or coauthored more than 150 papers and 10 books, published by national and international publishers. In PAK development, he contributed to core modules such as TL/UL geometric nonlinearity, solver partitioning system and buffering for large systems of equations, multipoint constraint equation methodology, solvers for substructured model and partial system reduction, and numerous finite element types in the PAK library. In the early 1990s, he developed graphical postprocessing modules and, later, with young collaborators, a graphical preprocessor that led to the integration of PAK-G into the software package. He also worked on the development and implementation of general soil and rock mechanics material models in PAK, particularly for dam foundation analysis, in cooperation with Jaroslav Černi National Institute for Water Resources. This marked a shift toward hydroinformatics, with a focus on modeling water flows in natural environments.

Building on this expertise, in 2001, Prof. Grujović established the Center for Information Technologies (CIT), employing a number of young researchers and implementing EU-funded, national scientific, educational, and industrial projects. CIT introduced 3D printing technology in Serbia. It was used to produce a custom breastbone implant (in 2008) that saved a patient's life, considered the first of its kind worldwide. This was followed by the development of reinforced ribs in cooperation with the Clinical Center Kragujevac. Research in new bioengineering materials and technologies naturally followed. He remained active in FEM development, leading the creation of a concrete material model for dam simulations, participating in AI research for water level prediction, and working on large-scale dam model parameter identification. With the CIT team, he designed the prototypes of alternative ventilators in Serbia within the first month of the COVID-19 pandemic. He took part in over 20 international and national projects, coordinating six. In 2004, he coordinated the EU project Course Development Program: Rapid Prototyping with WUS Austria and received the WUS Award for Best Course in Serbia.. He founded the eLearning Centre at the University of Kragujevac and joined the Serbian Task Force for eLearning in 2006. His career reflects a unique blend of research, innovation, and mentorship, with lasting impact on both science and education.

NENAD FILIPOVIĆ

Born on 02/23/1970 in Kragujevac. Graduated from high school in Kragujevac in 1989 with a scholarship for young talents. Completed undergraduate studies (1994) and earned a doctorate (1999) at the Faculty of Mechanical Engineering in Kragujevac. The scientific field of the doctorate is Applied Mechanics and Automatic Control, with the doctoral dissertation titled "Numerical Solution of Coupled Problems of Deformable Bodies and Fluids."

Employment and career advancement: Elected as an assistant professor in 2000 at the Technical Faculty in Čačak, later advancing in the Faculty of Mechanical Engineering in Kragujevac (2001-2005, assistant professor; 2005-2010, associate professor; since 2010, full professor) in the fields of Applied Mechanics, Applied Informatics, Computer Engineering, and Bioengineering.

Further education abroad: University of Vienna (2001), Harvard School of Public Health, USA (2003-2012, several months annually), Steinbeis, Stuttgart (2006-2007) with a Humboldt Scholarship.

Leadership roles at the University of Kragujevac: 2012-2015, Vice Rector for International Cooperation; from 2018 to 2024, he served as Rector. Since 2024, he has been acting Director of the Science Fund of the Republic of Serbia.

He has published more than 400 peer-reviewed papers and 8 books in a world-leading publisher. His H index is more than 45, and he has more than 10,000 citations. Patent US 11872080 B1: "Multi-modal Heart Diagnostics System and Method" (UAH-P-20024), approved in the USA (United States application number 17/187,172, filed February 26, 2021). The application pertains to automated echocardiography diagnostics with three-dimensional imaging based on an ultrasound probe that moves along a specified path on the patient's chest.

He is the founder, together with academician Miloš Kojić, of the Bioengineering Research and Development Center (BIOIRC) in Kragujevac. Since 2003, with 10-30 young researchers, He has been the leader of 9 international projects funded under FP7, Horizon2020, Horizon Europe, Erasmus+, CEI, SCOPES; 7 national projects funded by the relevant ministry; leader of the Serbian team in 8 bilateral projects; leader of the Serbian team in 1 trilateral project; leader of project teams in Serbia for more than 30 international projects. Professor Filipović led the establishment of the Center for Bioengineering at the Faculty of Engineering, which now houses the Laboratory for Stent Design and Testing according to ISO 25539-2 standard and the Electrospinning Laboratory. He initiated four master's programs at the University of Kragujevac: Bioengineering, Information Technologies, Game Development, and Artificial Intelligence Development. He is also the founder of the Software Engineering and Computer Technique program at the Faculty of Engineering. As Rector, he contributed to founding the Institute of Information Technologies (2019), which now has 80 young researchers. He mentored 16 doctoral theses, and is currently supervising many doctoral students. Professor Filipović organized and participated in numerous international and national conferences, worldwide and in Serbia.

His awards and recognitions include: the Young Scientists Award at MIT Conference on Computational Fluid & Solid Mechanics, Boston, 2003; Certificates of Appreciation from the Faculty of Engineering and the Clinical Center of Kragujevac for COVID-19 efforts, 2020; the "Captain Misha Anastasijevic" Award for promoting scientific research in Serbia, 2020. He authored numerous software packages based on the finite element method and discrete methods, and is one of the main authors of the software package PAK; initially in fluid mechanics and solid-fluid interaction, later in bioengineering. He is the main author of PAK-F and PAK-FS and a model of atherosclerosis for problems in cardiovascular biomechanics, and PAK-DPD software for the discrete modeling of the fluid-structure interaction problem. He connected PAK software with AI algorithms for hybrid computational AI analysis.



B. CONTRIBUTING AUTHORS

1. Starting year of work on the PAK software: 1975.

He graduated from the Faculty of Electrical Engineering of the University of Belgrade in 1963, in the Department for Telecommunication and Electronics, and completed his MS studies in 1982 at the same Faculty. His MS thesis was: Transactions in real time on the computer system IBM/7.

From 1964 to 1971, Mr. Marinković was working at the department for design and testing of electrical equipment for automobiles, in the car factory “Crvena Zastava” at Kragujevac, with a focus on the jamming of radio signals within the car (he was a member of the international commission for this field). From 1971 to 1985, he was the principal software engineer in the development of the IBM/360 and IBM/370 system programs at the Computer Center of the Automobile Factory “Crvena Zastava”. Until 2001, he continued his work in the same field at the Factory for Special Use, related to the CADAM and HELIX systems. He passed away in 2015.

Mr. Marinkovic joined the PAK group from the beginning and, as a talented and enthusiastic software engineer, made a fundamental contribution to the formulation of the PAK initial structure and organization. He supported other mechanical engineers of the PAK growing group over many years in coding within the PAK software and the software applications.



Milutin Marinković

2. Starting year of work on the PAK software: 1979.

Born on 01. June 1955 in Kragujevac. Graduated in 1979, MS studies completed in 1995, and Ph. D. degree earned in 2003, all at the Mechanical Engineering Faculty in Kragujevac. He was employed at Factory for Special Structures “Zastava” in Kragujevac, 1979-1997, as an engineer. He was elected in 1997 as a professor and was working at the Higher Education Technical School in Čačak till his retirement in 2020. Dr. Vlastelica was also elected as an associate professor at Metropolitan University in Belgrade in 2015. He was teaching numerous courses, from general mechanics to numerical methods and their applications. Dr. Vlastelica initiated and was the editor of the School journal, Technique and Application in Practice. Also, he co-authored two School textbooks.

Ivo Vlastelica was one of the first generation engaged in early PAK development. He significantly contributed for decades, till 2015, in the development and application of the finite element methodology and solution of various problems, starting from heat transfer modeling, to the methods for the stress integration of inelastic material models of metals, and nonlinear biological materials; the element-free Galerkin (EFG) method, and doublet mechanics. He is the co-author of numerous national and international papers.



Ivo Vlastelica

3. Starting year of work on the PAK software: 1993.

Born on 02/04/1966 in Kragujevac. Graduated from the Military Academy Zagreb in 1990 with the thesis "Stress and strain analysis of a rocket engine nozzle by thermal loading in software COSMOS-T". He earned a master's degree from the Faculty of Mechanical Engineering, University of Kragujevac, in 1995 with a thesis: "Implementation of the Finite Element Method in the Thermoplastic and Creep of Metals" under the leadership of Professor Miloš Kojić. His first task in the PAK development was related to implementing the model for rod structures. He was then involved in programming the model for thermoplastic and creeping metal behaviour developed by Professor M. Kojić. This methodology, based on the Kojić "Effective-Stress-Function" was built into the continuum finite elements, shells, and isoparametric beams. He was the co-author of one of the articles related to this topic.



Dušan Begović

4. Starting year of work on the PAK software: 1994

Born in Belgrade on April 4, 1961. He completed his studies at the Faculty of Mechanical Engineering (FME) in Kragujevac in 1986, graduating under the mentorship of Prof. Miloš Kojić 1986. He was enrolled in postgraduate studies at the FME, majoring in Mechanics, and passed the exams with an average grade of 10. He defended his master's thesis entitled PROBLEMS OF STATICS AND DYNAMICS OF STRUCTURES CONTAINING DEFORMABLE AND RIGID BODIES BY THE FINITE ELEMENT METHOD; Mentor M. Kojić. After completing his studies, as a scholarship holder of "ZASTAVA AUTOMOBILI", he was employed at the OJ "Technical Affairs", and in 1988 at OJ "Institute for Automobiles", where he was engaged in research and development projects of designing and calculating parts and assemblies of passenger cars. While working at the Automobile Institute until 2006, his career ranged from project engineer, head of department, head of direction, deputy director, and director of the Institute. In the period until 2010, he was the CEO of the company ZASTAVA AUTO, which was founded on the basis of a contract with the Fiat company for the license and the production of the Punto vehicles at the factory in Kragujevac. From 2010 to the present day, he has been working as the Manager of General Affairs in the company Stellantis. Zoran had his first encounters with the PAK and FE software package in 1983 as a student associate, where he was engaged in the preparation and control of test examples of the package. Later, in the preparation of his graduation thesis, he was involved in the development of a mathematical model for the automatic generation of finite element networks, which are significant for the analysis of thermo-voltage models that arise in butt welding of materials. A significant contribution to the PAK software was achieved through his Master's thesis in the period 1994-1998, where a theoretical model and relations were developed to describe the mutual interaction within a mechanical system containing deformable and rigid bodies. The mathematical model was implemented into the PAK, with numerical examples created to illustrate the accuracy of the developed procedures. Then the procedure was generalized to the system of rigid bodies that make up a chain, and finally, it was considered a chain of rigid bodies within 2D and 3D systems. The results of the work were presented at several domestic and international conferences and published in domestic and international journals.



Zoran Bogdanović

5. Starting year of work on the PAK software: 1995.

S. Vulović was born on 27/05/1970 in Kragujevac. She graduated from the Faculty of Mechanical Engineering in 1994 and defended her MSc thesis at the Faculty of Mechanical Engineering in 1998, and defended her Ph.D. thesis at the Faculty of Mechanical Engineering, University of Kragujevac in 2008. From 1995 to 2008, she was a research assistant, and between 2013 and 2019, she was a research associate at the Faculty of Engineering, University of Kragujevac. From 2008 to 2012, she worked as an assistant professor at the Faculty of Information Technology, Metropolitan University (Belgrade). Since 2019, she has worked at the Institute for Information Technologies, University of Kragujevac. She worked as a research associate from 2019 to 2024. Since 2024, she has worked as a senior research associate. She has authored over 90 scientific publications, including 15 papers in international journals and 4 registered technical solutions, and she has more than 100 citations.



Snežana Vulović

In 1995, she began working on the development of PAK software at the Engineering Software Lab (Faculty of Engineering, University of Kragujevac) under the leadership of M. Kojić. Her first task was developing a new program for fluid flow through porous media (later called PAK-C). She defended her thesis entitled Fluid Flow through Porous Media with Heat Transfer (PAK-CT).

6. Starting year of work on the PAK software: 1996.

Boban Stojanović earned his degree from the Faculty of Mechanical Engineering in Kragujevac, specializing in Applied Mechanics and Automation, graduating in 2002. He defended his final thesis, titled “Formulation of Solid Finite Elements Using the Assumed Strain Method,” with the highest grade. In 2007, he obtained his PhD in Computer Modeling from the Center for Interdisciplinary and Multidisciplinary Studies and Research at the University of Kragujevac, with a doctoral dissertation titled “Generalization of the Phenomenological Hill Model for the Study of Muscle Fatigue.” From 2003 to 2008, he worked as a research associate at the Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac. In 2003 and 2005, he held research positions at the Hong Kong Polytechnic University. In 2007, he was a visiting scholar at Harvard University. Since 2008, he has been employed at the Faculty of Science, University of Kragujevac, where he currently holds the position of Full Professor in the field of computer sciences. Boban Stojanović began contributing to the development of the PAK system as a freshman at the Faculty of Engineering, where he worked on PAK-G, a graphical user interface for finite element modeling. During his studies, he focused on the automated generation of 3D finite element meshes for various real-world problems. Alongside his research in numerical modeling, he was also involved—during his PhD studies—in the development of several software solutions for both academic and industrial applications. As part of his doctoral research, Boban developed a novel numerical material model of muscle tissue that incorporates fatigue and multifiber phenomena, successfully integrating it into the PAK finite element solver. In the years that followed, his team at the Faculty of Science developed an advanced multiscale muscle model, which has also been implemented within the PAK solver.



Boban Stojanović

7. Starting year of work on the PAK software: 2002

Born on 18/03/1967 in Čačak. Completed undergraduate studies (1990), master's studies (1996), and a doctorate (2005) at the Faculty of Mechanical Engineering – University of Kragujevac. She was elected at the Faculty of Mechanical Engineering in Kragujevac as: 2006-2011, assistant professor; 2011-2016, associate professor; since 2016, full professor, in Applied Mechanics. The doctorate was in Applied Mechanics and Computational Fracture Mechanics and Fatigue. Recently, her area of interest has been the application of Computational Fracture Mechanics and Fatigue in Biomedical Engineering. Her specialty is the application of fatigue to fracture analysis and integrity assessment in dental medicine, orthopedics, and also in the design of biomedical devices (such as cardiovascular stents).

She has authored over 100 scientific papers, one university textbook, and one monograph; and is co-author of two patented technical solutions resulting from the development of the PAK.

Within the PAK package, she has implemented and verified classical and advanced numerical methods of Computational Fracture Mechanics. The J-Equivalent Integral Method is used to calculate the basic parameters of linear elastic and elasto-plastic fracture mechanics, such as the J-integral and Stress Intensity Factors. Special attention in the development of the PAK software is devoted to automatic simulation of crack growth on a permanent mesh of finite elements. This was achieved by incorporating eXtended-Finite Elements in the PAK. By applying the XFEM method, it is possible to estimate the fatigue crack growth and the residual fatigue life of highly critical constructions in engineering.



Gordana Jovičić

8. Starting year of work on the PAK software: 2003.

Vladimir Ranković earned a B.Sc. in mechanical engineering at the Faculty of Mechanical Engineering, University of Belgrade, in 2002, and a Ph.D degree in Computer modeling (Applied computing) at the Center for Interdisciplinary and Multidisciplinary Studies and Research, University of Kragujevac, in 2007.

During his participation in the development of the PAK program, at Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac (2003-2008), he was involved in the development and implementation of material models of the arterial wall, the Nitinol material from which stents are made, and modeling and simulations of the behavior of complex structures composed of muscle tissue, an artery with an embedded stent, and blood. He was one of the leading members of the team that developed software for modeling groundwater flow with special application to the management of water sources based on wells with lateral screens. Since 2008, he has been employed at the Faculty of Economics, University of Kragujevac. He is currently a full professor of Applied Computer Science with expertise in the development of software based on multi-objective optimization, machine learning, and artificial intelligence techniques.



Vladimir Ranković

9. Starting year of work on the PAK software: 2003.

Miloš Ivanović (born 1978) obtained his MSc in Physics (2008) and PhD in Computer Science (2010) from the Faculty of Science, University of Kragujevac. From 2003 to 2008, he worked as a researcher in the University's Center for Supercomputing and later in the Bioengineering programme of the Serbian Academy of Sciences and Arts and the University of Kragujevac. His first supercomputing project involved adapting PAK-F to simulate airflow and particle deposition in lung structures. Since 2009, Ivanović has been teaching Computer Science at the Faculty of Science, where he is now a Full Professor. His courses cover Parallel and Distributed Computing, Operating Systems Design, Computer Networks, and Cybersecurity. During his PhD, he conducted research at Cranfield University with Professor Rade Vignjević and at the University of Braunschweig under Professor Hermann Matthies, focusing on distributed parallel computing for Smoothed Particle Hydrodynamics (SPH). After 2010, his research centered on high-performance computing (HPC), parallel algorithms for SPH, and dissipative particle dynamics. Between 2011 and 2013, he developed neutron track simulations and hybrid MPI-CUDA codes for muscle modeling and hydrological computing using PAK-P. From 2014 to 2016, he expanded into evolutionary algorithms, muscle contraction modeling, and graph optimizations. Since 2017, Ivanović has integrated machine learning with HPC, notably applying neural networks to optimize domain decomposition in muscle simulations. He also contributed to advancing PAK, incorporating surrogate models for cardiac and muscle simulations, widely used in several Horizon projects as a reliable biomedical modeling tool. Most recently, from 2023 onward, his research focuses on Physics-Informed Neural Networks (PINNs) for solving complex PDEs, such as Burgers' and Sine-Gordon equations, and soil diffusion. He has authored 45 publications, with 620 citations and an h-index of 13.



Miloš Ivanović

10. Starting year of work on the PAK software: 2011.

Born 28 June 1979 in Jagodina, Serbia. He earned a B.Sc. in Mechanical Engineering (2004), an M.Sc. (2009), and a Ph.D. in Computational Geomechanics (2014) at the University of Kragujevac. Since 2005, he has worked in the Engineering Software Lab, teaching mechanics and the finite-element method using PAK (assistant professor 2015; associate professor 2020). His focus is on numerical methods in geotechnical engineering, particularly constitutive modelling of geomaterials and implicit stress-integration algorithms. He has authored over 60 scientific papers, one university textbook, and one supplementary textbook, and is co-author of three patented technical solutions. He has participated in nationally funded projects (Ministry of Science and the Science Fund of the Republic of Serbia) and international collaborations. Within the PAK finite-element package, he has implemented and verified advanced soil and rock models and conducted PAK-based analyses for the Jaroslav Černi Water Management Institute, including stability studies of embankment and concrete dams, tunnels, and other geotechnical structures. Current interests include finite-element methods for material modelling and developing robust algorithms for large-scale geomechanical simulations.



Dragan Rakić

11. Starting year of work on the PAK software: 2006.

He graduated from the Faculty of Mechanical Engineering, University of Kragujevac, in 2006 and completed PhD studies in bioinformatics at the Faculty of Information Technology, University Metropolitan, Belgrade in 2012. Currently, he is an associate professor at the Faculty of Engineering - University of Kragujevac (former Faculty of Mechanical Engineering). His professional career started in 2006 at the Center of Scientific Research Work of the Serbian Academy of Sciences and Arts and the University of Kragujevac. During the next two years, he started to get involved in PAK development. The first significant project he worked on was the development of a module for modeling the motion of elastic bodies in a fluid flow using a strong coupling approach, which led to his Ph.D thesis. In parallel, he worked on the development of a module for modeling groundwater flow used for the analysis, assessment, and prediction of the state of the Sava River's alluvial aquifer – primary Belgrade's water source, under the authority of the Jaroslav Černi Water Institute LLC Belgrade. From 2008 to 2017, he continued the development of PAK in the Bioengineering Research and Development Center – BioIRC. He developed a new module for acoustics and acoustic–structure interaction, including solvers for the wave equation strongly coupled with solid dynamics using tetrahedral and prismatic finite elements. He also created a module for simulating stent deployment in blood vessels, addressing contact mechanics between independent solid domains. He has published over 10 SCI-indexed journal papers, more than 10 in national journals, five monograph chapters, two textbooks, and over 50 conference papers.



Velibor Isailović

12. Starting year of work on the PAK software: 2008.

V. Dunić was born on 12/09/1983 in Pirot. He earned his diploma from the Faculty of Mechanical Engineering in 2008 and completed his Ph.D. at the Faculty of Engineering, University of Kragujevac (FINK), in 2015. From 2008 to 2010, his work was supported by the Ministry of Science through a program for top Ph.D. students. Between 2011 and 2017, he served as a research assistant at FINK, becoming an assistant professor of Applied Mechanics in 2017 and an associate professor in 2021.

He began developing PAK software in 2008 under the mentorship of R. Slavković. His first task involved implementing the constitutive model for Shape Memory Alloys (SMA). In collaboration with N. Busarac and researchers from TU Braunschweig (B. Rosić, R. Niekamp, and H. Matthies), he worked on thermo-mechanical coupling between PAK-S and PAK-T. The SMA model was verified through comparison with experimental data obtained during his 2013 stay at IPPT, PAN in Warsaw, Poland. These results were published in journals and included in his Ph.D. thesis, co-supervised by R. Slavković and E. Pieczyska (IPPT, PAN). As a postdoc, he continued PAK-S development with N. Grujović, implementing the Concrete Damage Plasticity (CDP) model for projects with the Jaroslav Černi Institute. With his Ph.D. student J. Živković, he implemented the phase-field damage model (PFDM) into PAK-S. Within the DEEDS project (SANU–University of Kragujevac, 2023–24), he led the development of High- and Low-cycle fatigue procedures in PAK. In 2023, he spent three weeks at Aichi Institute of Technology, Japan, collaborating with R. Matsui on subloop cyclic behavior of SMA, following publication on SMA damage simulations. In the Science Fund of Serbia project PROMINENT (2024–26), he is work-package leader for PAK-DAM software, aimed at predicting structural damage. He has authored over 10 international journal papers and a textbook Dynamics of Structures (2025), featuring PAK examples.



Vladimir Dunić

13. Starting year of work on the PAK software: 2008.

Born on 11/01/1984 in Kragujevac. Graduated from high school in Kragujevac in 2003. Completed undergraduate studies (2008) and earned a doctorate (2012) at the Faculty of Engineering, University of Kragujevac. The scientific field of the doctorate is Informatics in engineering, with the doctoral dissertation titled "Numerical modelling of diffusion in composite systems". Elected as an assistant professor in 2013 at the Belgrade Metropolitan University (2013-2018, assistant professor; 2018-2023, associate professor; since 2023, full professor) in the fields of Information technologies and systems. He worked as a research associate and scientific associate in the Bioengineering Research and Development Center (BIOIRC) in Kragujevac (2008 – 2019), where he participated in a number of international scientific EU projects. He is currently an external collaborator of BIOIRC. He is also a research associate at the Institute for Information Technologies, University of Kragujevac (since 2019). Leadership roles at the Institute for Information Technologies: Head of the department for technical and technological sciences, 2021-2023; Assistant director for scientific and technical cooperation, since 2024. He has published more than 100 peer-reviewed papers and 1 book in world-leading publishers. His H index is 19, and he has more than 800 citations.



Miljan Milošević

Miljan Milošević participated in the development of numerous software packages based on the finite element method (PAKS, PAKE, PAKC, PAK-FS, PAKT, PAK-FIS, PAK-KTM). He participated in the development of computational methods devoted to nanomedicine and mass transport within tissue (PAK-KTM), which is summarized in the book published in 2022 by Elsevier, with Miljan Milosevic as a co-author. The central topic is the formulation and application of the multiscale smeared concept (Kojic Transport Model – KTM) for the gradient-driven physical fields within tissue. Miljan Milošević is currently a leading software developer of the user interface software CAD Field & Solid, for modelling and visualising the results of PAK FE software.

14. Starting year of work on the PAK software: 2009.

Marko Topalović was born on 01/01/1981 in Kragujevac. He graduated from the Faculty of Mechanical Engineering in 2006 and defended his Ph.D. thesis at the Faculty of Engineering, University of Kragujevac, in 2016. Between 2011 and 2017, he was a research assistant, and between 2017 and 2019, he was a research associate at the Faculty of Engineering, University of Kragujevac. Since 2019, he has worked as a research associate at the Institute for Information Technologies, University of Kragujevac.

He has authored over 70 scientific publications, including 15 papers in international journals and 5 registered technical solutions, and he has more than 100 citations.

In 2009, he began working on the development of PAK software at the Engineering Software Lab (Faculty of Engineering, University of Kragujevac) under the leadership of M. Živković. His first task was developing a new translator program that creates a PAK input file within the FEMAP pre-post processing software. With his mentor M. Živković, he improved the memory handling of large matrices. Another contribution was the development of new project solutions and makefiles for PAK compilation in Windows and Linux operating systems. His primary research focus is on meshless numerical methods like SPH, so for his Ph.D. thesis, he coupled the SPH solver MCM with PAKS, which improved PAK-Multiphysics capabilities.



Marko Topalović

15. Starting year of work on the PAK software: 2010

Born on 09/26/1985 in Kragujevac. Graduated from high school in Kragujevac in 2004 with the highest grades. Obtained the title of Graduated Mechanical Engineer in 2010, with achieved average grade of 9.91 (of 10), at the Faculty of Mechanical Engineering in Kragujevac, Department for Applied Mechanics and Automatic Control. Earned a doctorate (2017) at the Faculty of Engineering, University of Kragujevac, with achieved average grade of 10 (of 10). The doctoral dissertation title is "Electro-mechanical cochlea model and response analysis." Promoted by the Ministry of Education, Science and Technological Development of the Republic of Serbia with the title Research Associate in 2018. Elected as an Assistant Professor in 2024 at the Faculty of Mechanical and Civil Engineering, Kraljevo, University of Kragujevac, in the field of Applied Informatics in Engineering.



Milica Nikolić

Further education abroad: student exchange, Faculty of Electrical Engineering and Computer Science, FERI, University of Maribor, Slovenia (2009, two months), and postdoc at University of Technology, Department of Biomedical Engineering, Eindhoven, the Netherlands (2020-2022). She has published her scientific research in national and international journals and presented her work in conferences and scientific meetings of national and international significance. In total, more than 70 publications, H index 6, and 112 citations. She participated in national and international scientific EU projects.

Milica Nikolic develops numerical simulations based on the finite element method and discrete methods. She implemented in the software package PAK-C the acoustic wave equation. Also, she was developing in the PAK software package analysis of epicardium ablation process, sedimentation process, fluid behavior in bioreactors, and axonal elongation.

16. Starting year of work on the PAK software: 2011.

Born on 16/08/1983 in Kragujevac. Graduated from the Faculty of Mechanical Engineering in 2007 and defended his Ph.D. thesis at the Faculty of Engineering, University of Kragujevac, in 2018. From 2008 to 2011, he had a Ph.D. scholarship from the Ministry of Education, Sports, and Technological Development. Between 2011 and 2019, he was a research assistant at the Faculty of Engineering, University of Kragujevac. Since 2019, he has worked as a research associate at the Institute for Information Technologies, University of Kragujevac. Dr. Nikolić has authored more than 15 peer-reviewed journal articles, more than 20 conference papers, and 3 technical patents.



Aleksandar Nikolić

In 2011, he began working on the development of PAK software at the Engineering Software Lab (Faculty of Engineering, University of Kragujevac) under the leadership of Prof. M. Živković. The main role is to develop new computational models for the PAK-F software. The result was a Ph.D. thesis entitled: Simulation of laminar and turbulent flow on a realistic model of arterial bifurcation with stenoses. PAK-F software was expanded to new turbulent models as a part of his doctoral research, which is then used in various research projects. Recent research is focused on computational models in bioengineering, especially blood flow in arteries, finite element models for drug delivery in tumors, and organs.

17. Starting year of work on the PAK software: 2015.

Born on 08/18/1991 in Kragujevac. He completed his secondary education in Kragujevac in 2010 as a scholarship recipient from the Ministry of Education. He earned his bachelor's degree in 2016 and obtained his Ph.D. in 2023 from the Faculty of Mechanical Engineering, University of Kragujevac. His doctoral dissertation, titled "Analysis and Application of Smeared Methodology in the Modelling of Transport within Deformable Biological Systems", focused on advanced computational modeling in bioengineering. During his PhD studies, he worked as a Research Associate at the Research and Development Center for Bioengineering (BioIRC) in Kragujevac. He later held the position of Research Assistant at the Faculty of Mechanical Engineering (2017–2020), followed by a Research Associate role at the Institute for Information Technologies (2021–2023), where he is currently employed. Dr. Simić has authored over 30 peer-reviewed journal articles and more than 40 conference papers. His work has garnered an h-index of 11 and over 350 citations. He has participated in three international and one national research project. His primary research interests include the development and application of smeared methodologies for modeling diffusion, convection, and mechanical properties in large biological systems, as well as multi-scale finite element models for drug delivery in tumors.



Vladimir Simić

Dr. Simić has contributed to the development of several finite element-based software tools, notably as one of the key authors of PAKFiS (Fields and Solids), designed for numerical simulations in bioengineering. He is also a principal contributor to PAK-KTM (Kojić Transport Model), a software tool for modeling convective-diffusion transport in complex biological systems such as tumors and organs. Additionally, he developed the PAK Smeared Mechanics module as part of his doctoral research.

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18. Starting year of work on the PAK software: 2016.

Bogdan Milićević is a research associate at the Institute for Information Technologies, University of Kragujevac, Serbia. He received his M.Sc. degree in Computer Science (2016) from the Faculty of Science, University of Kragujevac, and completed his Ph.D. in Mechanical Engineering (2023) at the Faculty of Engineering, University of Kragujevac. Doctoral thesis, titled Surrogate Muscle Models Based on Artificial Neural Networks with Applications in Finite Element Analysis, reflects his expertise in coupling numerical analysis with machine learning.

Bogdan Milićević has contributed to the development and application of the PAK finite element solver in areas related to mass transport, fluid flow, and biomechanical modeling. His work includes creating and testing models for simulating diffusion- and convection-driven transport in biological tissues,

using the Kojić Transport Method (KTM). This framework allows simulation of transport across capillaries, interstitial space, and cellular membranes, and has been used in studies of drug delivery and tissue perfusion. In his work with the PAK solver, Bogdan Milićević contributed to the coupling of cardiac muscle micromodels with the macroscopic finite element model of the heart wall. To improve computational efficiency, Bogdan integrated surrogate muscle models into the solver, supporting the application in cardiac disease modeling. He has also developed modules for 3D modeling of the left ventricle based on echocardiographic imaging data, and he worked on several geometric algorithms within the CAD environment, which include procedures for surface fitting, mesh generation, geometric interpolation, etc.



Bogdan Milićević

C. COMPLETE LIST OF PARTICIPANTS IN THE PAK DEVELOPMENT AND APPLICATION

In this section it is given a list of contributors to the PAK development and/or application. For the PAK developers, the packages are displayed, while for PAK users it is shown the field of application. The order of contributors is set according to the year of the start of participation in the PAK development or its use.

	Name	Years of Engagement	Field/package	Comment
1.	Miloš Kojić	1975 –	PAK-S, PAK-T, PAK-F, PAK-FIS, PAK-KTM	Initiator and Principal Investigator of PAK
2.	Radovan Slavković	1975 –	PAK-S, PAK-T, PAK-F, PAK-FM, PAK-GEO	The main contributor (Co-author) in all fields in PAK-S M.S. and Ph.D. Mentor M. Kojić
3.	<u>Milutin Marinković</u>	1975 - 2000	PAK-S	The main contributor to software development
4.	Đuro Pavić	1975-1985	Engineering	First diploma work with R. Slavković, Mentor M. Kojić
5.	Dragoljub Grujović	1978-1985	Engineering	First M.S. thesis, use of PAK-S Mentor M. Kojić
6.	Gradimir Živković	1979-1985	Field models	Heat conduction M.S. thesis, Mentor M. Kojić
7.	Vera Nikolić	1979-1981	PAK-S	Automatic model of gear tooth M.S. thesis, Mentor M. Kojić
8.	Aleksandra Janković	1979-1983	PAK-S	Shell finite element M.S. thesis, Mentor M. Kojić Ph.D. Mentor Mladen Berković
9.	Ivo Vlastelica	1979-2000	PAK-S and PAK-F	Several material models, heat conduction, fracture mechanics
10.	Živadin Micić	1982-1986	PAK-S	Shell finite element M.S. thesis, Mentor M. Kojić
11.	Živomir Petronijević	1980-2000	Engineering	Acoustics in cars, use of PAK Ph.D. at ETF Belgrade
12.	Radmilo Savić	1980-1982	Engineering	Model of car joints M.S. Mentor M. Kojić
13.	Dragoljub Đorđević	1980-1982	Engineering	Dynamics in FE M.S. Mentor M. Kojić
14.	Milan Milovanović	1980-1990	Engineering	Models of cars supporting

			M.S. Mentor M. Kojić
15. Miroslav Žilvković	1983-	PAK-S, PAK-T, PAK-F, PAK-FS, PAK-PT, PAK-FM, PAK-FAT, PAK-E, PAK-EXP, PAK-GEO, PAK-G, PAKF-Turbulent, PAK-Test, SPH-LifeCycle, PAK-Multiphysics, PAK-DAM	The main contributor (Co-author) in all fields in PAK-S, PAK-PT M.S. and Ph.D. Mentor M. Kojić
16. Nenad Grujović	1983-	PAK-S, PAK-T, PAK-F, PAK-FM, PAK-GEO, PAK-G, PAK-Multiphysics.	Main contributor (Co-author) in all fields in PAK-S. M.S. and Ph.D. Mentor M. Kojić
17. Vladislav Manojlović	1988-2000	Engineering	Acoustics in cars, use of PAK. Ph.D. Mentor M. Kojić
18. Dušan Begović	1988-1995	PAK-S	Plasticity and creep model. M.S. Mentor M. Kojić
19. Zoran Bogdanović	1994-1998	PAK-S	Rigid body model. M.S. Mentor M. Kojić
20. Nenad Miloradović	1989	Engineering	Application to a special structure. M.S. Mentor M. Kojić
21. Milenko Jovanović	1989	Engineering	Application to a special structure. M.S. Thesis Mentor R. Slavković
22. Jugoslav Đorđević	1989	Engineering	Application to a special structure. M.S. Thesis Mentor R. Slavković
23. Radovan Petrović	1989	Engineering	Model of wagon supporting element. M.S. Thesis Mentor M. Kojić
24. Milan Stanković	1989	Engineering	Seismic analysis of a supporting structure. M.S. Thesis Mentor R. Slavković
25. Milica Todorović	1990	Engineering	Application of triangular elements. M.S. Thesis Mentor M. Kojić
26. Nenad Filipović	1992-	PAK-F, PAK-C, PAK-FS	Main contributor (Co-author) in fluid mechanics, coupled problems, and biomechanics. Ph.D. Mentor: M. Kojić
27. Nebojša Zdravković	1995-2000	PAK-FS	Biomechanical models. Ph.D. Mentor: M. Kojić

28.	Dejan Divac	1995–	PAK-S, Engineering	Cracked rock mass model, dams, tunnels
29.	Snežana Vulović	1995–	PAK-S, PAK-PT, PAK-FM, PAK-FAT, PAK-Multiphysics	Flow through porous media, Penalty Contact. M.S. Mentor: M. Kojić, Ph.D. Mentor: M. Živković
30.	Nikola Kojić	2001–2015	Bioengineering	Application of PAK in biomedical problems. Mentors: Gareth McKinley (MIT), Jeffrey Drazen (Harvard)
31.	Gordana Jovičić	2002–	PAK-FM, PAK-FAT	Fracture mechanics (XFEM). Ph.D. Mentor M. Živković
32.	Snežana Vrekić	2002	Engineering	Thin-walled beam structures. M.S. Mentor M. Živković
33.	Nikola Milivojević	2005–	PAK-G	Pre and post-processor, Ph.D. Mentor N. Grujović
34.	Boban Stojanović	2005–	PAK-FS, PAK-G	Biomechanical models, Muscle model, and molecular processes. Ph.D. Mentor: M. Kojić
35.	Miloš Ivanović	2005–	PAK-FS	Biomechanical models, coupled with other packages. Ph.D. Mentor: B. Stojanović
36.	Vladimir Ranković	2005–2008	PAK-FS	Biomedical models. Ph.D. Mentor: M. Kojić
37.	Dragan Rakić	2005–	PAK-S, PAK-GEO, PAK-Multiphysics, PAK-DAM	Geomechanical models. Ph.D. Mentor: M. Živković
38.	Rodoljub Vujanac	2005–	Engineering	Carrying elements, semi-rigid connections. M.S. Mentor R. Slavković, Ph.D. Mentor M. Živković
39.	Velibor Isailović	2006–	PAK-C, PAK-FS	Motion of deformable solids in fluid, remeshing, and contact models. Ph.D. Mentor M. Kojić
40.	Marina Vuković	2006–2007	Engineering	Spot-welded joints, gap element. M.S. Mentor M. Živković
41.	Nenad Bušarac	2006–2016	PAK-S, PAK-PT	Solvers, parallelization, coupling. M.S. Mentor M. Živković
42.	Dejan Petrović	2007–2013	PAK-C, CAD	Corrosion model. Ph.D.

				Mentor N. Filipović
43.	Vladimir Milovanović	2008–	Engineering	Special structures, fatigue problems. Ph.D. Mentor G. Jovičić
44.	Vladimir Dunić	2008–	PAK-S, PAK-T, PAK-Multiphysics, PAK-DAM	Material models with memory (SMA), damage plasticity, phase-field, thermo-mechanical coupling, and Dynamics. Ph.D. Mentor R. Slavković
45.	Miljan Milošević	2008–	PAK-FIS, PAK-KTM, CAD	Ph.D. Mentor: N. Filipović
46.	Vukašin Slavković	2009–	PAK-S	Crystal plasticity, polymer models. Ph.D. Mentor N. Grujović
47.	Dejan Krsmanović	2012–	PAK-F	Bioengineering problems. Ph.D. Mentor N. Filipović
48.	Dejan Veljković	2012–	Engineering	Hyperelastic material model. Ph.D. Mentor N. Filipović
49.	Arturas Ziemys	2012–2023	Bioengineering	Houston Methodist Research Institute, PAK-FIS, PAK-KTM
50.	Marko Topalović	2012–	PAK-Multiphysics, PAK-Test, SPH-LifeCycle	Coupling with the SPH method. Ph.D. Mentor: M. Živković
51.	Aleksandar Dišić	2012–2020	Engineering	Large strain analysis. Ph.D. Mentor M. Živković
52.	Aleksandar Nikolić	2012–	PAKF-Turbulent	Turbulent fluid flow. Ph.D. Mentor M. Živković
53.	Jelena Živković	2013–	PAK-S	Phase-field damage modeling. Ph.D. Mentor V. Dunić
54.	Tijana Djukić	2015–	PAK-F	Lattice-Boltzmann method. Ph.D. Mentor N. Filipović
55.	Radun Vulović	2015–	PAK-C, CAD	Image recognition. Ph.D. Mentor: N. Filipović
56.	Miloš Radović	2015–	PAK-C, PAK-F	Optimisation problem, AI. Ph.D. Mentor N. Filipović
57.	Vladimir Simić	2015–	PAK-FIS, PAK-KTM, CAD	Bioengineering problems. Ph.D. Mentor N. Filipović
58.	Bogdan Milićević	2016–	PAK-FIS, PAK-KTM, CAD	Bioengineering problems. Ph.D. Mentor N. Filipović

59.	Milica Nikolić	2017–	PAK-F, PAK-C	Bioengineering problems. Ph.D. Mentor N. Filipović
60.	Žarko Milošević	2018–	PAK-F, CAD	Bioengineering problems. Ph.D. Mentor N. Filipović
61.	Igor Saveljić	2018–	PAK-F	Bioengineering problems. Ph.D. Mentor N. Filipović
62.	Tijana Geroski (Šusterčić)	2023–	PAK-FS	Bioengineering problems. Ph.D. Mentor N. Filipović
63.	Aleksandra Vulović	2023–	PAK-C	Bioengineering problems. Ph.D. Mentor N. Filipović
64.	Smiljana Tomašević	2023–	PAK-FS	Bioengineering problems. Ph.D. Mentor N. Filipović

ELEMENTS IN PAK

Finite elements available in the PAK system are represented in this section. Elements are grouped according to their formulation. Minimal data are given to provide the basic information, without detailed descriptions, which, for a “standard” form, can be found in textbooks. For the elements that are unique and specific to the PAK program, additional brief comments are given; complete descriptions are available in the references of the PAK authors.

1. One-dimensional (1D) elements

These elements are displayed in Fig. 1. Fig. 1a shows a 1D (truss for the full cross-section, and a pipe for the circular hole in the cross-section). The nodal variables can be displacements or velocities. The 1D element in mechanics has a broad use, including modeling of contact between deformable bodies, or ropes (with resistance to stretch or to compression only). The 1D element in Fig. 1b displays a 1D

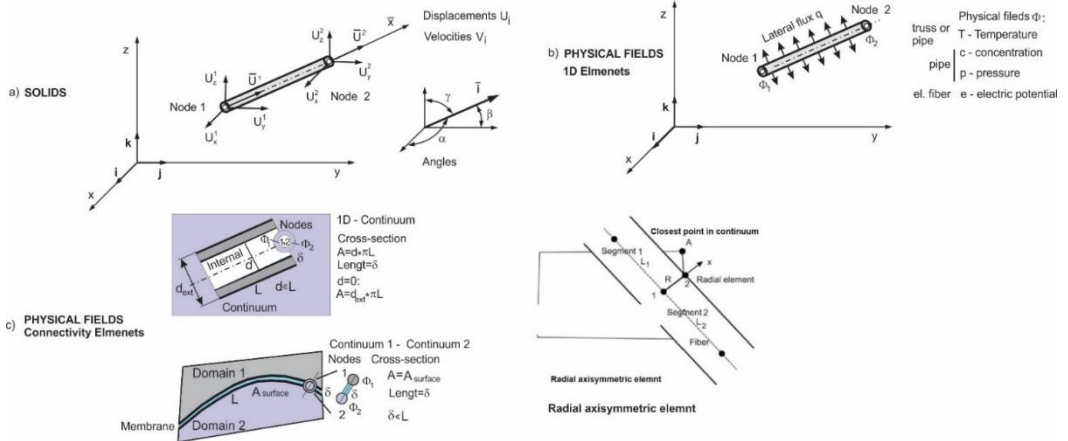


Fig. 1 One-dimensional finite elements. a) Truss (1D) element with 3 degrees of freedom per node (displacements or velocities); b) The element with the axial change of the physical field and radial flux; c) connectivity element used for modeling flux between two domains, in case of boundary between 1D and continuum, between two continua separated by a membrane, and radial axisymmetric element for mass release from fibers to continuum (according to [1],[3],[4])

element for gradient-driven physical fields. Note that there can be a lateral flow, including also mass transport in case of a hollow cross-section and porous wall. Besides the axial, the lateral flow is included in the element balance equation. The connectivity element is specific for the PAK, shown in the case of connecting 1D and continuum, and two continua. This element has a cross-section equal to the surface belonging to the node.

2. 3D and 2D continuum and 3D composite elements

Here, are first shown continuum elements in PAK, with a “standard” isoparametric formulation in the case of 3D and 2D conditions. These elements include mechanics for solid and fluid, and further physical fields. Then, two 3D composite elements are presented: the composite smeared finite element (CSFE) and the FE for the lung.

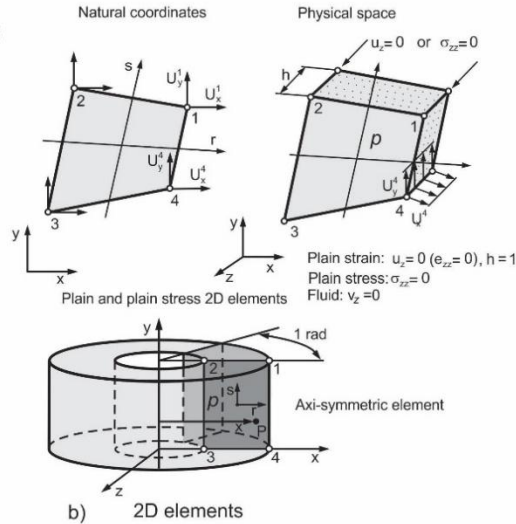
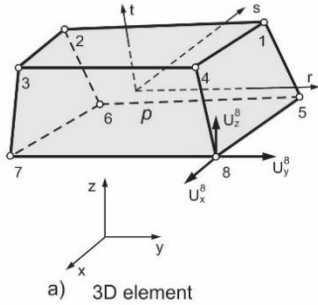
2.1 Isoparametric elements

Isoparametric elements for solids and fluids, according to a “standard” formulation present in textbooks on finite element method, are displayed in Fig. 2a for 3D and in Fig. 2b for 2D conditions. The 2D elements include plane strain, plane stress, and axisymmetric conditions in the case of the solid.

SOLID or FLUID

Nodal variables:

SOLID - displacements U_i and/or velocities V_i
 FLUID - velocities V_i and
 pressure p as element variable
 for mixed formulation



Physical fields Φ :
 T - Temperature
 p - pressure
 c - concentration
 e - electric potential

PHYSICAL FIELDS

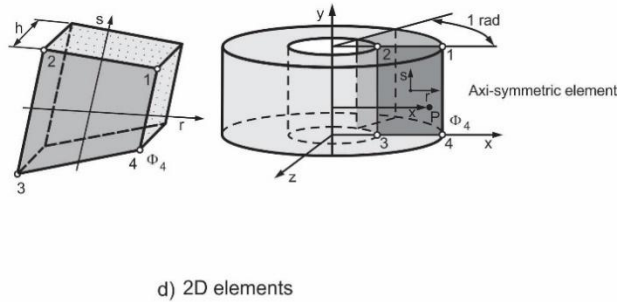
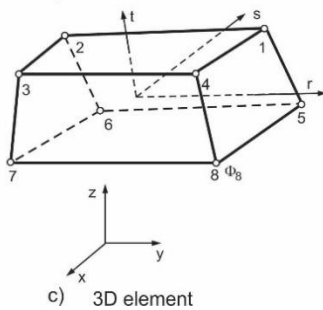


Fig 2 Continuum isoparametric elements. a) Eight-node element for solid and fluid. In the case of fluid and mixed formulation, an additional variable is the pressure p as the element quantity; b) 2D elements, for solid – plane strain, plane stress, and axisymmetric conditions; Physical fields: c) 3D element; d) 2D elements. (according to [3])

For the fluid and 2D elements, the fluid velocity is equal to zero in the z -direction, and circumferential direction in the case of the axisymmetric element. There is an additional variable – fluid pressure p when the mixed formulation is used. For the physical fields, the 3D element is shown in Fig. 2c, while in the case of 2D conditions (Fig. 2d) the field is uniform in the direction normal to the element, or the radial planes of the axisymmetric element.

2.2 Rigid body element

This element is designed for modeling parts of the structure whose deformations can be neglected with respect to the other parts. The element contains a reference (center) point and other necessary points for connection to other elements of the structure (Fig.3). The element formulation relies on the kinematics of rigid bodies and does not require numerical integration; hence the element is numerically efficient and only has translation of the center and rotation around the center as degrees of freedom.

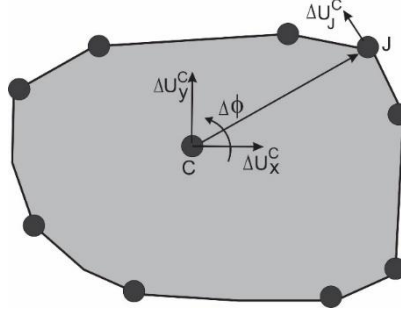


Fig. 3 Rigid body element. 2D representation of the increment of displacements of the body. A node J has a translational displacement of the center C and a relative rotational displacement. Hence, a 3D rigid body element has 6 degrees of freedom: 3 translational of the center C and 3 rotational of the body.

2.3 Composite smeared finite element (CSFE)

The CSFE is developed for modeling mass transfer and electrical signals in tissue, but it can be used for any gradient-driven physical field in a composite medium. It relies on the smeared concept and represents the basis for the Kojic Transport Model (KTM). The smeared fields are coupled by the connectivity elements at the FE nodes, as shown in Fig. 4. The CSFE has a multiscale character since it can have domains at different length scales. The volume of each domain K is specified by the volumetric fraction r_V^K while the cross-section of the connectivity element at a node J is determined by the surface area belonging to the node, as $A_{surface}^J = r_V^{K-1} r_{AV}^J V^J$, where r_{AV}^J and V^J are the area-to-volume ratio at node J , and V^J is the volume of the continuum associated with node J .

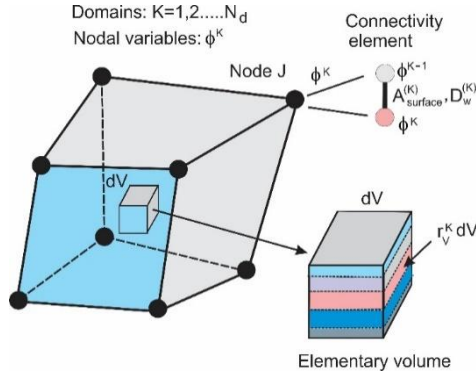


Fig. 4 Composite smeared finite element (CSFE). The element contains smeared physical fields coupled by the connectivity elements at each FE node. It is implemented in the PAK as the (KTM) (according to [4]).

2.4 Composite multiscale-multiphysics 3D finite element with internal microstructure for lung

This finite element is developed for modeling the lung. Regarding mechanics, it relies on the Wilson-Bachofen model of the balance between the internal and external supporting systems. The basic unit within the microstructure is a duct with external and internal fibers and alveoli. The surfactant covering the solids within the microstructure contributes to the internal force balance, crucial for lung function. The element is displayed in Fig. 5.

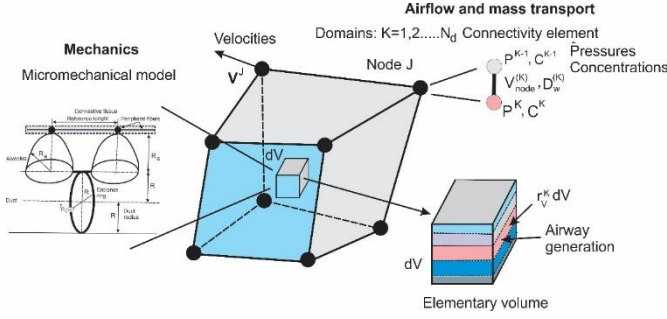


Fig. 5 Composite multiscale-multiphysics 3D finite element with internal microstructure for lung. On the left – micromechanical model, and on the right – smeared fields for the airway generations. (according to [5])

The KTM concept is included in the element for the airflow within the lung parenchyma, with element domains representing the airway generations.

3 Shell finite elements

We have in the PAK the isoparametric shell element based on the kinematics of deformation, where the initial straight line normal to the mid-surface remains straight during deformation, with displacement and rotation. Additionally, a four-node element with mixed interpolation of tensorial components MITC4 is built into the code. A shell element specific for the heart tissue is developed, where the fibrous helicoidal structure of the continuum is taken into account. The elements are displayed in Fig. 6.

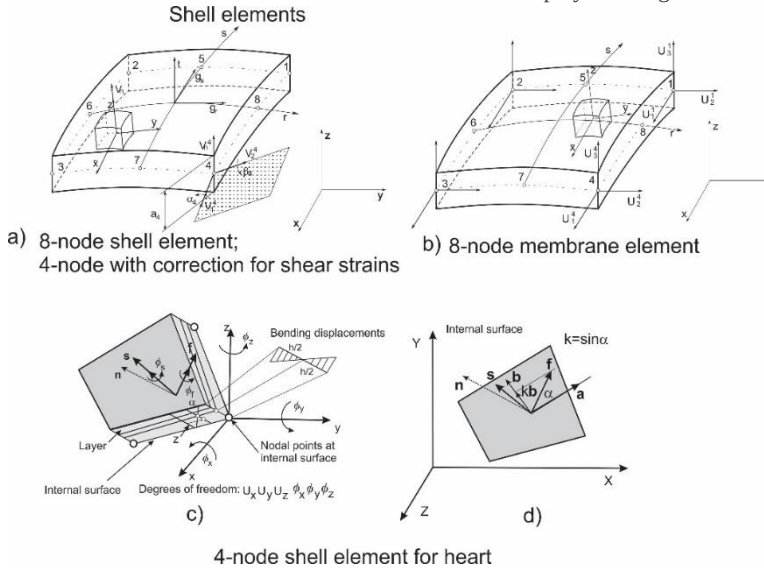


Fig. 6 Shell elements. a) 8-node isoparametric shell element with the “standard” kinematics of deformation. A 4-node shell element with mixed interpolation of the tensorial strains (MITC4) is built in the PAK code; b) Membrane element is a special case of shell, without rotational degrees of freedom and non-zero stresses only in the tangential plane of the membrane; c) 4-node shell element developed for heart walls, with helicoidal muscle fibers and orthotropy changing over the element thickness ; d) local coordinate system with unit vectors of the orthotropy directions. (according to [4])

4 Beam finite elements

We have PAK simple beam elements, usually present in the FE packages, and beam supplements specific to our PAK software.

4.1 Simple beam elements

There are several simple beam elements in PAK. The first type is a general isoparametric element shown in Fig. 7a. It relies on the same assumptions regarding kinematics of deformation as in the shell elements – a straight line normal to the beam axis in the two local principal directions remains straight during deformation, with translation and rotation. This element can be used for geometrical and material nonlinearities. The beam element shown in Fig. 7b has an analytically derived elastic stiffness matrix; hence, there is no need for numerical integration; it is efficient but cannot be used for material nonlinearity. A beam representing a circular arc is shown in Fig. 7c, as a specific beam. The elastic stiffness matrix is derived in analytical form.

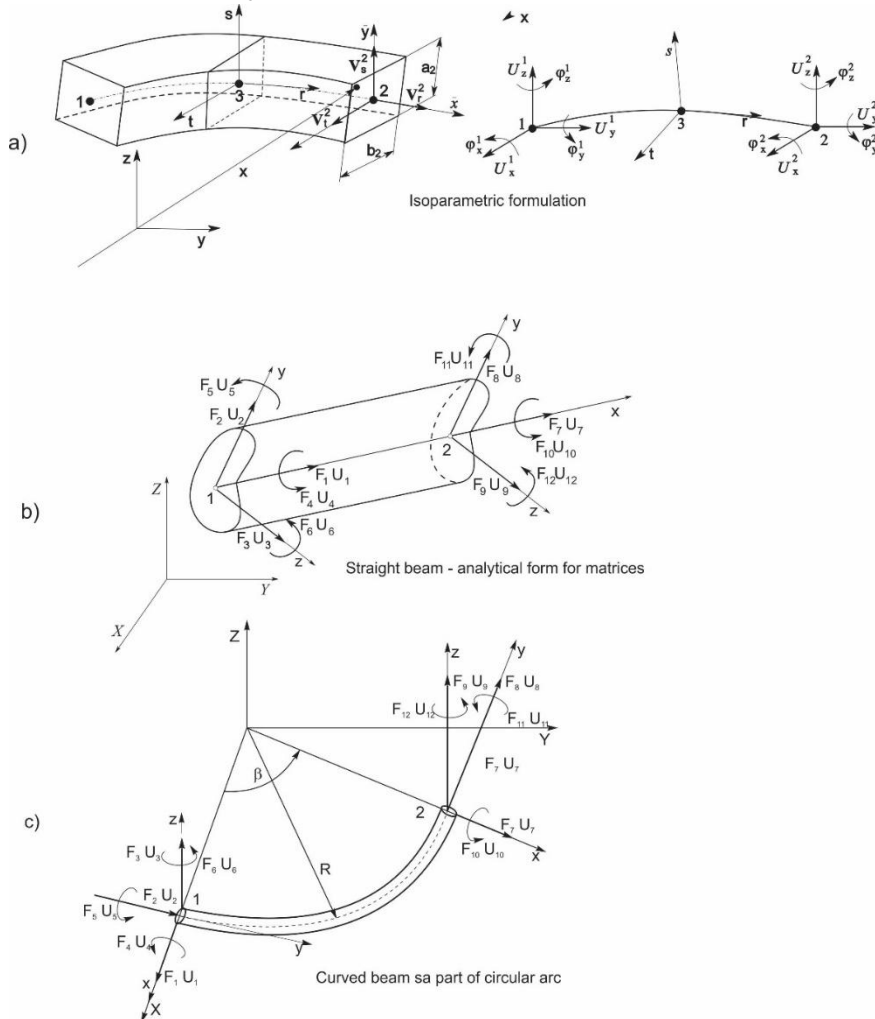


Fig. 7 Beam finite elements. a) Isoparametric beam element; b) A straight beam element with the analytically derived elastic stiffness matrix; c) Curved beam as part of circular arc lying in a plane. (according to [1]).

4.2 Beam superelements

These elements are formulated to simplify the modeling of beam-like structures with complex and deformable cross-sections. A typical example of this type of structure is a supporting system in the cars.

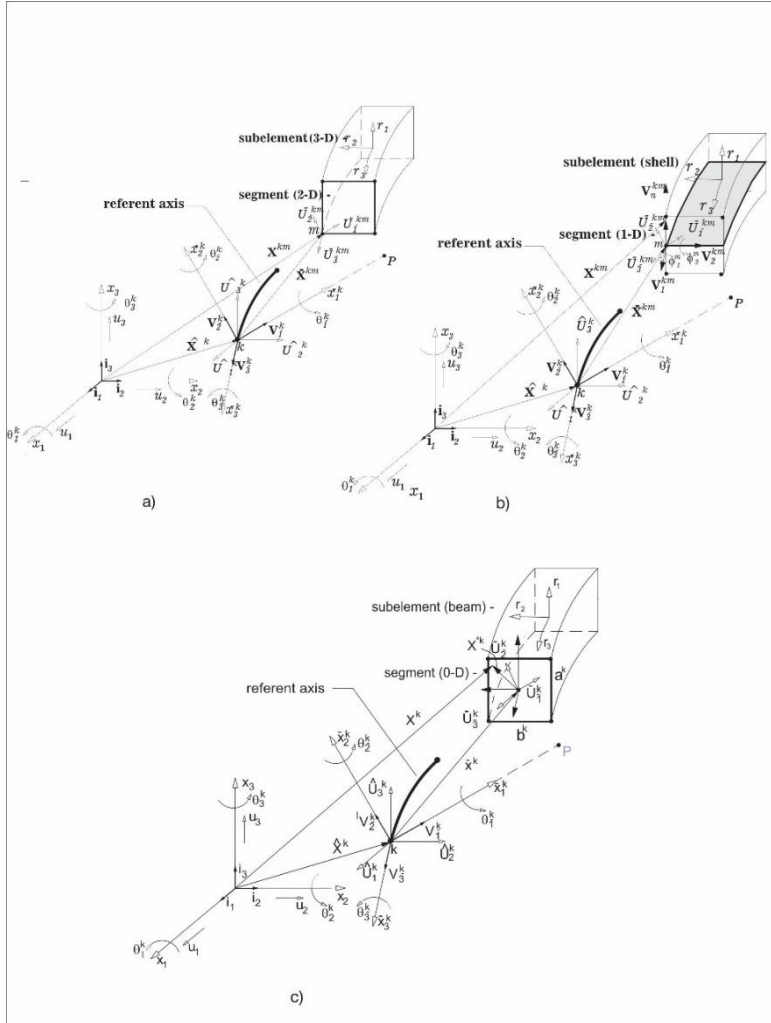


Fig. 8 Beam superelement. a) 2D element as the cross-section; b) Line element as the cross-section; c) Line pointwise definition of the cross-section. (according to [1],[6])

The idea is that the complex shape of a beam-like supporting element of a structure is specified simply by the main axis and the deformable cross-section. The element has 7 degrees of freedom, where, besides translation and rotation, an additional degree of freedom corresponds to the warping produced by torsion. The cross-section in Fig. 8a is a 2D element for the representation of a beam with a variable cross-section corresponding to the isoparametric element in Fig. 7a. The line element as a cross-section in Fig. 8b is used to generate a shell-type structure. Finally, the multilinear shape of the cross-section in Fig. 8c leads to a simple generation of a thin-wall beam with a variable and deformable cross-section.

References

1. Milos Kojic, Radovan Slavkovic, Miroslav Zivkovic and Nenad Grujovic, Finite Element Method I - Linear Analysis, Faculty of Mechanical Engineering, Kragujevac, 1998.
2. Milos Kojic and K. J. Bathe, Inelastic Analysis of Solids and Structures, Springer Verlag, 2005.
3. M. Kojic, N. Filipovic, B. Stojanovic, N. Kojic, Computer Modelling in Bioengineering – Theory, Examples and Software, J. Wiley and Sons, 2008.
4. Milos Kojic, Miljan Milosevic, Arturas Ziemys, Computational Models in Biomedical Engineering - Finite Element Models Based on Smeared Physical Fields: Theory, Solutions, and Software, Elsevier, 2022.
5. M. Kojic, A multiscale multiphysics finite element for lung, Journal of the Serbian Society for Computational Mechanics / Vol. 17 / No. 2, 2023 / pp 1-15.
6. M. Zivkovic, M. Kojic, R. Slavkovic, N. Grujovic, A general beam finite element with deformable cross-section, Comp. Meth. Appl. Mech. Engng., Vol. 190, pp. 2651-2680, 2001

MATERIAL MODELS IN PAK

The development and application of material models in the PAK system have been one of the main topics over the decades. There is a significant number of material models in engineering and bioengineering that have been implemented to solve various specific scientific and practical large-scale problems. Most of the models are nonlinear and the development of the right computational procedures with robust, accurate, and reliable characteristics has been a challenge. The material models are classified into two main groups: engineering and bioengineering models, although some of them apply to both fields.

1. ENGINEERING MATERIAL MODELS

A. LINEAR ELASTIC AND VISCOELASTIC MODELS

1.1 Elastic model

This is the model used in almost all modules of the PAK [1]). The constitutive relation is given as

$$\sigma_{ij} = C_{ij}^E e_j \quad (1.1)$$

where σ_{ij} are stresses, e_{ij} are engineering strains, and C_{ij}^E is the elastic constitutive matrix. The material parameters for the isotropic material are the Young modulus E and the Poisson ratio ν , while for the orthotropic conditions, the material parameters consist of the Young moduli E_i and the Poisson ratios ν_i .

1.2 Thermoelastic model

This model [1] is specified by the constitutive relation.

$$\sigma_{ij} = C_{ij}^E \left(e_j - e^{th} \delta_{ij} \right) \quad (1.2)$$

where e^{th} is the thermal strain and δ_{ij} is the Kronecker delta symbol.

1.3 Viscoelastic model

A linear viscoelastic model [4] is available, with constitutive relations.

$$\sigma_{ij} = \sigma_{ij}^E + \sigma_{ij}^\nu, \quad \sigma_{ii}^\nu = \lambda^\nu \dot{e}_{ii} \quad (1.3)$$

where σ_{ij}^E and σ_{ii}^ν are elastic and viscoelastic stresses; $\dot{e}_{ii}$ are strain rates, and λ^ν is a damping coefficient.

B. THERMO-ELASTIC-PLASTIC AND CREEP MODELS OF METALS

Elastic-plastic and elastic-plastic and creep models of metals are implemented into continuum, shell, and beam elements. The models rely on the Effective-Stress-Function (ESF)[2], or the governing parameter method (GPM) [3].

1.4 Elastic-plastic and viscoplastic model

The elastic-plastic deformation includes mixed hardening and perfect plasticity. The model definition and computational procedure in the PAK are illustrated in Fig. 1.1. The stress point remains on the yield surface, which changes the size and position in the deviatoric plane, while the effective stress remains on the yield curve. The governing parameter is the increment of the effective plastic strain Δe^P . An analytically derived constitutive elastic-plastic matrix provides a high convergence rate.

The computational scheme for the viscoplastic model is the same as for the elastic-plastic one, except that instead of plastic strains, the viscoplastic strains are used. The viscoplastic strain rate develops over time and is normal to the yield surface, while the stress exceeds the yield surface.

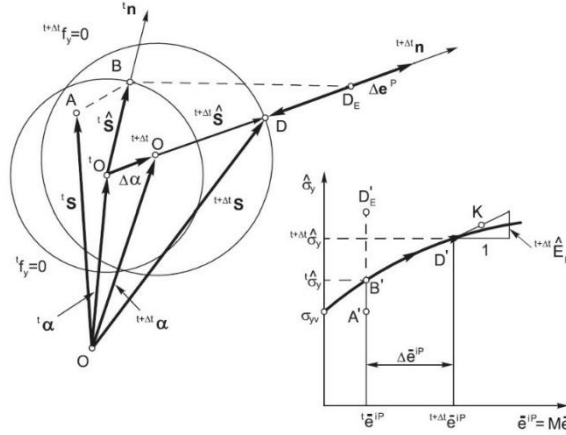


Fig. 1.1 Metal plasticity, mixed hardening. Yield surfaces at the start and end of the load step, and the corresponding points at the yield curve. The mixed hardening parameter is $0 \leq M \leq 1$ (according to [2])

1.5 Thermo-elastic-plastic and creep model

The model for thermoplasticity and creep of metals includes the GPM formulation for plasticity and creep laws for creep of material. Schematics of the stress integration is shown in Fig. 1.2. Material data consist of yield curves for different temperatures and creep laws. Plastic and creep deformation are incompressible.

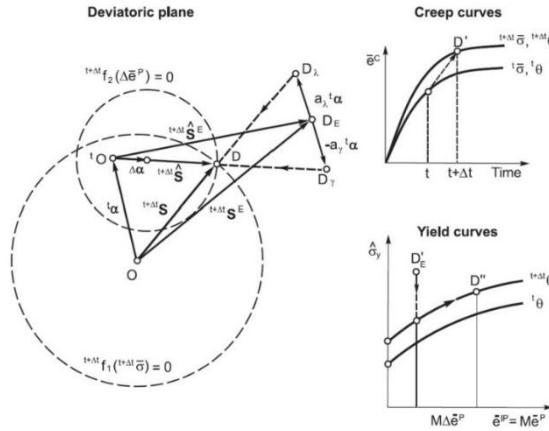


Fig. 1.2 Schematics of stress integration in the deviatoric plane for thermoplasticity and creep of metals. (according to [2])

C. ELASTIC-PLASTIC MODELS FOR GEOLOGICAL MATERIALS

There are several geological models in PAK that have been extensively used in modeling complex structures, such as dams and foundations. Some of these models are based on the GPM concept, while others are either extensions of these basic formulations or specific to meet the demands of practical large-scale engineering projects.

1.6 Cap Models

These models are defined in the plane $I_1 - \sqrt{J_{2D}}$, where I_1 is the first stress invariant and J_{2D} is the second invariant of the deviatoric stress. Fig. 1.3a shows the Drucker-Prager model with plane cap, while Fig. 1.3b displays increments of plastic strains for cap, vertex, and cap yielding regimes. A generalized cap model in Fig. 1.3c is defined by the yielding surface $f_1 = 0$, and cap surface $f_c = 0$; the increments of the plastic strain are displayed for the three loading regimes. Interestingly, the governing parameter for these models is the axial distance X along the I_1 axis.

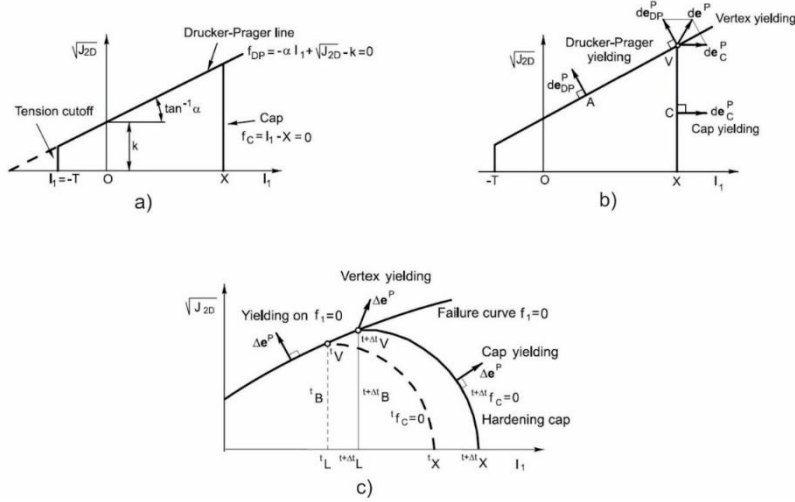


Fig. 1.3 Cap models. a) and b) show the Drucker-Prager model; c) The generalized cap model. (according to [2])

1.7 Cam-Clay Model

This model is defined according to experimental results in the plane $\sigma_m - \sqrt{3J_{2D}}$ by the ellipse with the horizontal axis equal to the hardening parameter p_0 which depends on the mean plastic strain e_m^P ; and the smaller axis defined by the critical state line and the parameter M , Fig. 1.4. The hardening, softening, and perfect plasticity regimes are shown in the figure. This model has been widely used in practical applications.

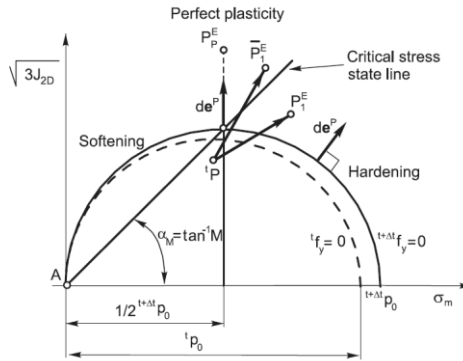


Fig. 1.4 Cam-clay model in the $\sigma_m - \sqrt{3J_{2D}}$, with the hardening, softening, and perfect plasticity regimes. Increments of the plastic strains are normal to the elliptical yield surface. (according to [2])

1.8 Mohr-Coulomb material model

The Mohr-Coulomb material model is one of the most commonly used models for simulating the mechanical behavior of granular materials [6, 7]. The model defines a direct dependence of the shear stress at failure τ_f on the normal stress σ , according to the following equation:

$$\tau_f = c + \sigma \tan \phi \quad (1.4)$$

The failure surface of this constitutive model in the principal stress space is shown in the following figure.

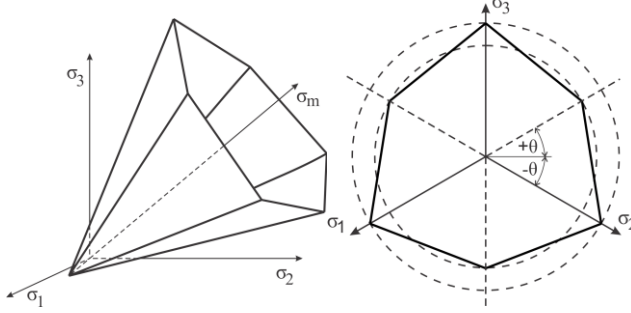


Fig. 1.5 Mohr-Coulomb failure surface

The equation of the failure surface of this material model [6] has the form:

$$f = \frac{I_1}{3} \sin \phi + \sqrt{J_{2D}} \left(\cos \theta - \frac{1}{\sqrt{3}} \sin \theta \sin \phi \right) - c \cos \phi \quad (1.5)$$

while the plastic potential equation in the general case differs from the yield surface (non-associative yield condition) and is defined by the equation:

$$g = \frac{I_1}{3} \sin \psi + \sqrt{J_{2D}} \left(\cos \theta - \frac{1}{\sqrt{3}} \sin \theta \sin \psi \right) \quad (1.6)$$

The quantities I_1 and J_{2D} in the previous equations represent the first stress invariant and the second stress deviator invariant, while the quantity θ represents the Lode angle. The quantities c , ϕ , and ψ represent the material model parameters, cohesion, the angle of internal friction in the material, and the dilatancy of the material, respectively.

1.9 Mohr-Coulomb material model with hardening

The Mohr-Coulomb material model with hardening is a modification of the Mohr-Coulomb material model described in the previous section. This model has a hardening feature that depends on the effective plastic strain [8, 9], expressed as:

$$\kappa = \bar{e}^P = \sum d\bar{e}^P \quad (1.7)$$

The increment of effective plastic strain can be calculated according to:

$$d\bar{e}^P = d\lambda \sqrt{\frac{2}{3} \frac{dg}{d\sigma} \frac{dg}{d\sigma}} \quad (1.8)$$

In elasto-plastic models without hardening, the failure function depends only on the stress state and does not take into account plastic strain. The failure surface equation of the Mohr-Coulomb material model with hardening also depends on the accumulated plastic strain, so the equation of the failure surface can be written in the form [10]:

$$f = f(\sigma, c(\kappa), \phi(\kappa)) \quad (1.9)$$

The hardening due to internal friction in the material [11, 12] is expressed as:

$$\phi(\bar{e}^P) = \sin^{-1} \left(2 \frac{\sqrt{\bar{e}^P \bar{e}_f^P}}{\bar{e}^P + \bar{e}_f^P} \sin(\phi_f) \right) \quad (1.10)$$

where $\bar{e}_f^P$ is the effective plastic deformation at the maximum angle of internal friction ϕ_f , which is shown in the following figure.

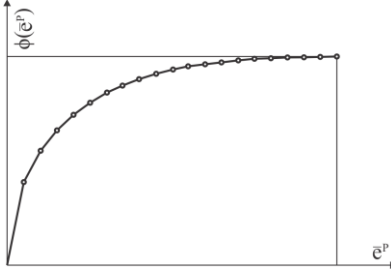


Fig. 1.6. Internal friction angle as a function of effective plastic strain

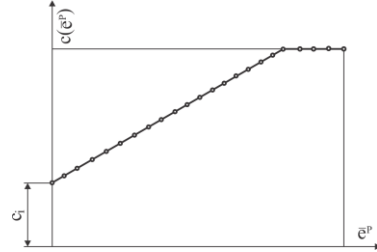


Fig. 1.7 Material cohesion as a function of effective plastic strain

Cohesive hardening in a material [11] is defined by the expression:

$$c(\bar{e}^P) = c_i + h_c \bar{e}^P \quad (1.11)$$

where c_i is the initial cohesion value, while h_c is the cohesion hardening parameter. The cohesion value can increase up to a maximum given value, after which it has a constant value:

$$c(\bar{e}^P) \leq c_f \quad (1.12)$$

as shown in the figure 1.7.

The dilatancy angle is also a function of the effective plastic strain [11] and is expressed by the equation:

$$\psi(\bar{e}^P) = \sin^{-1} \left(\sin \psi_i + (\sin \psi_f - \sin \psi_i) \frac{\sin \phi(\bar{e}^P) - \sin \phi_i}{\sin \phi_f - \sin \phi_i} \right) \quad (1.13)$$

where ψ_i is the initial dilatancy angle of the material, ϕ_i is the initial angle of internal friction, ψ_f is the maximum dilatancy angle, while ϕ_f represents the maximum angle of internal friction.

1.10 Matsuoka-Nakai material model

When integrating the constitutive relations of the Mohr-Coulomb material model, one of the frequent numerical problems is finding the derivative of the failure surface function at the boundary values of the Lode angle ($\theta = \pm 30^\circ$), because at the vertices of the hexagon it is not possible to determine a unique normal to the failure surface. For this reason, failure surfaces that do not contain these transitions have been introduced, and one of such surfaces is the Matsuoka-Nakai failure surface [13, 14, 15]. The equation of the failure surface of this material model is expressed by an equation of the form:

$$f = I_3 + \frac{\cos^2 \phi}{9 - \sin^2 \phi} I_1 I_2 = 0 \quad (1.14)$$

where ϕ represents the angle of internal friction of the material, while I_1 , I_2 and I_3 are the first, second, and third stress invariants, respectively. The failure surface of this model in the principal stress space and the deviatoric plane, along with the yield surface of the Mohr-Coulomb model, is shown in the following figure.

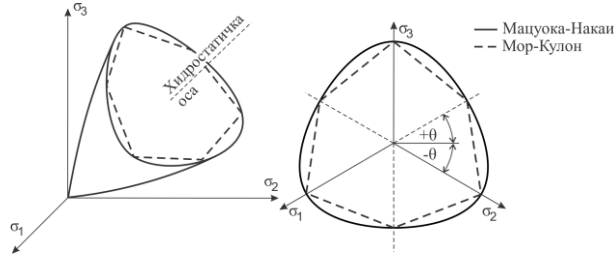


Fig. 1.8 Matsuoka-Nakai failure surface

The plastic potential of this material model [13] is defined by the expression:

$$g = I_3 + \frac{\cos^2 \psi}{9 - \sin^2 \psi} I_1 I_2 = 0 \quad (1.15)$$

where ψ represents the dilatancy angle of the material.

1.11 Drucker-Prager material model

The Drucker-Prager material model is one of the oldest material models in soil mechanics. It was created by simplifying the Mohr-Coulomb failure surface to overcome the problem of discontinuity [16, 17, 18, 19]. The failure surface of this model in the stress space is a cone whose axis coincides with the hydrostatic axis, as shown in the following figure.

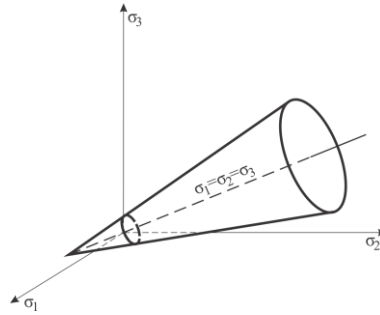


Fig. 1.9 Drucker-Prager failure surface

The failure surface equation of this model is defined as:

$$f = \alpha I_1 + \sqrt{J_{2D}} - k \quad (1.16)$$

while for the case of non-associative conditions the plastic potential surface is defined by the expression:

$$g = \beta I_1 + \sqrt{J_{2D}} \quad (1.17)$$

where I_1 represents the first stress invariant, while J_{2D} is the second stress deviator invariant. The quantities α , k and β represent the material model constants that can be calculated indirectly using the material constants of the Mohr-Coulomb model.

1.12 Hoek-Brown material model

The Generalized Hoek-Brown material model defines the relationship between the major and minor principal stresses in a material [20] by the following relation:

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left(m_b \frac{\sigma_3}{\sigma_{ci}} + s \right)^a \quad (1.18)$$

The failure surface of this model is a function of the stress state [20] and can be expressed by the following equation:

$$f = \frac{I_1}{3} m_b \sigma_{ci}^{\left(\frac{1}{a}-1\right)} - s \sigma_{ci}^{\frac{1}{a}} + 2^{\frac{1}{a}} \left(\sqrt{J_{2D}} \cos \theta \right)^{\frac{1}{a}} + m_b \sqrt{J_{2D}} \sigma_{ci}^{\left(\frac{1}{a}-1\right)} \left(\cos \theta - \frac{1}{\sqrt{3}} \sin \theta \right) \quad (1.19)$$

and is shown in the principal stress space, shown in the following figure.

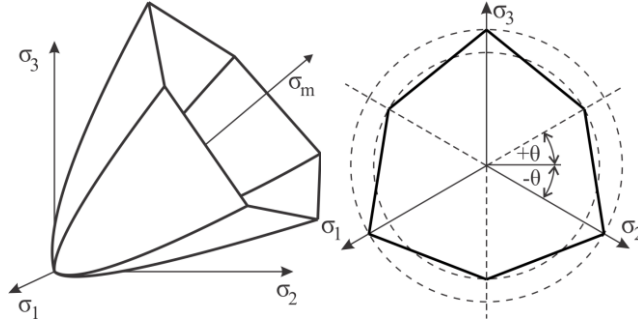


Fig. 1.10 Generalized Hoek-Brown failure surface

The plastic potential equation of this material model is:

$$g = \frac{I_1}{3} m_{bdil} \sigma_{ci}^{\left(\frac{1}{a}-1\right)} - s \sigma_{ci}^{\frac{1}{a}} + 2^{\frac{1}{a}} \left(\sqrt{J_{2D}} \cos \theta \right)^{\frac{1}{a}} + m_{bdil} \sqrt{J_{2D}} \sigma_{ci}^{\left(\frac{1}{a}-1\right)} \left(\cos \theta - \frac{1}{\sqrt{3}} \sin \theta \right) \quad (1.20)$$

The quantities I_1 and J_{2D} in the previous equations represent the stress invariants, while the quantity θ represents the Lode's angle. The quantities σ_{ci} , m_b , m_{bdil} , s and a are the parameters of the material model.

1.13 Hyperbolic failure surface material model

Due to the fact that granular unbound materials lack cohesion, the shear strength of soils can be described using effective stresses [21] by an equation of the form:

$$\tau_f = \sigma_n \tan \phi(\sigma_n) \quad (1.21)$$

where the maximum value of the angle of internal friction in the material is defined by the expression:

$$\phi_{CV_{max}} \quad (1.22)$$

where ψ is the contribution of dilatancy. The angle of internal friction in a material is a function of the normal effective stress represented as:

$$\phi(\sigma_n) = \phi_B + \delta\phi(\sigma_n) \quad (1.23)$$

The second term in the previous expression can be represented as:

$$\delta\phi(\sigma_n) = \frac{\Delta\phi}{1 + \frac{\sigma_n}{p_N}} \quad (1.24)$$

so the total angle of internal friction in the material is:

$$\phi(\sigma_n) = \phi_B + \frac{\Delta\phi}{1 + \frac{\sigma_n}{p_N}} \quad (1.25)$$

Shear strength of the material, using the previous equation [21], can be finally formulated as:

$$\tau_f = \sigma_n \tan \left(\phi_B + \frac{\Delta\phi}{1 + \frac{\sigma_n}{p_N}} \right) \quad (1.26)$$

representing the equation of failure shear stress envelope, as the function of normal stress. In the previous equation of the material model, the following parameters are used: ϕ_B - basic friction angle, $\Delta\phi$ - maximal angle difference $\phi_O - \phi_B$, p_N - normal stress of central secant angle.

Failure stress envelope is defined with the above equation are presented in following figure.

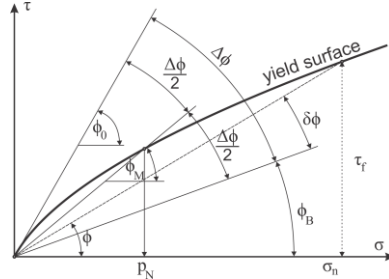


Fig. 1.11 Failure surface of the hyperbolic soil model

1.14 A temperature-dependent multilinear model with hysteresis

This is a model developed according to experimental uniaxial temperature-dependent cyclic curves recorded for a material specifically used for stents. The model relies on the equivalency of deformation work under uniaxial and general 3D conditions. The interpolation under the loading and unloading regimes is shown in Fig. 1.12. Interpolation is also performed with respect to temperatures.

Current at Gauss point $\dot{\epsilon} = C$, Temperature T

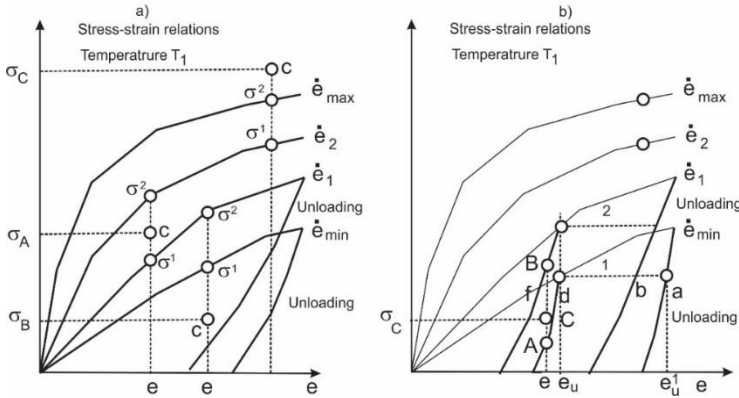


Fig. 1.12 A temperature-dependent multilinear model with hysteresis. Equivalent stress is interpolated using uniaxial experimental curves for various strain rates and temperatures. (according to [5])

1.15 Shape memory alloys

The constitutive model presented in Lagoudas's book [22] and its implementation in Finite Element Method (FEM) software is derived based on a Gibbs free energy function g , extended by a

transformation hardening function $f(\xi)$, which depends on a martensitic volume fraction ξ . It is given in an explicit form as in Qidwai and Lagoudas [23]:

$$g(\sigma, T, \xi, \mathbf{e}_{tr}) = -\frac{1}{2\rho} \sigma : \mathcal{M} : \sigma - \frac{1}{\rho} \sigma : [\alpha(T - T_0) + \mathbf{e}_{tr}] + c \left[(T - T_0) - T \ln \left(\frac{T}{T_0} \right) \right] - s_0 T + u_0 + \frac{1}{\rho} f(\xi) \quad (1.27)$$

The two internal state variables define the constitutive model: the martensitic volume fraction ξ and the martensitic transformation strain $\mathbf{e}_{tr}$. The variables σ and T are the total stress and the temperature of the material, while, $\mathcal{M}$ is the effective compliance tensor, α is the effective thermal expansion coefficient tensor, T is the current temperature of the material, T_0 is the reference temperature, c is the effective specific heat, s_0 is the effective specific entropy, u_0 is the effective specific internal energy, and ρ is the density of the material. The main assumption for the derivation of this constitutive model is that the change of martensitic transformation strain $\mathbf{e}_{tr}$ is strictly a result of a change in the amount of martensitic volume fraction ξ as follows [23]:

$$\dot{\mathbf{e}}_{tr} = H \mathbf{n}_{tr} \dot{\xi} \quad (1.28)$$

where H is the value of maximal transformation strain and $\mathbf{n}_{tr}$ is the transformation direction vector [22,23]. The explicit form of thermodynamic force Π is [22,23]:

$$\Pi(\sigma, T, \xi) = \sigma : \mathbf{n}_{tr} H + \frac{1}{2} \sigma : \Delta \mathcal{M} : \sigma + \sigma : \Delta \alpha (T - T_0) \rho \Delta c \left[(T - T_0) - T \ln \left(\frac{T}{T_0} \right) \right] + \rho \Delta s_0 T - \rho \Delta u_0 - \frac{\partial f(\xi)}{\partial \xi} \quad (1.29)$$

where $\Delta \mathcal{M} = \mathcal{M}^M - \mathcal{M}^A$, $\Delta \alpha = \alpha^M - \alpha^A$, $\Delta c = c^M - c^A$, $\Delta s_0 = s_0^M - s_0^A$, $\Delta u_0 = u_0^M - u_0^A$, where subscript A denotes austenitic and M martensitic phase.

1.16 Concrete damage plasticity model (CDPM)

The total strain can be additively decomposed into the elastic and the plastic strain component [24-26]:

$$\mathbf{e} = \mathbf{e}^E + \mathbf{e}^P \quad (1.30)$$

The elastic part is a recoverable part of the total strain, which defines the effective (undamaged) stress tensor as [24-26]:

$$\bar{\sigma} = \mathbf{D}_0 : \mathbf{e}^E \quad (1.31)$$

where $\mathbf{D}_0$ denotes the undamaged elastic stiffness tensor. The stress can be computed as follows:

$$\bar{\sigma} = \mathbf{D}_0 : (\mathbf{e} - \mathbf{e}^P) \quad (1.32)$$

Because the maximal eigen stress is necessary for the computation of yield function, the diagonal eigen stress tensor is transformed as [24-26]:

$$\hat{\sigma}^{tr} = \mathbf{P}^{-1} \bar{\sigma}^{tr} \mathbf{P}^{-T} \quad (1.33)$$

where $\mathbf{P}$ is the transformation matrix $\bar{\sigma}^{tr}$. Now, it is possible to calculate the yield function of CDP constitutive model by equation [24-26]:

$$F(\hat{\sigma}^{tr}, \kappa) = \frac{1}{1-\alpha} \left(\alpha (\hat{I}_1^{tr}) + \sqrt{\frac{3}{2}} \|\hat{\mathbf{S}}^{tr}\| + \beta(\kappa) \langle (\hat{\sigma}_1^{tr}) \rangle \right) - \bar{\sigma}_c(\kappa) \quad (1.34)$$

where, $\kappa = [\kappa_c, \kappa_t]$ is the damage vector which consist of compression and tension component, $\bar{\sigma}_c(\kappa)$ is the material cohesion, $\hat{I}_1^{tr} = \text{tr} \hat{\sigma}^{tr}$ is the first stress tensor invariant, $\|\hat{\mathbf{S}}^{tr}\| = \sqrt{\hat{\mathbf{S}}^{tr} : \hat{\mathbf{S}}^{tr}}$ is the stress tensor deviator norm, $\hat{\mathbf{S}}^{tr} = \hat{\sigma}^{tr} - \hat{\sigma}_m \mathbf{I}$ is the deviator of effective stress, $\hat{\sigma}_m = \frac{1}{3} \text{tr} \hat{\sigma}^{tr}$ is the mean effective stress, and $\hat{\sigma}_1^{tr}$ is the algebraic maximum of eigenvalues of effective stress tensor. In the case when $F(\hat{\sigma}^{tr}, \kappa) <$

0, the solution is elastic and there is no damage or plastic strain increase. However, if the condition is not satisfied, the stress integration procedure is employed.

2. BIOENGINEERING MATERIAL MODELS

2.1 Bone model

This is a linear model built in PAK [4] by using a specific Young's modulus E_{bone} as $E_{\text{axial}} = E_c \dot{\epsilon}^{0.06} \left(\frac{\rho}{\rho_c} \right)^3$

where E_{axial} is the elastic modulus of bone of apparent density ρ , tested at strain rate of $\dot{\epsilon} [s^{-1}]$; and E_c is elastic modulus of bone with an apparent density of ρ_c tested at strain rate of $1.0 s^{-1}$.

2.2 Uniaxial tissue model, without and with hysteresis

The constitutive curves for the uniaxial tissue model are shown in Fig. 2.1a, while in case of hysteresis is displayed in Fig. 2.1b. In the case without hysteresis, the computational procedure consists of the discretization of the constitutive curves into a number of linear segments. When there is hysteresis, the computation methodology includes scaling of the hysteresis (shown as “Numerical”).

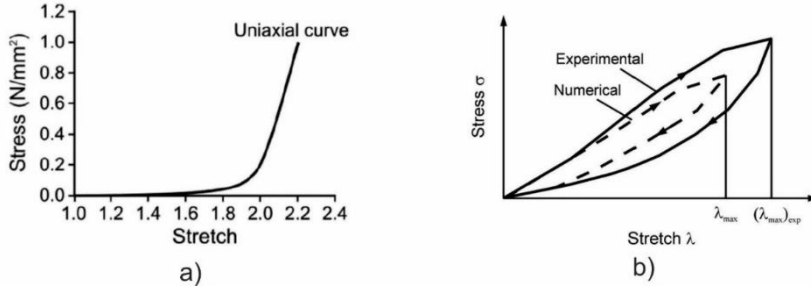


Fig. 2.1 Uniaxial tissue model. a) Without hysteresis; b) With hysteresis. The numerical curve corresponds to a maximum stretch at an integration point, scaled with respect to the experimental curve. (according to [4])

2.3 Biaxial model for biological membranes

A specific computational procedure has been developed for a biaxial tissue model, applicable to tissue membranes. The uniaxial and biaxial (with equal loading in two membrane directions) are used as the reference curves. The stresses at the current stretches are computed at the interpolated curve between the two reference ones.

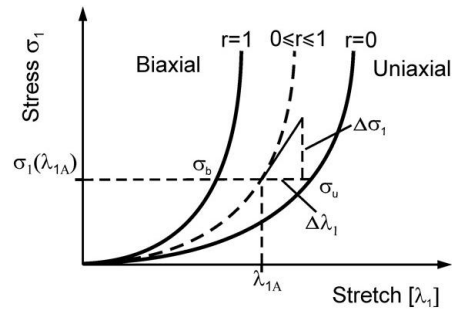


Fig. 2.2 Biaxial model for biological membranes. Interpolation of stresses between the uniaxial and biaxial curves. (according to [4])

2.4 Model of surfactant

The lung microstructure is covered by a biological liquid called surfactant. Lung function is not possible without the action of surfactant. The load of tissue produced by surfactant has a hysteretic character since the deformation of the lung has cycles. A special computational procedure has been implemented to PAK for modeling the surfactant action. Fig. 2.3 shows the experimental and “Numerical” curves for the dependence of the surface tension on the surface ratio (of the surface covered by surfactant). The “Numerical” curve corresponds to the area ratio at a point on the surface during the iterative force balance solution of the computational model.

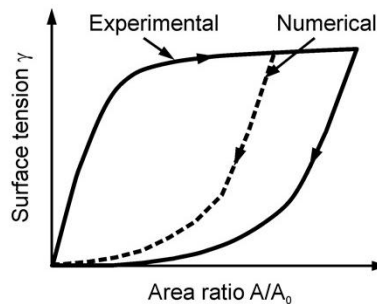


Fig. 2.3 Constitutive curve for surfactant covering the internal microstructure of the lung. The hysteretic character of the dependence between the surface tension and the area ratio of the surface covered by surfactant. (according to [4])

2.5 Skeletal muscle model

The muscle model built into the PAK is based on Hill’s phenomenological formulation Fig. 2.4c). It relies on two characteristics of the muscle’s mechanical behavior: the tension-velocity curve (Fig. 2.4a) and the isometric tension-length curve (Fig. 2.4b).

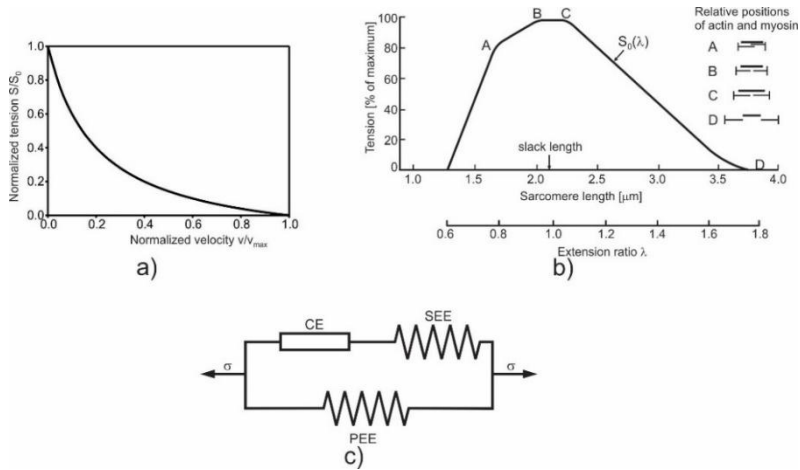


Fig. 2.4 Muscle model. a) Tension-velocity curve; b) Isometric tension-length curve; c) Hill's three-component model: CE is the contractile element, SEE is the series elastic element and PEE is the parallel elastic element, σ is the stress in the muscle fiber direction (according to [4])

2.6 Cartilage mechanics model

Multiphysics mechanical model of cartilage is implemented in the PAK software. As shown in Fig. 2.5, this model assumes a composite structure with solid and fluid. The porous solid is filled with biological fluid, and fluid flow is produced by the solid deformation, and also due to electrokinetic coupling. The formulation of the methodology is according to a mixture theory.

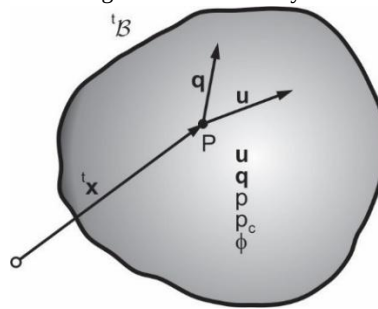


Fig. 2.5 Configuration tB of cartilage at time t , considered as a continuous medium (mixture), and variables at a material point P whose position vector is $\mathbf{r}$. The variables are: $\mathbf{u}$ - displacement of the mixture, $\mathbf{q}$ - relative fluid velocity with respect to the solid (Darcy's velocity), p - fluid pressure, p_c - swelling pressure;; ϕ - electrical potential if the swelling pressure effects are interpreted by electrokinetic coupling. (according to [4])

2.7 Cell model

A special model for cell mechanics is implemented in the PAK software. It is a biphasic model with solid and fluid domains, as shown in Fig. 2.6. The solid contains fibers with the passive stress of the solid phase and active stress in the direction of fibers. The caption of the figure provides the basic information of the model.

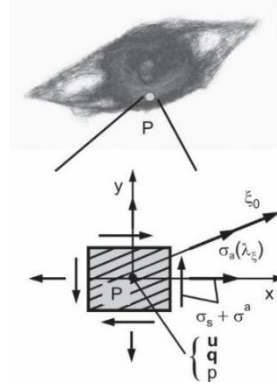


Fig. 2.6 Biphasic model of cell. Stresses at a material point P within the solid phase include the passive and active parts σ_s and σ^a (2D representation of stresses in the figure). The stress σ^a is acting along the fibers (direction ξ_0) and depend on the fiber stretch λ_ξ . The field variables of the model are: displacement of solid $\mathbf{u}$, relative fluid velocity with respect to solid (Darcy's velocity) $\mathbf{q}$, and fluid pressure p . (according to [4])

2.8 Heart tissue model

We have developed and implemented in the PAK code a material model for the heart tissue with direct use of the hysteretic experimental biaxial curves and shear curves, Fig.2.7. The curves are recorded for several ratios of the strains in the fiber and sheet directions. Instead of the usual 3 shear strains, here other measures are used – 6 amount of shear values corresponding to 6 shear modes on the local fiber-sheet-normal planes; these shear measures are built into the code.

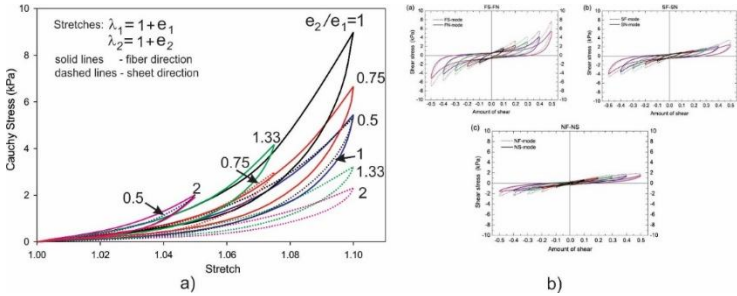


Fig. 2.7 Experimentally determined constitutive curves for heart tissue. a) Biaxial curves for the radial-sheet -directions; b) Amount of shear curves on the radial-sheet-normal planes. (according to [5])

The constitutive curves are directly used within the code, with interpolations among the curves analogous to those displayed for the temperature-dependent multilinear model with hysteresis.

References

1. Milos Kojic, Radovan Slavkovic, Miroslav Zivkovic and Nenad Grujovic, Finite Element Method I - Linear Analysis, Faculty of Mechanical Engineering, Kragujevac, 1998.
2. Milos Kojic and K. J. Bathe, Inelastic Analysis of Solids and Structures, Springer Verlag, 2005.
3. M. Kojic, The governing parameter method for implicit integration of viscoplastic constitutive relations for isotropic and orthotropic metals, Computational Mechanics, Vol. 19, No. 1, pp. 49-57, 1996.
4. M. Kojic, N. Filipovic, B. Stojanovic, N. Kojic, Computer Modelling in Bioengineering – Theory, Examples and Software, J. Wiley and Sons, 2008.
5. Milos Kojic, Miljan Milosevic, Arturas Ziemys, Computational Models in Biomedical Engineering - Finite Element Models Based on Smeared Physical Fields: Theory, Solutions, and Software, Elsevier, 2022.
6. Smith i V. Griffiths, Programming the Finite Element Method, England: John Wiley & Sons Ltd, 2004.
7. G. Balmer, "A general analytical solution for Mohr's envelope," Proceedings of American Society for Testing and Materials, vol. 52, pp. 1260-1271, 1952.
8. P. Vermeer i R. Borst, "Non-associated plasticity for soils, concrete and rock," HERON, t. 29, br. 3, 1984.
9. G. T. Dounias, D. M. Potts i V. P. R., "Finite element analysis of progressive failure: Two case studies," Computers and Geotechnics, pp. 155-175, 1988.
10. D. Conte, F. Silvestri i T. A., "Stability analysis of slopes in soils with strain-softening behaviour," Computers and Geotechnics, pp. 710-722, 2010.
11. R. Woods i A. Rahim, "CRISP2D Geotechnical finite element analysis software - Technical reference manual," The CRISP Consortium Limited, 2007.
12. B. Muhunthan i S. Murugaiah, "Numerical simulation of the performance of sand columns," Washington State University, Department of Civil & Environmental Engineering, Pullman, 2006.
13. H. Matsuoka i T. Nakai, "Stress-deformation and strength characteristics of soil under three different principal stresses," Proceedings of the Japan Society of Civil Engineers, t. 232, pp. 59-70, 1974.
14. Rocscience inc, "Plasticity Models in Phase2," [Na mreži]. Available: http://www.rocscience.com/downloads/phase2/webhelp/phase2.htm#phase2_model/.
15. TNO DIANA BV, "DIANA - Finite Element Analysis," [Na mreži]. Available: <http://tnodiana.com/upload/files/DIANA/942/HTML/GeoTech/GeoTech.html>.
16. D. Drucker i W. Prager, "Soil mechanics and plastic analysis for limit design," Quarterly of Applied Mathematics, t. 10, br. 2, pp. 157-165, 1952.
17. D. Drucker, "A more fundamental approach to plastic stress-strain relations," Proceedings of the First US National Congress of Applied Mechanics-ASME, pp. 487-497, 1951.
18. D. Drucker, "Extension of the stability postulate with emphasis on temperature changes in plasticity," Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 170-184, 1960.
19. D. Rakić, Razvoj Drucker-Prager materijalnog modela sa primenom u geomehanici-Magistarska teza, Kragujevac: Mašinski fakultet u Kragujevcu, 2009.
20. E. Hoek, C. Carranza-Torres i B. Corkum, "Hoek-Brown failure criterion - 2002 edition," u Proceedings of the 5th North American Rock Mechanics Symposium and the 17th Tunnelling Association of Canada: NARMS-TAC 2002, Toronto, Canada, 2002.
21. M. Maksimović, Mehanika tla, četvrto izdanje, Beograd: AMG knjiga, 2008.
22. Lagoudas D. Shape Memory Alloys: Modeling and Engineering Applications. Springer, 2010.
23. Qidwai M and Lagoudas D. Numerical implementation of a shape memory alloy thermomechanical constitutive model using return mapping algorithms, *Int J Numer Meth Eng* 2000; 47: 1123-1168.
24. Lee J. Theory and implementation of plastic-damage model for concrete structures under cyclic and dynamic loading, PhD Dissertation. - Berkeley: University of California, 1996.
25. Lee Jeeho and Fenves Gregory Plastic-Damage Model for Cyclic Loading of Concrete Structures, *Journal of Engineering Mechanics*. - 1998. - 8: Vol. 124.
26. Lubliner Jacob, Oliver Javier and Onate Eugenio A plastic-damage model for concrete, *International Journal of Solids and Structures*. - 1989. - 3 : Vol. Volume 25. - pp. 299-326.

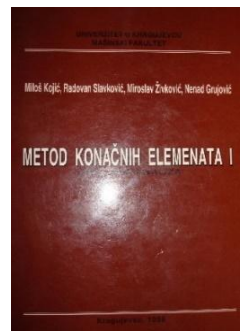
BIBLIOGRAPHY

This concluding section gives a summary of the references related to the PAK development and applications over five decades. The summary illustrates the achievements documented within the computational mechanics community through the selected publications. The publications only refer to the authors present in the review papers within this special issue. Quite a large number of books or monographs and scientific papers published by the most prestigious publishers all over the world demonstrate the scientific level of research and versatility of topics in Engineering and Bioengineering built into the PAK software, as well as the PAK applications. Without overstatement, it could be concluded, that the involvement of tens of researchers in Serbia and abroad also proves that the extraordinary efforts over decades contributed to society and particularly to the computational community.

A - Books and Monographs

1. **Miloš Kojić, Radovan Slavković, Miroslav Žiković, Nenad Grujović**
Finite Element Method (in Serbian: Metod konačnih elementa) I
– Linear Analysis, Faculty of Mechanical Engineering,
Kragujevac, 1998, 2010.

This is the first textbook (480 pages) related to the courses on the Finite Element Method at the Mechanical Engineering Faculty and the software package PAK. It is a self-contained textbook, adjusted for courses relying on the basic knowledge of mechanics and physics. The book summarizes the basic relations in solid mechanics, fluid flow, and heat transfer, and introduces the formulation of the FEM in the linear analysis. The finite elements built in the PAK are: continuum 2D and 3D elements, 1D truss and beams, beam superelement, shells, finite elements for fluid flow, and heat transfer. Each chapter contains examples with detailed descriptions and comments on the numerical results. Linear material models, statics, and dynamics are covered within the book. The final chapter addresses some specifics related to the numerical procedures for the FEM.



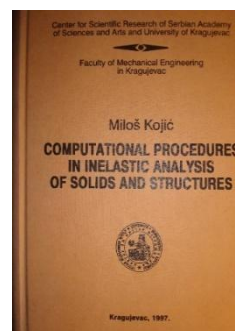
2. **Miloš Kojić,**
A General Concept of Implicit Integration of Constitutive
Relations for Inelastic Material Deformation, (in Serbian) Center
for Scientific Research of Serbian Academy of Sciences and Art
and University of Kragujevac, Kragujevac, 1993.

A short monograph (92 pages) presents the concept of the implicit stress integration for inelastic materials, initially introduced by the author, as the “effective stress function”, for thermoplasticity and creep of metals. Selected examples illustrate the application of this concept, demonstrating robustness, accuracy, and efficiency.



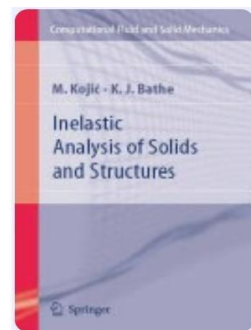
3. **Miloš Kojić,**
Computational Procedures in Inelastic Analysis of Solids and
Structures, (monograph, in English), Center for Scientific
Research of Serbian Academy of Sciences and Arts and University
of Kragujevac, Kragujevac, 1997.

This large monograph (441 pages) spans from the fundamentals of the mechanics of solids to computational procedures for implicit stress integration of inelastic materials. The captions of the chapters are: Introduction to the mechanics of solids, Incremental-iterative solution procedures for nonlinear structural problems, Metal plasticity, Thermo-plasticity and creep, Plasticity of geological materials, and Large strain inelastic deformation. This monograph served as the basis for the later book with K. J. Bathe, Springer, 2005. All models are built into the PAK program.



4. Miloš Kojić and Klaus-Jurgen Bathe
Inelastic Analysis of Solids and Structures, Springer, 2005.

The book (monograph, 414 pages) provides a short background and detailed description of the implicit stress integration (based on the “effective stress function” generalized further to the Governing parameter method – GPM) of the material models: metal plasticity; thermoplasticity, creep, and viscoplasticity; several basic geological plasticity models (Cap models, Cam-clay model, General soil plasticity model). Typical examples are designed to illustrate the computational methodology. The last chapter is devoted to the large strain elastic-plastic deformation.



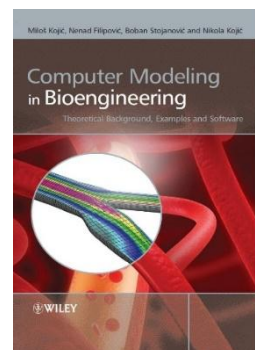
5. Miroslav Živković,
Nonlinear analysis of structures, Faculty of Mechanical Engineering, Kragujevac, 2006.

This monograph (180 pages) considers the basics of continuum kinematics, finite deformation tensors, corresponding stress tensors, as well as incremental equilibrium equations for nonlinear analysis. A detailed derivation of the equations for the isotropic von Mises plasticity for the 3d, shell, and beam elements with the theory for incompatible displacements in natural coordinates is given. The super-beam finite element of deformable section and general geometry is also presented. An algorithm for solving equilibrium incremental equations of finite elements for geometric and material nonlinear analysis is described. Descriptions and solutions for 17 complex examples are given, covering all types of nonlinear analysis and all types of finite elements.



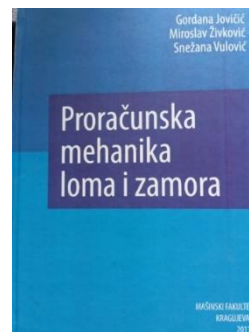
6. Miloš Kojić, Nenad Filipović, Boban Stojanović, Nikola Kojić,
Computer Modelling in Bioengineering – Theory, Examples and Software, J. Wiley and Sons, 2008.

This book (446 pages) represents a summary of the research in Bioengineering in the period 1995-2008 and the implementation of the developed computational methods into the PAK. The research has been supported by the Ministry of Science of Serbia, the City of Kragujevac, and through joint grants between the University of Kragujevac (Center for Scientific Research of the Serbian Academy of Sciences and Arts and the University of Kragujevac, and Harvard School of Public Health). Besides the authors, 20 contributors are listed for their direct participation in the earlier development of the PAK or the laboratory investigations (medical doctors Mirko Rošić and Božidar Novaković, and others). The book is written as a self-contained textbook, covering I – Theoretical Background, II – Fundamentals of Computational Methods, III – Computational Methods in Bioengineering. The book is accompanied by the PAK-based software with 80 examples, with a detailed explanation of the problem formulation and exploration of the effects of the model parameters.



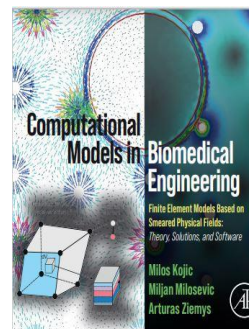
7. **Gordana Jovičić, Miroslav Živković, Snežana Vulović, Computational Fracture Mechanics and Fatigue, Faculty of Mechanical Engineering, Kragujevac, 2011.**

The monograph has 223 pages. Numerical Methods for Fracture Mechanics and Fatigue implemented in the PAK software were processed within it. Special attention in the publication is devoted to eXtended Finite Element Method (XFEM), J-Equivalent Domain Integral Method (J-EDI), Fatigue crack growth, Low-cycle, high-cycle fatigue of materials. Within the Monograph, the results obtained using the PAK software are presented.



8. **Milos Kojic, Miljan Milosevic, Arturas Ziemys, Computational Models in Biomedical Engineering - Finite Element Models Based on Smeared Physical Fields: Theory, Solutions, and Software, Elsevier, 2022.**

This recent book (391 pages) covers the research and results achieved in the period 2010-2024 in collaboration between Houston Methodist Research Institute (HMRI) and Bioengineering R&D Center BioIRC. The laboratory investigations were realized at the HMRI, while the computational models were developed at the HMRI (Milos Kojic and Arturas Ziemys) and executed and supported by the PAK developers at BioIRC (Miljan Milosevic, Vladimir Simic, and Bogdan Milicevic). The book is written in the form of a self-contained textbook with a review of the relevant physical laws followed by finite element modeling. The main topic of the book is mass transport within tissue and tumors and includes a chapter devoted to heart electrophysiology and mechanics. The original methodologies, such as numerical homogenization and multiscale smeared concept are described in detail and applied to typical examples, including those based on the imaging recorded in laboratories. The composite smeared finite element, shown on the cover page represents the basis of the Kojic Transport Model (KTM) applicable to any physical field within a composite medium as tissue. The book includes a chapter describing the generation of problems addressed in the book, using the pre-and post-processor CAD which supports the PAK-solver; the selected examples are available in the accompanying software.



9. **Vladimir Dunić, Dynamics of Structures, (in Serbian: Dinamika konstrukcija), Faculty of Engineering, Kragujevac, 2025.**

This is the textbook (206 pages) represent a combination of analytical and numerical approach for solving of structural dynamics problems. It is related to the Master degree course of Dynamics of structures at the Faculty of Engineering University of Kragujevac. It contains the theory background necessary for the analytical solution of dynamic systems modeled as single-degree of freedom and multi-degree of freedom problems as well as fundamentals of Finite Element Method theory for implicit and explicit dynamic analysis of structures. The instructions for solving structural dynamics problems by using the FEM software package PAK are given through several examples.



B - Journal Papers

1. Anic,M., Savic,S., Milovanovic,A., Milosevic,M., Milicevic,B., Simic,V., Filipovic,N., (2020) Solution of fluid flow through left heart ventricle, *Applied Engineering Letters: Journal of Engineering and Applied Sciences*, 5(4), 120–125, <https://doi.org/10.18485/aeletters.2020.5.4.2>, ISSN: 2466-4847.
2. Blagojevic, M., Nikolic, A., Zivkovic, N.M., Zivkovic, M., Stankovic, G. (2013). Influence of block topologies on endothelial shear stress observed in CFD analysis of artery bifurcation. *Acta Bioengineering and Biomechanics*, Vol. 15, No. 1, 97-104.
3. Blagojevic, M., Nikolic, A., Zivkovic, M., Savic S., (2013) Fluid Structure Interaction on the Example of Real Artery Bifurcation of Random Selected Patient, *Technics – Mechanical Engineering*, Vol.68, No.1, pp. 59-66.
4. Blagojevic, M., Nikolic, A., Zivkovic, N.M., Zivkovic, M., Stankovic, G., Pavlovic A., (2013) Role of oscillatory shear index in predicting the occurrence and development of plaque, *Journal of the Serbian Society for Computational Mechanics*, Vol. 7, No. 2, pp. 29-37.
5. Blagojevic, M., Nikolic, A., Zivkovic, M., Zivkovic, N. M., Stankovic, G., (2014) A novel framework for fluid/structure interaction in rapid subject-specific simulations of blood flow in coronary artery bifurcations, *Vojnosanit Pregl*; 71(3): 285–292.
6. Blagojević, M., Erić, J., Knežević, Lj., Živković, M., Tihaček-Šojić, Lj., (2015) Numerical Modeling of the Edentulous Mandible with a Complete Denture Using Multiblock Method, *Machine Design*, Vol.7, No.1, pp. 19-22.
7. Cirkovic, B., Isailovic, V., Milosevic, Z., Radulovic, J., Sofla, A., Radisic, M., Kojic,M. Filipovic, N., (2012) Analytical and numerical analysis of magnetic separation of cardiomyocytes, *JSSCM*, 6(2), 145-159.
8. Cukanovic, D., Zivkovic, M., Jakovljevic, A., Savic, S., (2013) Applying numerical method in the strength calculation of high-pressure steamline, *Journal of Applied Engineering Science*, 11(2), 254, 99-105.
9. Dimkic, M., Rankovic, V., Filipovic, N., Stojanovic, B., Isailovic, V., Pusic, M., Kojic, M., (2013) Modeling of radial well lateral screens using 1D finite elements, *J. of Hydroinformatics*, 15 (2), 405-415.
10. Dunić, V., Busarac, N., Rakić, D., Slavković, V., Slavković, R., Živković M., (2012) Thermo-mechanical coupling procedure using partitioned approach - Application to arc welding simulation, *Journal of the Serbian Society for Computational Mechanics*, Vol. 6, No. 1, pp. 29-40.
11. Dunić, V., Pieczyska, E., Tobushi, H., Staszczak, M., Slavković, R. (2014) Experimental and numerical thermo-mechanical analysis of shape memory alloy subjected to tension with various stress and strain rates, *Smart Materials and Structures*, 23 (5), 055026 (11pp)
12. Dunić, V., Busarac, N., Slavković, V., Rosić, B., Niekamp, R., Matthies, H., Slavković, R., Živković, M. (2016) A thermo-mechanically coupled finite strain model considering inelastic heat generation, *Continuum Mechanics and Thermodynamics*, 28(4), 993-1007
13. Dunić, V., Pieczyska, E., Kowalewski, Z., Matsui, R., Slavković, R. (2019) Experimental and Numerical Investigation of Mechanical and Thermal Effects in TiNi SMA during Transformation-Induced Creep Phenomena, *Materials*, 12(6), 883
14. Dunić,V., Slavković, R. (2020) Implicit stress integration procedure for large strains of the reformulated Shape Memory Alloys material model, *Continuum Mechanics and Thermodynamics*, 32 (5), 1287-1309
15. Dunić, V., Živković, J., Milovanović, V., Pavlović, A., Radovanović, A., Živković, M. (2021) Two-Intervals Hardening Function in a Phase-Field Damage Model for the Simulation of Aluminum Alloy Ductile Behavior, *Metals*, 11 (11) 1685
16. Dunić, V., Gubelj, N., Živković, M., Milovanović, V., Jagarinec, D., Djordjevic, N. (2024) Experimental Characterization and Phase-Field Damage Modeling of Ductile Fracture in AISI 316L, *Metals*, 14(7) 787
17. Dunić, V., Matsui, R., Takeda, K., Živković, M. (2024) Phase-field damage simulation of subloop loading in TiNi SMA, *International Journal of Damage Mechanics*, 33(8) 577-604

18. Filipović, N., Kojić, M. (2004) Computer simulations of blood flow with mass transport through the carotid artery bifurcation, *Theoret. Appl. Mech. (Serbian)*, 31(1), 1-33.
19. Filipovic, N., Mijailovic, S., Tsuda, A., Kojic, M. (2006) An implicit algorithm within the arbitrary Lagrangian-Eulerian formulation for solving incompressible fluid flow with large boundary motions, *Comp. Meth. Appl. Mech. Eng.*, 95, 6347-6361.
20. Filipovic, N., Ravnica, D., Kojic, M., Mentzer, S.J., Haber, S., Tsuda, A., (2008) Interactions of blood cell constituents: Experimental investigation and computational modeling by discrete particle dynamics algorithm, *Microvascular Research*, 75, 279-284.
21. Filipović, N., Petrović, D., Jovanović, A., Jovanović, S., Balos, D. M. Kojić, (2008) DPD simulation of self-healing process of nanocoating, *J. Serb. Soc. Comp. Mech.*, 2(2), 42-50.
22. Filipovic, N., Haber, S., Kojic, M., Tsuda, A. (2008) Dissipative Particle Dynamics Simulation of Flow Generated by Two Rotating Concentric Cylinders. Part II: Lateral Dissipative and Random Forces. *J. Phys. D: Appl. Phys.* 41:035504.
23. Filipovic, N., Kojic, M., A. Tsuda, A. (2008) Modeling of thrombosis using dissipative particle method, *Philosoph. Trans. Roy. Soc. London, A* 366(1879).
24. Filipovic, N., Ivanovic, M., Kojic, M., (2008) A comparative numerical study between dissipative particle dynamics (DPD) and smooth particle dynamics (SPH) when applied to simple unsteady flows, *Microfluidics and Nanofluidics*, 1613-4982.
25. Filipovic, N., Cvetkovic, A., Isailovic, V., Matovic, Z., Rosic, M., Kojic, M., (2009) Computer simulation of flow and mixing at the duodenal stump after gastric resection, *World Journal of Gastroenterology*, ISSN1007-9327, {wjg.wjgnet.com/ doi:10.3748/wjg.15.0000}.
26. Filipović, N., Petrović, D., Isailović, V., Jovanović, A., Kojić, M. (2010) Modeling of self-healing process in new nanocoating of surfaces by material with containers filled with healing agents, *Contemporary materials*, 1(2), 129-132.
27. Filipovic, N., Kojic, M., Ferrari, M., (2011) Dissipative particle dynamics simulation of circular and elliptical particles motion in 2D laminar shear flow, *Journal of Microfluidics and Nanofluidics*, 1127-1134.
28. Filipovic N., Ivanovic, M., Krstajic, D., Kojic, M., (2011) Hemodynamic Flow Modeling through an Abdominal Aorta Aneurysm Using Data Mining Tools, *IEEE - Transactions on Information Technology in Biomedicine*, B 15, 189-194.
29. Filipovic N., M. Ivanovic, D. Krstajic and M.Kojic, Hemodynamic Flow Modeling through an Abdominal Aorta Aneurysm Using Data Mining Tools, *IEEE - Transactions on Information Technology in Biomedicine*, B 15, 189-194, 2011.
30. Filipovic, N., Kojic, M., Ferrari, M., (2011) Dissipative particle dynamics simulation of circular and elliptical particles motion in 2D laminar shear flow, *Journal of Microfluidics and Nanofluidics*, 1127-1134.
31. Filipovic, N., Rosic, M., Isailovic, V., Milosevic, Z., Nikolic, D., Milasinovic, D., Radovic, M., Stojanovic, B., Ivanovic, M., Tanaskovic, I., Saveljic, I., Milosevic, M., Petrovic, D., Obradovic, M., Themis, E., Sakellarios, A., Siogkas, P., Marraccini, P., Vozzi, F., Meunier, N., Teng, Z., Fotiadis, D., Parodi, O., Kojic, M. (2011) ARTREAT project: computer, experimental and clinical analysis of three-dimensional plaque formation and progression in arteries; *Journal of the Serbian Society for Computational Mechanics*, 5(2), 129-146.
32. Filipović, N., Petrović, D., Obradović, M., Jovanović, A., Jovanović, S., Baloš, D., Kojić, M., (2011) Modeling of self-healing materials using nanocontainers, *Contemporary materials*, 2(1) 18-26.
33. Filipovic, N., Isailovic, V., Djukic, T., Ferrari, M., Kojic, M., (2012) Multi-scale modeling of circular and elliptical particles in laminar shear flow, *IEEE Trans Biomed Eng.*, 59 (1):50-53.
34. Filipovic, N., Jovanovic, A., Petrovic, D., Obradovic, M., Jovanovic, S., Balos, D., Kojic, M., (2012) Modelling of self-healing materials using discrete and continuum methods, *Surface Coatings International*, 95(2), 74-79.
35. Filipovic, N., Radovic, M., Isailovic, V., Milosevic, Z., Nikolic, D., Saveljic, I., Milosevic, M., Petrovic, D., Obradovic, M., Krsmanovic, D., Themis, E., Sakellarios, A., Siogkas, P., Marraccini, P., Vozzi, F., Meunier, N., Teng, Z., Fotiadis, D., Parodi, O., Kojic, M., (2012) Plaque formation and stent deployment with heating thermal effects in arteries, *JSSCM – Special Issue Vol. 6(1)*, 11-28.

36. Filipovic,N., Gibney, B., Kojic, M., Nikolic, D., Isailovic, V., Ysasi, A., Konerding, M., Mentzer, S., Tsuda,A., (2013) Mapping cyclic stretch in the postpneumectomy murine lung, *J. Applied Physiology*, 115(9), 1370-1378.
37. Filipovic, N., Meunier, N., Fotiadis, D., Parodi, O., Kojic, M., (2014) Three-dimensional numerical simulation of plaque formation in arteries, in M. Garbey, B. Lee Bass, S. Berceli, C. Collet, and P. Cerveri, Eds: *Computational Surgery and Dual Training*, Springer, 257-264.
38. Filipovic,N., Nikolic,D., Isailovic,V., Milosevic,M., Geroski, V., Karanasiou, G., Fawdry,M. , Flanagan, A., Fotiadis, D., Kojic,M., (2021) In vitro and in silico testing of partially and fully bioresorbable vascular scaffold, *J. Biomechanics*, DOI 10.1016/j.jbiomech.2020.110158, *J Biomech.* 2021 Jan 22;115:110158.
39. Filipovic,N., Saveljic,I., Sustersic,T., Milosevic, M., Milicevic,B., Simic,V., Ivanovic,M., Kojic,M., (2022) *In Silico* Clinical Trials for Cardiovascular Disease, *JoVE*, PMID:35695532, DOI:10.3791/63573.
40. Filipovic,N., Sustersic,T., Milosevic,M., Milicevic,B., Simic,V., Prodanovic,M., Mijailovic,S., Kojic,M.,(2022) SILICOFM platform, multiscale modeling of left ventricle from echocardiographic images and drug influence for cardiomyopathy disease, *Computer Methods and Programs in Biomedicine*, 227, 107124.
41. Filipovic, N., Nikolic, M., Sustersic, T. (2020). Simulation of organ-on-a-chip systems, Book: *Biomaterials for Organ and Tissue Regeneration – New Technologies and Future Prospects* (ed. Nihal Vrana, Helena Knopf-Marques and Julien Barthes), Elsevier, Chapter 28, pp. 753-790. ISBN: 9780081029060
42. Fine, D., Grattoni, A., Hosali, S., Ziemys, A., De Rosa, Gill J., Medema, R., Hudson, L., Kojic, M., Milosevic, M., Brousseau, L. III, Goodall, R., Ferrari, M., Liu, X., (2010) A robust nanofluidic membrane with tunable zero-order release for implantable dose specific drug delivery, *Lab on a Chip*, DOI: 10.1039/c0lc00013b.
43. Geroski, T., Filipović, N. (2024). Artificial Intelligence Empowering Medical Image Processing. In *In Silico Clinical Trials for Cardiovascular Disease: A Finite Element and Machine Learning Approach* (pp. 179-208). Cham: Springer Nature Switzerland. https://doi.org/10.1007/978-3-031-60044-9_7
44. Geroski, T., Gkaintes, O., Vulović, A., Ukaj, N., Barrasa-Fano, J., Perez-Boerema, F., Filipović, N. (2024). SGABU computational platform for multiscale modeling: Bridging the gap between education and research. *Computer methods and programs in biomedicine*, 243, 107935. <https://doi.org/10.1016/j.cmpb.2023.107935>
45. Geroski, T., Jakovljević, D., & Filipović, N. (2023). Big Data in multiscale modelling: from medical image processing to personalized models. *Journal of Big Data*, vol. 10, no 1, pp. 1-22. ISSN: 2196-1115, <https://doi.org/10.1186/s40537-023-00763-y>
46. Geroski,V., Milosevic,M., Simic,V., Milicevic,B., Filipovic,N., Kojic,M., (2020) Composite Smeared Finite Element – Application to Electrical Field, *Computational Bioengineering and Bioinformatics, ICCB 2019, Learning and Analytics in Intelligent Systems*, Springer Cham, 11, 35–43, https://doi.org/10.1007/978-3-030-43658-2_4, ISBN: 978-3-030-43657-5.
47. Grujovic, N., Divac, D., Zivkovic, M., Slavkovic, R., Milivojevic, N., Milivojevic, V., Rakic, D., (2013) An inelastic stress integration algorithm for a rock mass containing sets of discontinuities, *Acta Geotechnica*, Vol.8, No.3, pp. 265-278, ISSN 1861-1125, Doi 10.1007/s11440-012-0194-3.
48. Isailovic, V., Kojic, M., Milosevic, M., Filipovic, N., Kojic, N., Ziemys, A., Ferrari, M., (2014) A computational study of trajectories of micro- and nano-particles with different shapes in flow through small channels, *J. Serb. Soc. Comp. Mech.*, 8 (2), 14-28.
49. Jovicic, G., Zivkovic, M., Jovicic, N., (2005) Numerical Modeling of Crack Growth Using the Level Set Fast Marching Method, *FME Transactions* 33, 11-19.
50. Jovićić, G., Živković, M., Maksimović, K., Đorđević, N., (2008) The crack Growth Analysis on a Real Structure Using the X-FEM and EFG Methods, *Scientific Technical Review*, Vol.LVIII, No.2, 21-26.
51. Jovicic, G., Zivkovic, M., Jovicic, N., (2009) Numerical Simulation of Crack Modeling using Extended Finite Element Method, *Strojniški vestnik - Journal of Mechanical Engineering*, 55(9), 549-554.

52. Jovicic, G., Grabulov, V., Maksimovic, S., Zivkovic, M., Jovicic, N., Boskovic, G., Maksimovic, K., (2009) Residual life estimation of a thermal power plant component: The high-pressure turbine housing case, *Thermal Science*: Vol. 13, No. 4, pp. 99-106.
53. Jovicic, G., Zivkovic, M., Jovicic, N., Milovanovic, D., Sedmak, A., (2010) Improvement of algorithm for numerical crack modelling, *Archives of Civil and Mechanical Engineering*, Vol. 10(3), 19-35.
54. Jovicic G., Nikolic R., Zivkovic M., Milovanovic D., Jovicic N., Maksimovic S., Djokovic J., (2013) An estimation of the high-pressure pipe residual life, *Archives of Civil and Mechanical Engineering*; 13(1), 36-44.
55. Kai, M., Ziemys, A., Liu, Y. T., Kojic, M., Ferrari, M., Yokoi, K., (2019) Tumor-Site Dependent Transport Properties Determine Nanotherapeutics Delivery and Its Efficacy, *Translational Oncology*, 12(9), 1196–1205.
56. Kiseliovas, V., Milosevic, M., Kojic, M., Mazutis, L., Kai, M., Liu, Y. T., Yokoi, K., Ferrari, M., Ziemys, A., (2017) Tumor progression effects on drug vector access to tumor-associated capillary bed, *J. Controlled Release*, <http://dx.doi.org/10.1016/j.jconrel.2017.05.031>.
57. Kojić, M., Slavković, R., Pavić, Dj., Marinković, M. (1997), Application of the finite element method on thin-walled beams, (in Serbian) *Motors and Motor Vehicles (MVM)*, 15 (entire issue).
58. Kojić, M., Slavković, R., Pavić, Dj., Marinković, M. (1978), The results achieved in the development of computational programs applied to automobiles in the factory Zavodi "Crvena Zastava", (in Serbian) *MVM*, 22.
59. Kojić, M., Petronijević, Ž. (1978), The fundamentals of the finite element method, applications in technical practice, and perspectives of the application of computers in structural analysis (in Serbian) *MVM*, 22.
60. Kojić, M., Grujović, D., Janković, A., Nikolić, V. (1980), Analysis of the stress-state and displacements of a gear tooth, *Tehnika, Mechanical Engineering*, 9.
61. Kojić, M., Živković, G. (1981), Numerical determination of the temperature field within the disc of the brakes during braking period of the automobile, (in Serbian) *MVM*, 39-40.
62. Kojić, M., Nikolić, V. (1981), Analysis and design of gears and automobile transmission by application of computers, (in Serbian) *MVM*, 39-40.
63. Kojić, M., Savić, R., Nikolić, V., (1982), Analysis of the stress-state and deformation within a joint of the connecting bar of a passenger car by application of the finite element method and experimental method, in case of static loading, (in Serbian) *MVM*, 46-47.
64. Kojić, M., Petronijević, Ž., Manojlović, V., (1982), Analysis of acoustic characteristics of automobile interior by application of 3D isoparametric finite elements, (in Serbian) *Analiza akustičkih karakteristika unutrašnjosti automobila primenom prostornih izoparametarskih konačnih elemenata*, *Tehnika, Mechanical Engineering*.
65. Kojić, M., Milovanović, M., Slavković, R. (1982) A comparative analysis of the stiffness of the front structure and the floor of a car, for different design solutions (in Serbian) *MVM*, 82.
66. Kojić, M., Petronijević, Ž., Manojlović, V., (1983), Determination of the influence of the design parameters of the car body on the internal acoustic characteristics by application of 3D isoparametric finite elements, (in Serbian), *MVM*, 50-51.
67. Kojić, M., Nikolić, V., (1983), Application of the finite element method for determining deformation and stress-state of a gear of a motor vehicle, (in Serbian), *MVM*, 50-51.
68. Kojić, M., Petronijević, Ž., Manojlović, V., (1983), Acoustic analysis of the car interior with moving walls by application of the finite element method, (in Serbian), *MVM*, 50-51.
69. Kojić, M., Grujović, D., Živković, G., Živulović, M., (1983), Program package for structural analysis and its application on a plastic block, (in Serbian) *Scientific-Technical Review*, Vol. XXXIII.
70. Kojić, M., Micić, M., (1983), A contribution to solving linear and nonlinear shell structures by application of the finite element method, (in Serbian), *Zastava*, 1.
71. Kojić, M., Milovanović, M., Slavković, R., (1984), A contribution to the application of the FEM in the design of the car floor at the initial design stage, (in Serbian), *Zastava*, 2.

72. Kojić, M., Milovanović, M., (1984), Analysis of the characteristics of the cross-section of a lateral supporting element of a car on the maximal stress within the supporting element, (in Serbian), MVM, 54-55.
73. Kojić, M., Nikolić, V., (1984), A contribution to the study of the stress-state within a conical gear by application of the FEM, (in Serbian), 59.
74. Kojic, M, Petronijevic, Z., Manojlovic, V., (1984), Influence of car body constructive parameters on acoustic characteristics of car cavity, *Int. J. of Vehicle Design*, 5(6).
75. Kojic, M., Bathe, K. J., (1987), The "effective stress-function" algorithm for thermo-elasto-plasticity and creep, *Int. J. Num. Meth., Engng.*, 24,1509-1532.
76. Kojic, M., Bathe, K. J., (1987), Thermo-elastic-plastic and creep analysis of shell structures, *Computers and Structures*, 26(1/2),135-143.
77. Kojic, M., Bathe, K. J., (1987), Studies of finite element procedures - stress solution at a closed elastic strain path with stretching and shearing using updated Lagrangian Jaumann formulation, *Computers and Structures*, 26(1/2), 175-179.
78. Kojic, M., (1993), Implicit stress integration for elastic-plastic deformation of von Mises material with mixed hardening, *Theoretical and Applied Mechanics*, 19, 59-71.
79. Kojic, M., Grujovic,N., Slavkovic,R., Kojic, A., (1995) Solution procedure for elastic-plastic orthotropic multilayered pipe deformation under internal and external pressure, *AIAA Journal*, 34(12), 2354-2358.
80. Kojic,M., Zivkovic,M., Kojic,A., (1995) Elastic-plastic analysis of orthotropic multilayered beam, *Computers and Structures*, 57(2), 205-211.
81. Kojic,M., Grujovic,N., Slavkovic,R., Zivkovic,M., (1996) A general orthotropic von Mises plasticity material model with mixed hardening - model definition and implicit stress integration procedure, *Transactions of ASME J. Applied Mechanics*, 63, 376-382.
82. Kojic,M., (1996) The governing parameter method for implicit integration of viscoplastic constitutive relations for isotropic and orthotropic metals, *Computational Mechanics*, 19(1), 49-57.
83. Kojic,M., Filipovic,N., Mijailovic, S.,(1997) A general formulation for finite element analysis of flow through a porous deformable medium, *Theoretical and Applied Mechanics (Yugoslavian)*, 23, 67-81.
84. Kojic, M., Mijailovic, S., Zdravkovic,N., (1998) A numerical algorithm for stress integration of a fiber-fiber kinetics model with Coulomb friction for connective tissue, *Computational Mechanics*, 21(2), 189-198.
85. Kojic, M., Mijailovic, S., Zdravkovic, N., (1998) Modelling of muscle behavior by the finite element method using Hill's three-element model, *Int. J. Num. Meth. Engng.*, 43, 941-953.
86. Kojic, M., Filipovic, N., Vulovic,S., Mijailovic, S., (1998) A finite element solution procedure for porous medium with fluid flow and electromechanical coupling, *Comm. Num. Meth. Engng*, 14, 381-392.
87. Kojic, M., Filipovic, N., Mijailovic, S., (2001) A large strain finite element analysis of cartilage deformation with electrokinetic coupling, *Comp. Meth. Appl. Mech. Engng.*, 190, 2447-2464.
88. Kojic, M., Vlastelica, I., Zivkovic, M., (2002) Implicit stress integration procedure for large strain deformation of Gurson model, *Int. J. Num. Meth. Engng.*, 53, 2701-2720.
89. Kojic, M., (2002) Stress integration procedures for inelastic material models within finite element method, review paper, *J. Appl. Mech. Reviews*, 55, 389-414.
90. Kojic, M. (2002), An extension of 3-D procedure to large strain analysis of shells, *Comp. Meth. Appl. Mech. Engng.*, 191, 2447-2462.
91. Kojic, M., Zdravkovic,N., Mijailovic, S., (2003) A numerical stress calculation procedure for a fiber-fiber kinetics model with Coulomb and viscous friction of connective tissue, *Computational Mechanics*, 30, 185-195.
92. Kojic, M., Vlastelica, I., Zivkovic, M., (2004) Implicit stress integration algorithm for Gurson model in case of large strain shell deformation, *Facta Universitatis (Nis)*, 4(16), 85-99.
93. Kojic, M., Tsuda, A.,(2004) Gravitational deposition of aerosols from oscillatory laminar pipe flows, *J. Aerosol Science*, 35, 245-261.
94. Kojic, N., Kojic, M., Gudlavalleti, S., McKinley, G., (2004) Solvent removal during synthetic and Nephila fiber spinning, *Biomacromolecules*, 5(5), 1698-1707.
95. Kojic, M.,Vlastelica,I., Stojanovic, B., Rankovic, V., Tsuda, A., (2006) Stress integration procedures for a biaxial isotropic material model of biological membranes and for hysteretic

- models of muscle fibers and surfactant, *Int. J. Num. Meth. Engng.*, 68, 893-909.
96. Kojic, N., Kojic, M., Tschumperlin, D., (2006) Computational modeling of extracellular mechanotransduction, *Biophysical Journal*, 90 (4, 11), 4261-4270.
 97. Kojic, N., Kojic, A., Kojic, M., (2006) Numerical determination of the solvent diffusion coefficient in a concentrated polymer solution, *Comm. Num. Meth. Eng.*, 22, 1003-1013.
 98. Kojic, M., (2007) Serbian Society for Computational Mechanics – a brief overview, *IACM Expressions*, 21, 18-21.
 99. Kojić, M., Filipović, N., Stojanović, B., Ranković, V., Krstić, M., Otašević, L., Ivanović, M., Nedeljković, M., Dimkić, M., Tričković, M., Pušić, M., Boreli-Zdravković, Đ., Đurić, D., (2007) Finite element modeling of underground water flow with Ranney wells, *Water Science & Technology: Water Supply*, 7(3), 41–50.
 100. Kojić, M., Isailović, V., Stojanović, B., Filipović, N., (2007) Modeling of cell mechanical response by biphasic models with activation, *J. Serb. Soc. Comp. Mech.*, 1, 135-143.
 101. Kojic,M., Filipovic, N., Tsuda, A. (2008) A mesoscopic bridging scale method for fluids and coupling dissipative particle dynamics with continuum finite element method, *Comp. Meth. Appl. Mech. Eng.*, 197, 821-833.
 102. Kojic, M., (2010) On the stress integration for nonlinear material models within the finite element method, *Bulletin CXLII de l'Académie serbe des sciences et des arts Classe des sciences techniques* 32, 47-82.
 103. Kojić,N., Huang, A., Chung, E., Ivanović, M., Filipović, N., Kojić,M., Tschumperlin, D. J., (2010) A 3-D model of ligand transport in a deforming extracellular space, *Biophysical Journal*, 99, 3517–3525.
 104. Kojic, M., Vlastelica, I., Decuzzi, P., Granik, V. T. , Ferrari, M. (2011) A finite element formulation for the doublet mechanics modeling of microstructural materials, *Comp. Meth. Appl. Mech. Engng.*, 200, 1446–1454.
 105. Kojic, M., Milosevic,M., Kojic, N., Ferrari,M., Ziemys,A., (2011) On diffusion in nanospace, *J. Serbian Soc. Comp. Mechanics*, 5(1), 84-109.
 106. Kojic, M., Ziemys, A., Milosevic, M., Isailovic, V., Kojic,N., Rosic,M., Filipovic, N., Ferrari, M., (2011) Transport in biological systems, *J. Serbian Soc. Comp. Mechanics*, 5(2), 101-128.
 107. Kojic, M., Vlastelica, I., Decuzzi, P., Gentile, V., Ferrari, M., (2011) A Microstructural Doublet Mechanics Finite Element Formulation, *Monograph of South Slavic Academy for Nonlinear Sciences*, (3), 2011.
 108. Kojic, M., Butler, J. P., Vlastelica, I., Stojanovic, B., Rankovic, V., Tsuda, A., (2011) Geometric hysteresis of alveolated ductal architecture, *ASME J. Biomechanics*, 133 / 111005-1- 11.
 109. Kojic, N., Panzer, M. J., Leisk, G. G., Raja, W. K., Kojic, M., Kaplan, D.L. , (2012) Ion electrodiffusion governs silk electrogelation, *Soft Matter* (cover article), 8, 6897-6905.
 110. Kojic, M., Filipovic,N., (2012) Computational mechanics in science, applications and teaching, *Theoretical and Appl. Mechanics*, 40(S1),147-162.
 111. Kojic, M., (2013) Simple concepts in computational mechanics – do they really work?, *J. Serbian Soc. Comp. Mech.*, 7(1), 1 – 15.
 112. Kojić, M., Milošević, M., Kojić, N., Ferrari, M., Ziemys, A. (2013) Numerical modeling of diffusion in complex media with surface interaction effects, *Contemporary Materials*, 2, 153-166.
 113. Kojic, M., Milosevic,M., Kojic,N., Kim, K., Ferrari, M., Ziemys, A., (2014) A multiscale MD–FE model of diffusion in composite media with internal surface interaction based on numerical homogenization procedure, *Comput. Methods Appl. Mech. Engng.* 269, 123–138.
 114. Kojic, M., Milosevic, M., Simic,V., Ferrari, M., (2014) A 1D pipe finite element with rigid and deformable walls, *J. Serb. Soc. Comp. Mech.*, 8 (2), 38-53.
 115. Kojic,M, Milosevic,M., Kojic,N., Isailovic, V., Petrovic, D., Filipovic, N., Ferrari,M., Ziemys, A., (2015) Transport phenomena: Computational models for convective and diffusive transport in capillaries and tissue, in: Suvranu De, Wonmuk Hwang, Ellen Kuhl, Eds., *Multiscale Modeling in Biomechanics and Mechanobiology*, Springer, Chapter 7, 131-156.
 116. Kojic, M., Milosevic, M., Kojic, N., Starosolski, Z., Ghaghada, K., Serda, R., Annapragada, A., Ferrari, M., Ziemys, A.,(2015) A multi-scale FE model for convective-diffusive drug transport within tumor and large vascular networks, *Comput. Methods Appl. Mech. Engng.*, 294,100–122.
 117. Kojic,M., Milosevic,M., Wu,S., Blanco,E., Ferrari,M., Ziemys,M., (2015) Mass partitioning effects in diffusion transport, *Phys. Chem. Chem. Phys.*, 17, 20630-20635.

118. Kojić, N., Milošević, M., Petrović, D., Isailović, V., Sarioglu, A. F., Haber, D. A., Kojić, M., Toner, M., (2015) A computational study of circulating large tumor cells traversing microvessels, *Computers in Biology and Medicine* (among 10 the best in 2015), 63, 187–195.
119. Kojić, M. (2016) , A review of models in bioengineering developed by group lead by Miloš Kojić in period 2006-2016, *J. Serb. Soc. Comp. Mech.*, 10 (1), 5-19.
120. Kojic, M., Milosevic, M., Simic, V., Koay, E. J., Fleming, J. B., Nizzero, S., Kojic, N., Ziemys, A., Ferrari, M., (2017) A composite smeared finite element for mass transport in capillary systems and biological tissue, *Comp. Meth. Appl. Mech. Engrg.*, 324, 413–437, <https://doi.org/10.1016/j.cma.2017.06.019>.
121. Kojic, M., Milosevic, M., Simic, V., Stojanovic, D., Uskokovic, P. (2017) A radial 1D Finite Element for Drug Release from Drug Loaded Nanofibers, *J. Serb. Soc. Comp. Mech.*, 11(1), 82-93.
122. Kojic, M., Simic, V., Milosevic, M., (2017) Incremental Finite Element Formulation for Large Strains Based on the Nodal Force Increments, *J. Serb. Soc. Comp. Mech.*, 11(1), 97-109.
123. Kojic, M., Milosevic, M., Simic, V., Koay, E. J., Kojic, N., Ziemys, A., Ferrari, M., (2017) Extension of the Composite Smeared Finite Element (CSFE) to Include Lymphatic System in Modeling Mass Transport in Capillary Systems and Biological Tissue, *J. Serb. Soc. Comp. Mech.*, 11(2), 108-120.
124. Kojic, M.; Simic, V.; Milosevic, M., (2017) Composite smeared finite element – some aspects of the formulation and accuracy, *IPSI Transactions on Advanced Research*, 13(2), 1820 – 4511.
125. Kojic, M., Milosevic, M., Kojic, N., Koay, E. J., Fleming, J. B., Ferrari, M., Ziemys, A. (2018) Mass release curves as the constitutive curves for modeling diffusive transport within biological tissue, *Computers in Biology and Medicine*, 92, 156–167.
126. Kojic, M., Milosevic, M., Simic, V., Koay, E. J., Kojic, N., Ziemys, A., Ferrari, M., (2018) Multiscale smeared finite element model for mass transport in biological tissue: from blood vessels to cells and cellular organelles, *Computers in Biology and Medicine*, 99, 7-23, 2018.
127. Kojic, M., (2018) Smeared concept as a general methodology in finite element modeling of physical fields and mechanical problems in composite media, *J. Serb. Soc. Comp. Mech.*, 12(2), 1-16.
128. Kojic, M., Milosevic, M., Simic, V., Ziemys, A., Filipovic, N., Ferrari, M., (2019) Smeared multiscale finite element model for electrophysiology and ionic transport in biological tissue, *Computers in Biology and Medicine*, 108, 288-304.
129. Kojic, M., Milosevic, M., Simic, V., Milicevic, B., Geroski, V., Nizzero, S., Ziemys, A., Filipovic, N., Ferrari, M., (2019) Smeared multiscale finite element models for mass transport and electrophysiology coupled to muscle mechanics, *Frontiers in Bioengineering*, doi: 10.3389.fbioe.2019.00381.
130. Kojic, M., (2020) Multiscale composite 3d finite element for lung mechanics, *J. Serbian Soc. Comp. Mech.* 14(1), 1-11.
131. Kojic, M., Milosevic, M., Milicevic, B., Simic, V., (2020) Heart mechanical model based on Holzapfel experiments, *Computational Bioengineering and Bioinformatics, ICCB 2019, Learning and Analytics in Intelligent Systems*, Springer Cham, 11, 12–21, https://doi.org/10.1007/978-3-030-43658-2_2, ISBN: 978-3-030-43657-5.
132. Kojic, M., Milosevic, M., Simic, V., Geroski, V., Milicevic, B., Ziemys, A., Filipovic, N., (2020) Finite Element Models with Smeared Fields Within Tissue – A Review of the Current Developments, *Computational Bioengineering and Bioinformatics, ICCB 2019, Learning and Analytics in Intelligent Systems*, Springer Cham, 11, 22–34, https://doi.org/10.1007/978-3-030-43658-2_3, ISBN: 978-3-030-43657-5.
133. Kojic, M., Milosevic, M., Milicevic, B., Geroski, V., Simic, V., Trifunovic, D., Stankovic, G., Filipovic, N. (2021) Computational model for heart tissue with direct use of experimental constitutive relationships, *J. Serb. Soc. Comput. Mech.*, 15(1), 1-23.
134. Kojic, M., (2023) A multiscale multiphysics finite element for lung, *Journal of the Serbian Society for Computational Mechanics*, 17 (2), 1-15.
135. Kojic, M., Milosevic, M., Simic, V., Milicevic, B., Terracciano, R., Filgueira, C. S. (2024), On the generality of the finite element modeling physical fields in biological systems by the multiscale smeared concept (Kojic transport model), *Heliyon* 10 (2024) e26354.
136. Kostić, S., Rakić, D., Bodić, A., Nedeljković, S., Milivojević, N., (2024) Dynamic response of existing embankment dams in specific geotechnical and seismological conditions: contemporary

- framework for Serbian national guidelines, *Frontiers in Earth Science*, Vol.12, No.2024, pp. 1-19, ISSN 2296-6463, Doi 10.3389/feart.2024.1358928.
137. Mahadevan, T. S. , Milosevic,M., Kojic, M., Hussain, F., Kojic,N., Serda, R., Ferrari, M., Ziemys, A., (2013), Diffusion transport of nanoparticles at nanochannel boundaries, *J. of Nanoparticle Research*, 15, 1477-1487.
 138. Mahadevan, T. S. , Kojic, M. , Ferrari, M., Ziemys, A., (2013) Mechanisms of reduced solute diffusivity at nanoconfined solid-liquid interface, *Chemical Physics*, 421, 15-21.
 139. Martinez, J. O., Chiappini, C. , Ziemys, A., Faust, A. M., Kojic, M., Liu, X., Ferrari, M., Tasciotti, E. , (2013) Engineering multi-stage nanovectors for controlled degradation and tunable release kinetics, *J. Biomaterials*, 34, 8469-8477.
 140. Martino, A., Terracciano,R., Milićević,B., Milošević,M., Simić,V., Fallon, B. C., Carcamo-Bahena, Y., Royal, A.L.R., Carcamo-Bahena,A.A.E., Butler, B., Willson, R. C., Kojić, M., Filgueira, C.S.,(2024), An Insight into Perfusion Anisotropy within Solid Murine Lung Cancer Tumors, *Pharmaceutics*,doi: 10.20944/preprints202406.1818.v1.
 141. Mijailovich, S., Kojic, M., Zivkovic,M., Fabry, B. , Fredberg, J., (2002) A finite element model of cell deformation during magnetic bead twisting, *J. Appl. Physiol.*, 93, 1429-1436.
 142. Mijailovich, S.,Kojic, M., Tsuda, A., (2010) Particle-induced indentation of the alveolar epithelium caused by surface tension forces, *Journal of Applied Physiology*, 109: 1179–1194.
 143. Mijailovich, S.M., Stojanovic, B., Kojic, M., Liang, A., Wedeen, V.J., Gilbert, R.J., (2020) Derivation of a finite-element model of lingual deformation during swallowing from the mechanics of mesoscale myofiber tracts obtained by MRI, *J Appl Physiol.*,109,1500–1514.
 144. Milićević,B., Ivanović,M., Stojanović,B., Milošević,M., Kojić,M., Filipović,N., (2022) “Huxley muscle model surrogates for high-speed multi-scale simulations of cardiac contraction,” *Computers in Biology and Medicine*, (149) Elsevier BV, p. 105963.
 145. Milicevic,B., Milosevic,M., Simic,V., Trifunovic,D., Stankovic,G., Filipovic,N., Kojic,M., (2023), Cardiac hypertrophy simulations using parametric and echocardiography-based left ventricle model with shell finite elements, *Computers in Biology and Medicine* 157 106742.
 146. Milićević,B., Milošević,M., Simić,V., Preveden, A., Velicki,L., Jakovljević, Đ., Bosnić, Z., Pičulin,M., Žunković,B., Kojić,M., Filipović,N., (2023) Machine learning and physical based modeling for cardiac hypertrophy, *HELIYON*, doi: <https://doi.org/10.1016/j.heliyon.2023.e16724>.
 147. Milicevic,B., Filipovic,N., (2024) Use Case: AI-Based Surrogate Muscle Models for Cardiac Cycle Simulations of the Left Ventricle, In *Silico Clinical Trials for Cardiovascular Disease: A Finite Element and Machine Learning Approach*, Springer Nature Switzerland, 363–398, https://doi.org/10.1007/978-3-031-60044-9_13.
 148. Milosevic,M., Simic,V., Milicevic,B., Koay, E.J., Ferrari,M., Ziemys,A., Kojic,M., (2018) Correction function for accuracy improvement of the Composite Smeared Finite Element for diffusive transport in biological tissue systems, *Comp. Meth. Appl. Mech. Engrg.*, <https://doi.org/10.1016/j.cma.2018.04.012>.
 149. Milosevic,M., Stojanovic,D., Simic ,V., Milicevic ,B., Radisavljevic,A., Uskokovic,P., Kojic,M., (2018) A Computational Model for Drug Release from PLGA Implant, *Materials* 11(12), 2416.
 150. Milosevic,M., Stojanovic, D. B., Simic,V., Grkovic, M., Bjelovic, M., Uskokovic, P. S., Kojic,M. (2020) Preparation and modeling of three-layered PCL/PLGA/PCL fibrous scaffolds for prolonged drug release, *Scientific Reports*, <https://doi.org/10.1038/s41598-020-68117-9>.
 151. Milosevic,M., Anic,M., Nikolic,D., Geroski,V., Milicevic,B., Kojic,M., Filipovic,N., (2021) Application of in silico Platform for the Development and Optimization of Fully Bioresorbable Vascular Scaffold Designs, *Frontiers in Medical Technology*, 3(55), –, <https://doi.org/10.3389/fmedt.2021.724062>, ISSN: 2673-3129.
 152. Milosevic,M., Anic,M., Nikolic,D., Milicevic,B., Kojic,M., Filipovic,N., (2022) InSilc Computational Tool for In Silico Optimization of Drug-Eluting Bioresorbable Vascular Scaffolds, *Computational and Mathematical Methods in Medicine*, 2022(-), 1–14, <https://doi.org/10.1155/2022/5311208>, ISSN: 1748-6718.
 153. Milosevic,M., Simic,V., Nikolic,A., Shao,N., Hashimoto, C. K., Godin,B., Leonard, F., Liu,X., Kojic,M.,(2023) Modeling critical interaction for metastasis between circulating tumor cells

- (CTCs) and platelets adhered to the capillary wall, *Computer Methods and Programs in Biomedicine* 242,107810.
154. Milosevic M., Milicevic, M., Simic, V., Anic, M., Kojic, M., Jakovljevic, Dj., Filipovic, N. (2023) Application of In Silico Trials for the Investigation of Drug Effects on Cardiomyopathy-Diseased Heart Cycle Properties, *Appl. Sci.*, 13,11780.
 155. Milovanović, V., Dunić, V., Rakić, D., Živković, M., (2013) Identification causes of cracking on the underframe of wagon for containers transportation – Fatigue strength assessment of wagon welded joints, *Engineering Failure Analysis*, Vol 31, 118-131.
 156. Milovanovic V., Rakic D., Zivkovic M., (2013) Thermo-mechanic analysis of cement transport wagon - identification of the cause of cracks. *Annals of Faculty Engineering Hunedoara-International Journal of Engineering*, 11(4), 309–314.
 157. Milovanovic, V., Zivkovic, M., Disic, A., Rakic, D., Zivkovic, J., (2014) Experimental and Numerical Strength Analysis of Wagon for Transporting Bulk Material, *IMK-14 – Research & Development in Heavy Machinery* 20 (2), EN61-66.
 158. Milovanović, V., Živković, M., Jovičić, G., Kozak D., (2016) The Analysis of Choice Influence in Fatigue Failure Criteria on Integrity Assessment of Wagon Structure, *Tehnički vjesnik* 23 (3), 701-705.
 159. Miljković, O., Ivanović, M., Filipović, N., Kojić, M. (2008) AI models of the hemodynamic simulation, *J. Serb. Soc. Comp. Mech.*, 2(2), 59-72.
 160. Nikolic, A., Blagojevic, M., Zivkovic, M., Aleksic, A., Savic, S. (2012). Software Technologies for the Analysis of Blood Flow in the Human Body, *International Journal of Industrial Engineering and Management*, 3(2), 99–104.
 161. Nikolic, R., Radovanovic, M., Zivkovic, M., Nikolic A., Rakic, D., Blagojevic, M., (2014) Modeling of thermoelectric module operation in inhomogeneous transient temperature field using finite element method, *Thermal Science*, Vol. 18(1), 239-250.
 162. Norvaisas, P., Kojic, M., Milosevic, M., Ziemys, A., (2013) Prediction and Analysis of Drug Delivery Systems: From Drug-Vector Compatibility to Release Kinetics, *CRS_Newsletter* 09, 30,14-15.
 163. Pavlovic, A., Zivkovic, M., Vulovic S., (2022) Self-Adjusting Handbrake Mechanism Design, *J. Eng. Manag. Syst. Eng.*, vol. 1, no. 2, pp. 51-57.
 164. Pieczyska, P., Staszczak, M., Dunić, V., Slavković, R., Tobushi, H., Takeda, K. (2014) Development of stress-induced martensitic transformation in TiNi Shape Memory Alloy, *Journal of Materials Engineering and Performance*, 23(7), 2505-2514
 165. Pieczyska, E., Kowalewski, Z., Dunić, V. (2017) Stress Relaxation Effects in TiNi SMA During Superelastic Deformation: Experiment and Constitutive Model, *Shape Memory and Superelasticity*, 3(4), 392-402
 166. Radlović, S., Kojić, M., Petrović, Z., Vlastelica, I., Stojanović, B. (1998). Finite element method and possibility of its application in implantology, (in Sebian), *Stom. Glasnik*, (45), 137-140.
 167. Rakić, D., Živković, M., Slavković, R., Kojić, M., (2008) Stress integration for the Drucker-Prager material model without hardening using the incremental plasticity theory, *J. Serb. Soc. Comp. Mech.*, 2(1), 80-89.
 168. Rakić, D., Živković, M., (2015) Stress integration of the Drucker-Prager material model with kinematic hardening, *Theoretical and applied mechanics*, Vol.42, No.3, pp. 201-209, ISSN 1450-5584, Doi 10.2298/TAM1503201R.
 169. Rakic, D., Dunic, V., Zivkovic, M., Grujovic, N., Divac, D., (2019) Modeling of damaged concrete using initial degradation parameter, *Journal of the Serbian society for computational mechanics*, Vol.13, No.2, pp. 8-18, ISSN 1820-6530, Doi 10.24874/jsscm.2019.13.02.02.
 170. Rakić, D., Živković, M., (2020) Elastoplastic constitutive model for granular soil based on hyperbolic failure surface, *Građevinar*, Vol.72, No.2, pp. 115-125, ISSN 0350-2465, Doi 10.14256/JCE.2272.2017.
 171. Rakić D., Bodić, A., Milivojević, N., Dunić, V., Živković, M., (2021) Concrete damage plasticity material model parameters identification, *Journal of the Serbian Society for Computational Mechanics*, Vol.15, No.2, pp. 111-122, ISSN 1820-6530, Doi 10.24874/jsscm.2021.15.02.11.
 172. Rakić, D., Stojković, M., Ivetić, D., Živković, M., Milivojević, N., (2022) Failure Assessment of Embankment Dam Elements: Case Study of the Pirot Reservoir System, *Applied Sciences*, Vol.12, No.2, pp. 558, ISSN 2076-3417, Doi <https://doi.org/10.3390/app12020558>.

173. Rakić, D., Dunić, V., Živković, M., Radovanović, S., Divac, D., Šumarac, D., (2023) Strength reduction method for a factor of safety determination of damaged concrete structures, *International Journal of Damage Mechanics*, Vol.32, No.10, pp. 1125-1143, ISSN 1056-7895, Doi 10.1177/10567895231183469.
174. Ranković, V., Ristić, B., Kojić, M., (2007) Internal fixation of femoral bone comminuted fracture - FE Analysis, *J. Serb. Soc. Comp. Mech.*, 1, 120-128.
175. Rosić, B., Matthies, H., Živković, M., (2011) Uncertainty Quantification of Infinitesimal Elastoplasticity, *Scientific Technical Review*, Vol.61, No.2, pp.3-9.
176. Rosic, M., Pantovic, S., Rankovic, V., Obradovic, Z., Filipovic, N., Kojic, M. (2008) Evaluation of dynamic response and biomechanical properties of isolated blood vessels, *J. Biochem. Biophys. Methods*, 70, 966-972.
177. Ruiz-Esparza, G. U., Wu, S., Segura-Ibarra, V., Cara, F. E., Evans, K. W., Milosevic, M., Ziemys, A., Kojic, M., Meric-Bernstam, F., Ferrari, M., Blanco, E. (2014) Polymer Nanoparticles Encased in a Cyclodextrin Complex Shell for Potential Site- and Sequence-Specific Drug Release, *Adv. Funct. Mater.*, 24, 4753–4761.
178. Santagiuliana, R., Milosevic, M., Milicevic, B., Sciumè, V., Simic, V., Ziemys, A., Kojic, M., Schrefler, B. A., (2019) Coupling tumor growth and bio distribution models, *Biomedical Microdevices*, <https://doi.org/10.1007/s10544-019-0368-y>.
179. Simic, V., Milosevic, M., Milicevic, V., Filipovic, N., Kojic, M. (2023) A novel composite smeared finite element for mechanics (CSFEM): Some applications, *Technology and Health Care* 31, 719-733.
180. Simic, V., Nikolic, A., Shao, N., Milosevic, M., Leonard, F., Liu, X., Kojic, M., (2025), A parametric study of motion and attachment to capillary walls of circulating tumor cells (CTCs) interacting with non-activated and activated platelets, *Computer Methods and Programs in Biomedicine*, (264), DOI: <https://doi.org/10.1016/j.cmpb.2025.108699>, ISSN: 0169-2607,.
181. Slavković, R., Živković, M., Kojić, M., (1994), Enhanced 8-node three-dimensional solid and 4-node shell elements with incompatible generalized displacements, *Communications in Numerical Methods in Engineering*, 10(9), pp. 699-709, ISSN -, DOI: 10.1002/cnm.1640100904,
182. Stamenović, D., Kojić, M., Stojanović, B., Hunter, D., (2008), A finite element analysis of an osteoarthritis knee brace, *J. Serb. Soc. Comp. Mech.*, 2(2), 29-41
183. Stamenović, D., Kojić, M., Stojanović, B., Hunter, D. (2009) Pneumatic osteoarthritis knee brace, *J. Biomech. Engng. Transactions ASME*, 131, 045001-1,6.
184. Stanojević Pirković, M., Pavić, O., Filipović, F., Saveljić, I., Geroski, T., Exarchos, T., & Filipović, N. (2023). Fractional Flow Reserve-Based Patient Risk Classification. *Diagnostics*, vol. 13, no. 21, 3349. <https://doi.org/10.3390/diagnostics13213349>
185. Stepanović, Ž., Živković, M., Vulović, S., Aćimović, L.J., Ristić, B., Matić, A., Grujović, Z., (2011) High, open wedge tibial osteotomy: Finite element analysis of five internal fixation modalities, *Vojnosanit Pregl*, 68(10), 867–871.
186. Stevanović, D., Topalović, M., Živković, M., (2021) Improvement of the Sparse Matrices Storage Routines for Large FEM Calculations, *Journal of the Serbian Society for Computational Mechanics*, Vol. 15, No. 1, pp 81-97.
187. Stojanovic, B., Kojic, M., Rosic, M., Tsui, C.P., Tang, C.Y. (2007) An extension of Hill's three-component model to include different fiber types in finite element modelling of muscle, *Int. J. Num. Meth. Eng.*, 71, 801-817.
188. Stojanović, B., Kojić, M., (2007) Modeling of musculoskeletal systems using finite element method, *J. Serb. Soc. Comp. Mech.*, 1, 110-119.
189. Šušteršič, T., Nikolić, M., Barthes, J., Vrana, N.E. and Filipović, N. (2020). In Silico Modelling of Epithelial Barrier Formation with A549 Cancerous Cell Line, 11th World Biomaterials Congress, 11 - 15 December 2020 (Virtual conference)
190. Šušteršič, T., Kovačević, V., Ranković, V., Rasulić, L., & Filipović, N. (2022). Computational Modelling and Machine Learning Based Image Processing in Spine Research. In *Personalized Orthopedics*. (Ed. O. Canciglieri Junior, M. D. Trajanovic), Chapter 16, Springer, Cham., pp. 441-501, ISBN: 978-3-030-98279-9, https://doi.org/10.1007/978-3-030-98279-9_16
191. Tang, C. Y. , Stojanovic, B., Tsui, C. P., Kojic, M. (2005) Modeling of muscle fatigue using Hill's model, *Bio-Medical Materials and Engng.*, 15(5), 2005.
192. Tang, C.Y., Tsui, C.P., Stojanovic, B., Kojic, M. (2007) Finite element modelling of skeletal

- muscles coupled with fatigue, *Int. J. Mech. Sciences*, 49, 1179-1191.
193. Tomasevic,S., Milosevic,M., Milicevic,B., Simic,V., Prodanovic,M., Mijailovich,S., Filipovic,N., (2023) Computational Modeling on Drugs Effects for Left Ventricle in Cardiomyopathy Disease, *Pharmaceutics*, 15(3), 793, <https://doi.org/10.3390/pharmaceutics15030793>, ISSN: 1999-4923.
 194. Tomasevic,S., Milosevic,M., Milicevic,B., Simic,V., Prodanovic,M., Mijailovich,S.M., Filipovic,N., (2024) Finite Element Analysis of Myocardial Work in Cardiomyopathy, *Disruptive Information Technologies for a Smart Society, Lecture Notes in Networks and Systems*, 872, 23–29, https://doi.org/10.1007/978-3-031-50755-7_3.
 195. Topalovic, M., Nikolic, A., Milovanovic, V., Vulovic, S., Ivanovic, M. (2022) Smoothed particle hydrodynamics for blood flow analysis: development of particle lifecycle algorithm, *Comp. Part. Mech.*, 9, 1119–1135.
 196. Veljković, D., Kojić, M., (2010) Prediction of planar uniaxial and constrained biaxial state of deformation by commonly used anisotropic constitutive models in arterial mechanics, *J. Serb. Soc. Comp. Mech.*, 4(2), 54-74.
 197. Veljkovic, D., Filipovic, N., Kojic, M. (2012) The effect of asymmetry and axial prestraining on the amplitude of mechanical stresses in abdominal aortic aneurism, *J. of mechanics in Medicine and Biology*, 12(5), 1250089.
 198. Veljkovic, D., Rankovic,V., Pantovic, S., Rosic, M., Kojic,M., (2014) Hyperelastic behavior of porcine aorta segment under extension-inflation tests fitted with various phenomenological models, *Acta of Bioeng. Biomech.*,16(3), 37-45.
 199. Vlastelica, I., Veljković, D., Ranković, V., Stojanović, B., Rosić, M., Kojić, M., (2007) Modeling of urinary bladder deformation within passive and active regimes, *J. Serb. Soc. Comp. Mech.*,1, 129-134.
 200. Vlastelica,I., Isailović, V., Djukić, T., Filipović,N., Kojić, M., (2008) On accuracy of the element-free Galerkin (EFG), method in modeling incompressible fluid flow, *J. Serb. Soc. Comp. Mech.*, 2(1), 90-99.
 201. Vulovic, S., Zivkovic, M., Grujovic, N., Slavkovic, R., (2007) A comparative Study of Contact Problems Solution Based on the Penalty and Lagrange Multiplier Approaches, *Journal of the Serbian Society for Computational Mechanics*, Vol. 1, No. 1, 174-183.
 202. Vulovic, S., Zivkovic, M., Grujovic, (2008) Contact Problems Based on the Penalty Method, *Scientific Technical Review*, Vol.LVIII, No.3-4, 33-37.
 203. Vulovic, S., Topalovic, M., Zivkovic, M., Divac, D., Milivojevic, V., (2024) Advancement of Finite Element Method Solver Used in Dam Safety Monitoring System by Interpolation of Pore Pressure and Temperature Values, *Appl. Sci.* 2024, 14(21), 9680.
 204. Yazdi, K., Ziemys, A., Evangelopoulos, M., Martinez, J. O., Kojic, M., Tasciotti, E., (2015) Physicochemical properties affect the synthesis, controlled delivery, degradation and pharmacokinetics of inorganic nanoporous materials, *Nanomedicine*, ISSN 1743-5889.
 205. Yokoi, K., Kojic M., (equal contribution), Milosevic, M., Tanei, T., Ferrari, M., Ziemys, A., (2014) Capillary-wall collagen as a biophysical marker of nanotherapeutic permeability into the tumor microenvironment, *Cancer Research*, 74(16), 4239-4246.
 206. Yokoi,K., Chan, D., Kojic, M., Milosevic,M., Engler, D., Matsunami, R., Tanei, T., Saito, Y., Ferrari, M., Ziemys, A. (2015) Liposomal doxorubicin extravasation controlled by phenotype-specific transport properties of tumor microenvironment and vascular barrier, *Journal of Controlled Release*, 217, 293–299.
 207. Ziemys, A., Kojic, M., Milosevic, M., Kojic, N., Hussain, F., Ferrari,M., Grattoni, A., (2011) Hierarchical modeling of diffusive transport through nanochannels by coupling molecular dynamics with finite element method, *Journal of Computational Physics*, 230, 5722–5731.
 208. Ziemys, A., Kojic, M., Milosevic, M., Ferrari, M. (2012) Interfacial effects on nanoconfined diffusive mass transport regimes, *Physical Review Letters*, 108(23), 236102.
 209. Ziemys, A., Filipovic, N., Ferrari, M., Kojic, M. (2014) Transport in nanoconfinement and within blood vessel wall, in M. Garbey, B. Lee Bass, S. Berceci, C. Collet, and P. Cerveri, Eds: *Computational Surgery and Dual Training*, Springer, 273-288.
 210. Ziemys, A., Klemm, S., Milosevic, M., Yokoi, K., Ferrari, M., Kojic, M., (2015) Computational analysis of drug transport in tumor microenvironment as a critical compartment for nanotherapeutic pharmacokinetics, *Drug Delivery*, DOI: 0.3109/10717544.2015.1022837.

211. Ziemys, A., Kojic, M., Ferrari, M. (2015) Physics-based multiscale mass transport model in drug delivery and tumor microenvironment, *Handbook of Mathematical Methods in Cancer Biology*, NCI.
212. Ziemys, A., Klemm, S., Milosevic, M., Yokoi, K., Ferrari, M., Kojic, M. (2016) Computational analysis of drug transport in tumor microenvironment as a critical compartment for nanotherapeutic pharmacokinetics, *Drug Delivery*. 23(7), 2524-2531.
213. Ziemys, Z., Yokoi, K., Kai, M., Liu, Y. T., Kojic, M., Simic, V., Milosevic, M., Holder, A., Ferrari, M., (2018) Progression-dependent transport heterogeneity of breast cancer liver metastases as a factor in therapeutic resistance, *Journal of Controlled Release*. 291,99–105.
214. Ziemys, A., Kojic, M., Milosevic, M., Schrefler, B., Ferrari, M., (2018), Multiscale models for transport and biodistribution of therapeutics in cancer, *quantitative systems pharmacology: models and model-based systems with applications*, 42, 209-237, Book Series: Computer Aided Chemical Engineering.
215. Ziemys, A., Simic, V., Milosevic, M., Kojic, M., Liu, Y.T., Yokoi, K. (2021) Attenuated Microcirculation in Small Metastatic Tumors in Murine Liver, *Pharmaceutics*,13, 703, 2021, <https://doi.org/10.3390/pharmaceutics13050703>.
216. Zivkovic, M., Kojic, M., Slavkovic, R., Grujovic, N., (2001) A general beam finite element with deformable cross-section, *Comp. Meth. Appl. Mech. Engng.*, 190, 2651-2680.
217. Živković, M., Vulović, S., Vujanac, R., (2010) Assessment of the drum remaining lifetime in thermal power plant, *Thermal Science*, Vol. 14, S313-S321.
218. Zivkovic, M., Nikolic, A., Slavkovic, R., Zivic, F., (2010) Non-linear transient heat conduction analysis of insulation wall of tank for transportation of liquid aluminum, *Thermal Science*, Vol. 14, S299-S312.
219. Živković, M., Vuković, M., Lazić, V., Milovanović V., Dunić V., Kozak D., Rakić D., (2019) Experimental and FE Modeling Investigation of Spot Welded Thin Steel Sheets, *Technical Gazette*, Vol. 26, No.1, pp. 217-221.
220. Živković, J., Dunić, V., Milovanović, V., Pavlović, A., Živković, M. (2021) A Modified Phase-Field Damage Model for Metal Plasticity at Finite Strains: Numerical Development and Experimental Validation, *Metals*, 11(1), 47

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