

## Effect of surface roughness on a magnetic squeeze film for a sphere in a spherical seat

G.M. Deheri<sup>1</sup>, S. J. Patel<sup>1\*</sup>, J. R. Patel<sup>1</sup>

<sup>1</sup> Department of Mathematics, Sardar Patel University, V.V.Nagar, Gujarat, India

\* *Corresponding author* sej.al.patel85@yahoo.com

### Abstract

This article aims to analyze the performance of a magnetic fluid based squeeze film for a sphere in a rough spherical seat. Neuringer-Rosensweig model has been adopted to get the magneto hydrodynamic equations. The stochastic averaging model of Christensen and Tonder has been used to evaluate the effect of surface roughness. The associated Reynolds type equation is derived and solved to obtain the pressure distribution resulting in the calculation of load carrying capacity. The results presented in graphical forms suggest that the adverse effect of transverse surface roughness can be minimized by the positive effect of ferrofluid lubrication at least in the case of negatively skewed roughness. Equally important is the role of variance in augmenting the performance characteristics.

**Keywords:** Spherical bearing, magnetic fluid, roughness, load carrying capacity, porosity

### 1. Introduction

It is well established that ferrofluid lubrication has turned out to be an important tool for improving the performance of a squeeze film. The use of magnetic fluid as a lubricant has led to the development of energy devices such as magnetically cooled high-fidelity speakers, computer disc drives and semiconductors etc. The most important property of the magnetic fluid is that they can be retained at any location with the aid of magnets and precisely, for this property these fluids are widely employed in space vehicles.

Bhat and Deheri (1993) analyzed the effect of magnetic fluid on the action of a curved squeeze film existing between two circular disks. The effect of magnetization was independent of the curvature of the upper disk. Burcan (2004) analyzed the experimental data for the lubricated bearings of precision and concluded that magnetic field altered the energetic state of equilibrium occurring in spinning friction of slides. Lin et al. (2013) investigated the squeeze film characteristics of parallel circular disks lubricated by a ferrofluid with non-Newtonian couple stresses. The non-Newtonian ferrofluid lubrication was found to provide a higher load carrying capacity.

By now, it is well established that the roughness of the bearing surfaces significantly affects the performance characteristics. Several methods have been discussed to deal with the effect of surface roughness and the performance characteristics of squeeze films. Tzeng and Saibel (1967) employed a stochastic approach to model the random roughness which in turn, was extended by Christensen and Tonder (1969 (a), 1969 (b), 1970) to study the effect of surface roughness in general. These studies recognized the random character of surface roughness. A

number of investigations deployed the stochastic model of Christensen and Tonder to analyze the effect of surface roughness.

Pajapati (1992) investigated the combined effect of surface roughness and elastic deformation on the squeeze film performance between rotating porous circular plates in the presence of a concentric circular pocket. The larger size of the pocket reduced the load carrying capacity. Gupta and Deheri (1996) discussed the behavior of a hydrodynamic squeeze film between a non-rotating spherical surface and a hemispherical bearing under a steady load. It was found that the composite roughness of the surfaces affected the performance characteristics adversely. Andharia et al. (1999) dealt with the effect of transverse surface roughness on the behavior of a squeeze film in a spherical bearing. It was observed that the composite roughness of the surfaces significantly affected the performance of the bearing system. Andharia et al. (2001) discussed the longitudinal surface roughness effects on the behavior of a squeeze film in a spherical bearing. The composite roughness of the surfaces modified the performance of the bearing system considerably. Nanduvnamani et al. (2005) presented a theoretical study on the effect of surface roughness on the couple stress squeeze film between a sphere and a flat plate. For azimuthal roughness pattern the load carrying capacity and squeeze film time were found to be increased.

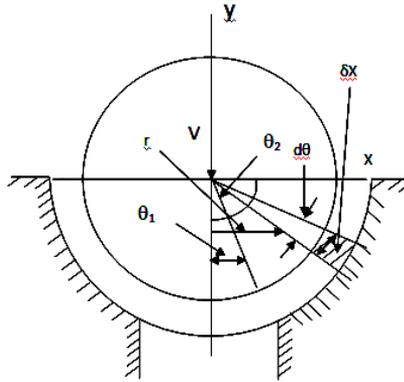
Patel and Deheri (2003) studied the effect of transverse surface roughness on the behavior of a ferrofluid squeeze film between porous circular plates with a concentric circular pocket. The adverse effect of surface roughness was found to be reduced by the ferrofluid lubrication with a suitable choice of pocket size. Patel et al. (2011) discussed the effect of surface roughness on the behavior of a ferrofluid-based squeeze film between circular plates considering porous matrix of variable thickness. A significant observation was that with a proper selection of thickness ratio parameter, a magnetic fluid based squeeze film bearing with variable porous matrix thickness could be made to perform better than that of a conventional porous bearing with an uniform porous matrix thickness, at least in the case of negatively skewed roughness.

Zueco and Beg (2010) analyzed numerically the hydrodynamic squeeze film between two rotating disks making use of numerical network simulation methods. Lin (2013) analyzed the effect of inertia forces on the non-Newtonian couple stress squeeze film between a sphere and a flat plate. The consideration of fluid inertia forces provided a longer squeeze film time especially, for the squeeze film operating with a lower film height and a larger non-Newtonian parameter. Patel and Deheri (2014) analyzed the performance of ferrofluid lubrication on a squeeze film in rough porous parallel circular disks, considering slip velocity. It was found that the variance (-ve) provided assistance to magnetization in overcoming the negative effect of porosity and standard deviation associated with roughness.

Here, it has been proposed to study the effect of transverse surface roughness on a magnetic squeeze film for a sphere in a rough spherical seat.

## 2. Analysis

The geometrical configuration of the bearing system is presented below.



**Fig. 1.** A Sphere in a Spherical seat

It consists of a sphere with radius  $r$  in a hemispherical seat. The amount of flow from the control volume due to Poiseuille and Couette flow can be expressed as

$$\frac{-\pi}{6\eta} \frac{dp}{dx} \left( rh^3 \frac{dp}{dx} \right) \delta x - \pi V \frac{d}{dx} (rh \sin \theta) \delta x \quad (1)$$

where  $\eta$  is the viscosity of the lubricant. The rate of reduction of flow due to squeeze action turns out to be

$$2\pi V \cos \theta \delta x \quad (2)$$

Therefore, on equating the total flow rate from the control section to the rate of reduction in volume of control space, one arrives at

$$\frac{d}{dx} \left( rh^3 \frac{dp}{dx} \right) + 6\eta V \frac{d}{dx} (rh \sin \theta) = -12\eta V r \cos \theta \quad (3)$$

As

$$6\eta V \frac{d}{dx} (rh \sin \theta)$$

is smaller than the other terms, it can be disregarded. Again, since  $x = R\theta$  and  $r = R \sin \theta$  one concludes that

$$\frac{d}{d\theta} \left( h^3 \sin \theta \frac{dp}{d\theta} \right) = -6\eta R^2 V \sin 2\theta \quad (4)$$

The porosity effect transforms this equation into

$$\frac{d}{d\theta} \left( \sin \theta (h^3 + 12\varphi H) \frac{dp}{d\theta} \right) = -6\eta R^2 V \sin 2\theta$$

where  $\varphi$  is the permeability and  $H$  is the thickness of porous facing.

Incorporation of the magnetic field effect under Neuringer-Rosensweig model based fluid flow, leads to (Bhat (2003), B. L. Prajapati (1995) and Bhat and Deheri (1993))

$$\frac{d}{d\theta} \left( \sin \theta (h^3 + 12\phi H) \frac{d}{d\theta} \left( p - \frac{\mu_0 \bar{\mu}}{2} H^2 \right) \right) = -6\eta R^2 V \sin 2\theta \quad (5)$$

where,  $\mu_0$  is magnetic permeability,  $\bar{\mu}$  is magnetic susceptibility and

$$M^2 = KR^2 \cos \theta$$

wherein,  $K$  is a suitably chosen constant for obtaining a magnetic field of desired strength.

In order to evaluate the effect of surface roughness the stochastic averaging model of Christensen and Tonder (1969 (a), 1969 (b), 1970) has been employed. Stochastically averaging the above equation (5) in the light of Christensen and Tonder's method one is led to

$$\frac{d}{d\theta} \left( \sin \theta g(h) \frac{d}{d\theta} \left( p - \frac{\mu_0 \bar{\mu}}{2} H^2 \right) \right) = -6\eta R^2 V \sin 2\theta \quad (6)$$

where

$$g(h) = h^3 + 3(\sigma^2 + \alpha^2)h + 3h^2\alpha + \alpha^3 + 3\sigma^2\alpha + \varepsilon + 12\phi H$$

while  $\sigma$  is the standard deviation,  $\alpha$  is the mean and  $\varepsilon$  is the skewness. The details regarding these parameters can be obtained from Christensen and Tonder (1969 (a), 1969 (b), 1970). The following non-dimensional quantities are introduced:

$$\bar{h} = \frac{h}{C} = 1 - e \cos \theta, \quad P = \frac{C^3 e}{\eta R^2 V} p, \quad \mu^* = \frac{k\mu_0 \mu C^3 e}{\eta V}, \quad \bar{\alpha} = \frac{\alpha}{C},$$

$$\bar{\sigma} = \frac{\sigma}{C}, \quad \bar{\varepsilon} = \frac{\varepsilon}{C^3}, \quad \psi = \frac{\phi H}{C^3}, \quad a = 3\bar{\alpha}, \quad b = 3(\bar{\alpha}^2 + \bar{\sigma}^2),$$

$$c = 3\bar{\sigma}^2 \bar{\alpha} + \bar{\alpha}^3 + \bar{\varepsilon} + 12\psi, \quad J = \sqrt[3]{-2a^3 + 3\sqrt{3}k_1 + 9ab - 27c}$$

$$k_1 = \sqrt{4a^3c - a^2b^2 - 18abc + 4b^3 + 27c^2}, \quad J_1 = \frac{J}{3\sqrt[3]{2}} - \frac{\sqrt[3]{2}(3b - a^2)}{3J} - \frac{a}{3}$$

$$J_2 = -2\frac{J}{6\sqrt[3]{2}} + \frac{2(3b - a^2)}{3(2)^{2/3}J} - \frac{2a}{3}$$

$$J_3 = 4\left(\frac{J}{6\sqrt[3]{2}}\right)^2 + 4\left(\frac{J}{6\sqrt[3]{2}}\right)\left(\frac{3b - a^2}{3(2)^{2/3}J}\right) + 4\left(\frac{3b - a^2}{3(2)^{2/3}J}\right)^2 + \frac{2a}{3}\frac{J}{6\sqrt[3]{2}} - 2\left(\frac{3b - a^2}{3(2)^{2/3}J}\right)\frac{2}{3} + \frac{a^2}{9}$$

where  $C$  is radial clearance and  $e$  is the eccentricity ratio.

With these above non-dimensional quantities, equation (6) assumes the form

$$\frac{d}{d\theta} \left( \sin \theta g(\bar{h}) \frac{d}{d\theta} \left( p - \frac{\mu^*}{2} \cos \theta \right) \right) = -6e \sin 2\theta$$

Integration of this equation with the boundary conditions

$$P = 0, \quad \theta = \frac{\pi}{2},$$

and

$$\frac{dP}{d\theta} = 0, \quad \theta = 0$$

gives the expression for non-dimensional pressure distribution as

$$P = \frac{\mu^*}{2} \cos \theta + 6 \left[ A_1 L_1 + \left( \frac{2C_1 + B_1 J_3}{\sqrt{4J_3 - J_2^2}} \right) T_1 + \frac{B_1}{2} L_2 \right] \quad (7)$$

where

$$A_1 = \frac{1}{J_1^2 - J_1 J_2 + J_3}, \quad B_1 = \frac{-1}{J_1^2 - J_1 J_2 + J_3}, \quad C_1 = \frac{J_2 - J_1}{J_1^2 - J_1 J_2 + J_3},$$

$$L_1 = \ln \left( \frac{1 - J_1}{\bar{h} - J_1} \right), \quad L_2 = \ln \left( \frac{J_3 - J_2 + 1}{J_3 - J_2 \bar{h} + \bar{h}^2} \right)$$

and

$$T_1 = \tan^{-1} \left( \frac{2(1 - \bar{h}) / \sqrt{4J_3 - J_2^2}}{1 + \frac{(2 - J_2)(2\bar{h} - J_2)}{(4J_3 - J_2^2)}} \right)$$

Then the expression for dimensionless load carrying capacity is derived as

$$W = \frac{C^3 e}{\pi \eta R^4 V} w = \frac{\mu^*}{6} + 6 \left[ A_1 \int_0^{\frac{\pi}{2}} L_1 \sin 2\theta d\theta + \left( \frac{2C_1 + B_1 J_3}{\sqrt{4J_3 - J_2^2}} \right) \int_0^{\frac{\pi}{2}} T_1 \sin 2\theta d\theta + \frac{B_1}{2} \int_0^{\frac{\pi}{2}} L_2 \sin 2\theta d\theta \right] \quad (8)$$

### 3. Results and Discussion

It is easily seen that the dimensionless pressure distribution is obtained from equation (7). while equation (8) determines the load carrying capacity for the bearing. It is observed that the pressure distribution gets increased by

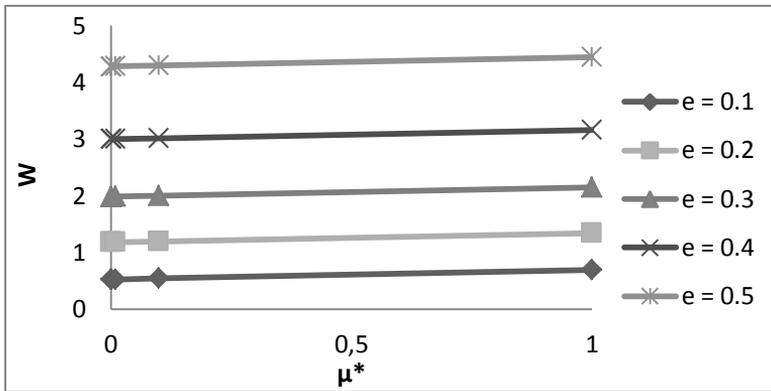
$$\frac{\mu^*}{2} \cos \theta$$

while the increase in the load carrying capacity is found to be

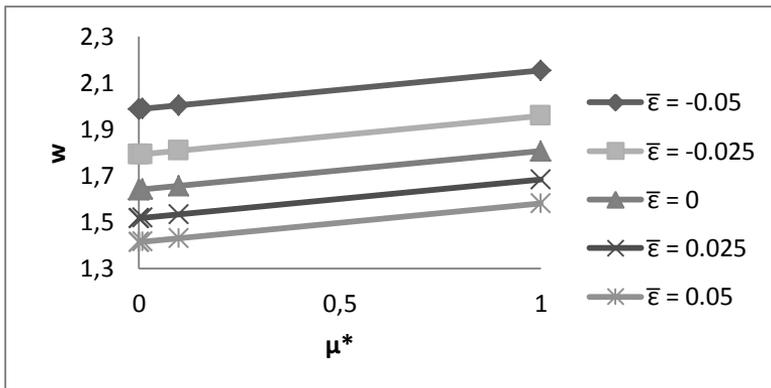
$$\frac{\mu^*}{6}$$

as compared to the case of conventional lubricant based bearing system. Probably, this may be due to the fact that the magnetization enhances the viscosity of the lubricant. Besides, one can see that the expression involved in equation (8) is linear with respect to the magnetization parameter. Accordingly, an increase in the magnetization would lead to increased load carrying capacity.

As seen below in figures 2-6, the load carrying capacity rises sharply with the increase in magnetization.



**Fig. 2.** Variation of Load carrying capacity with respect to  $\mu^*$  and  $e$ .



**Fig. 3.** Variation of Load carrying capacity with respect to  $\mu^*$  and  $\bar{\epsilon}$ .

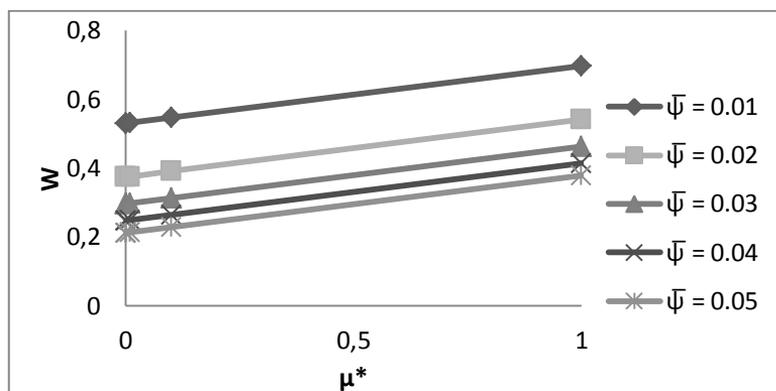


Fig. 4. Variation of Load carrying capacity with respect to  $\mu^*$  and  $\bar{\psi}$ .

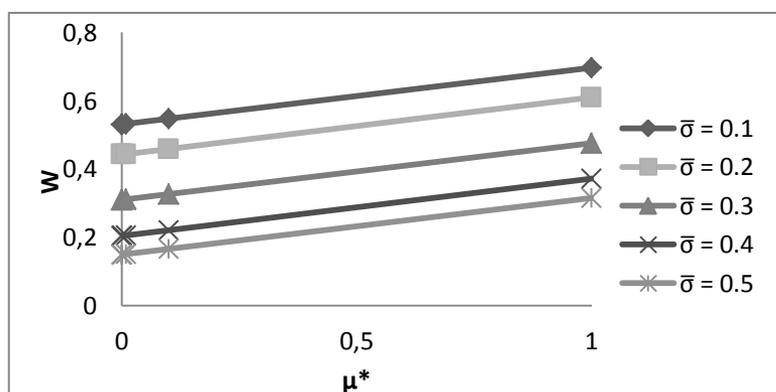


Fig. 5. Variation of Load carrying capacity with respect to  $\mu^*$  and  $\bar{\sigma}$ .

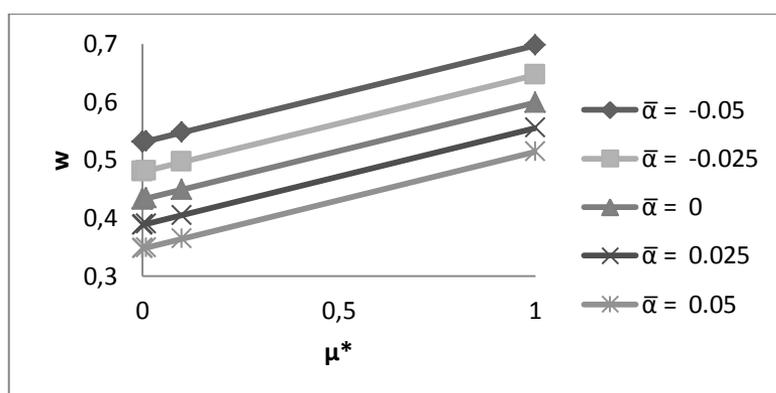


Fig. 6. Variation of Load carrying capacity with respect to  $\mu^*$  and  $\bar{\alpha}$ .

The variation of load carrying capacity with respect to  $e$  presented in figures 7-10 makes it clear that the load carrying capacity gets increased sharply owing to the eccentricity.

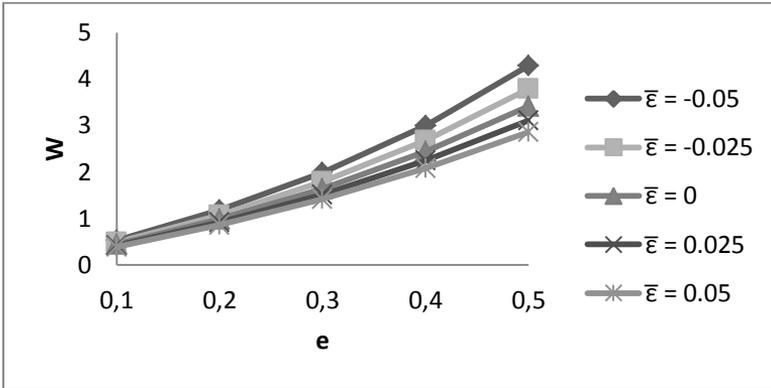


Fig. 7. Variation of Load carrying capacity with respect to  $e$  and  $\bar{\epsilon}$ .

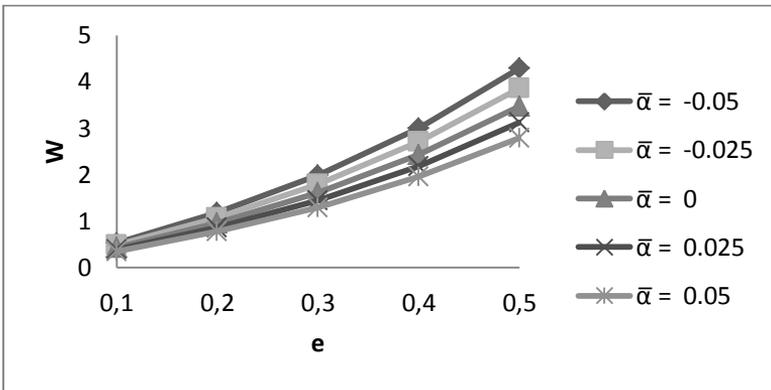


Fig. 8. Variation of Load carrying capacity with respect to  $e$  and  $\bar{\alpha}$ .

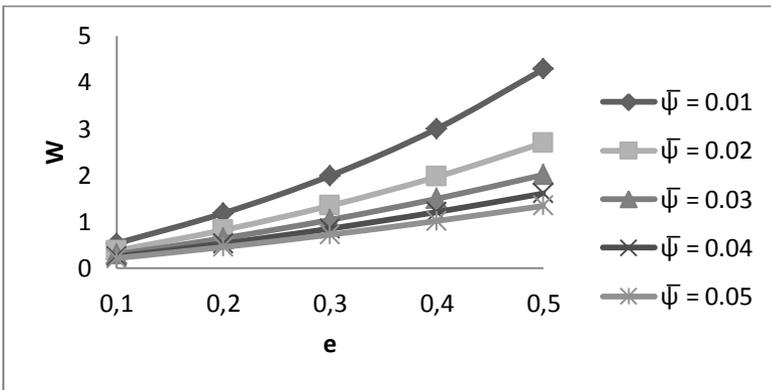
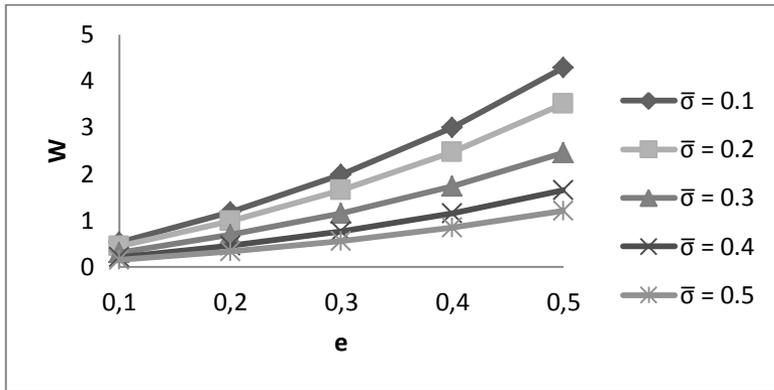
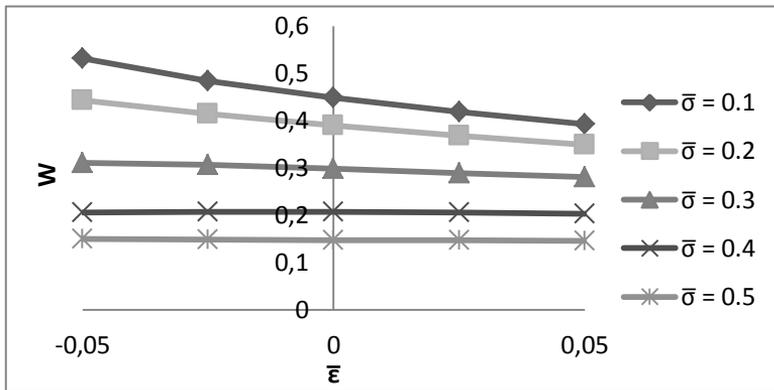


Fig. 9. Variation of Load carrying capacity with respect to  $e$  and  $\bar{\psi}$ .

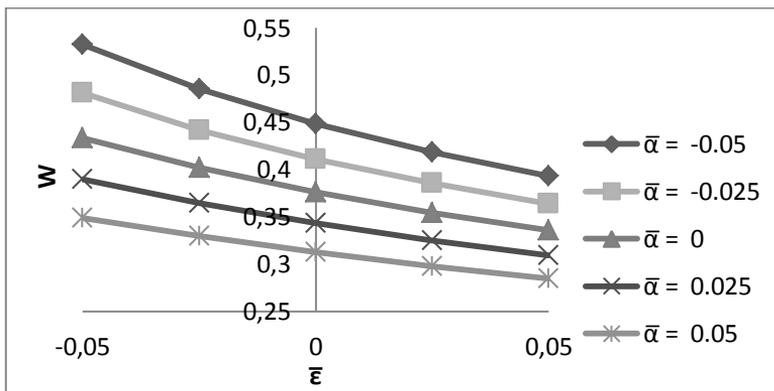


**Fig. 10.** Variation of Load carrying capacity with respect to  $e$  and  $\bar{\sigma}$ .

The fact that negatively skewed roughness increases the load carrying capacity while the load carrying capacity gets decreased due to positively skewed roughness, can be availed from figures 11-13.



**Fig. 11.** Variation of Load carrying capacity with respect to  $\bar{\epsilon}$  and  $\bar{\sigma}$ .



**Fig. 12.** Variation of Load carrying capacity with respect to  $\bar{\epsilon}$  and  $\bar{\alpha}$ .

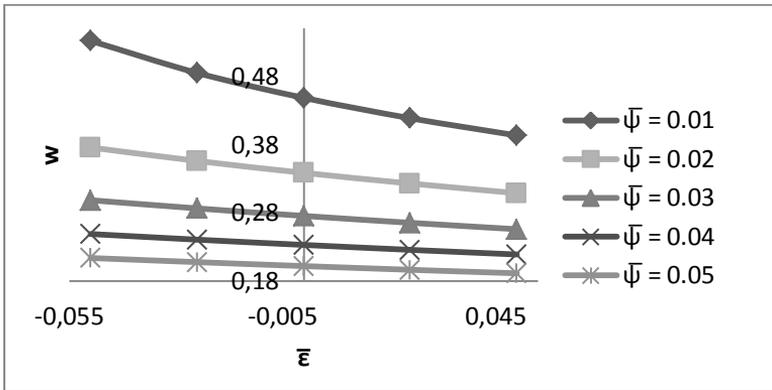


Fig. 13. Variation of Load carrying capacity with respect to  $\bar{\epsilon}$  and  $\bar{\psi}$ .

Figures 14-15 establish that the trends of load carrying capacity with respect to variance are almost similar to that of skewness. But the rate of increase in the load carrying capacity is registered to be more in the case of negatively skewed roughness.

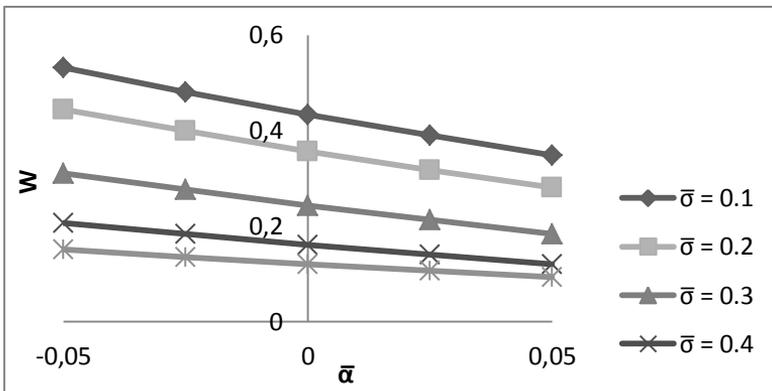


Fig. 14. Variation of Load carrying capacity with respect to  $\bar{\alpha}$  and  $\bar{\sigma}$ .

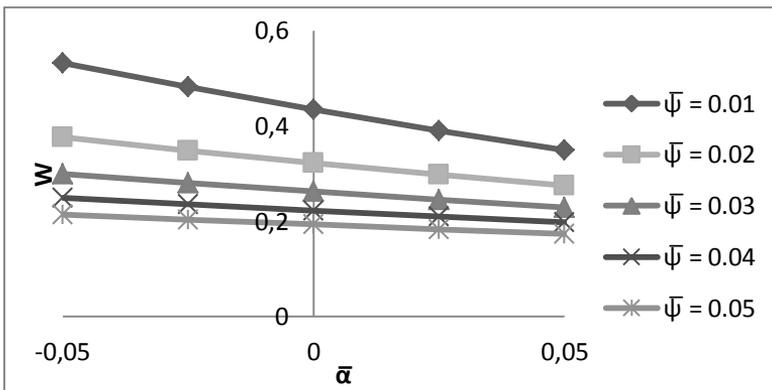


Fig. 15. Variation of Load carrying capacity with respect to  $\bar{\alpha}$  and  $\bar{\psi}$ .

Lastly, Figure 16 indicates that the combined effect of porosity and standard deviation is to reduce the load carrying capacity.

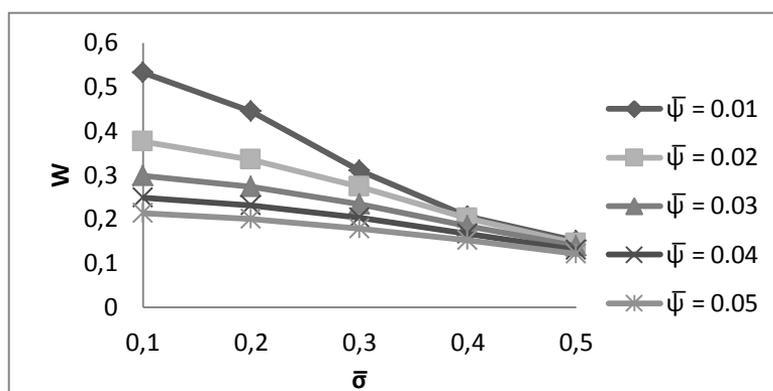


Fig. 16. Variation of Load carrying capacity with respect to  $\bar{\sigma}$  and  $\bar{\psi}$ .

Some of the figures presented here reveal that the adverse effect of standard deviation and porosity can be compensated to a large extent by the magnetization at least in the case of negatively skewed roughness. Of course, variance (-ve) adds to this compensation.

#### 4. Conclusions

The bearing registers an improved performance due to magnetization. Although, the effect of transverse surface roughness is adverse, in general, the situation could be retrieved by the positive effect of magnetization, in view of the combined positive effect of negatively skewed roughness and variance (-ve). However, this investigation strongly suggests that the roughness aspects must be duly considered while designing the bearing system. In addition, this type of bearing system supports certain amount of load even when there is no flow which does not happen in the case of conventional lubricant based bearing system.

Извод

**Ефекат храпавости површине на магнетизовани притиснут филм код сфери у сферичном лежишту**

G.M. Deheri<sup>1</sup>, S. J. Patel<sup>1\*</sup>, J. R. Patel<sup>1</sup>

<sup>1</sup>Department of Mathematics, Sardar Patel University, V.V.Nagar, Gujarat, India

\* Главни аутор sejal.patel85@yahoo.com

## Резиме

Циљ овог рада је да анализира перформансе магнетизованог филма флуида под притиском у случају сфере у храпавом сферичном лежишту. Модел Нојрингер-Розенсвајга је усвојен да се добију магнетно-хидродинамичке једначине. Модел Кристенсена и Тондера за стохастичко усредњавање је коришћен да се одреди ефекат храпавости површине. Одговарајућа једначина Рејнолдсовог типа је изведена и решена да се добије расподела притиска која води ка срачунавању носивости лежаја. Резултати представљени у графичкој форми сугерирају да непожељни ефекат трансверзалне површинске храпавости може бити минимизован позитивним ефектом подмазивања ферофлуидом, бар у случају негативно закошене храпавости. Подједнако је важна улога варијансе при разматрању карактеристика перформанси.

**Кључне речи:** Сферично лежиште, магнетизовани флуид, храпавост, носивост, порозност

## References

- Andharia PI, Deheri GM, Gupta JL (2001). Effect of longitudinal surface roughness on the behavior of squeeze film in a spherical bearing, *International Journal of Applied Mechanics and Engineering*, 6(4), 885-897.
- Andharia PI, Gupta JL, Deheri GM (1999). Effect of transverse surface roughness on the behavior of squeeze film in a spherical bearing, *International Journal of Applied Mechanics and Engineering*, 4, 19-24.
- Bhat MV (2003). Lubrication with a magnetic fluid, Team spirit Pvt. Ltd., India.
- Bhat MV, Deheri GM (1993). Magnetic –fluid based squeeze film in curved porous circular discs, *Journal of Magnetism and Magnetic Materials*, 127(1-2), 159-162.
- Burcan Jan (2004). Investigation on magnetic fluids as lubricant, *Indian Journal of Engineering and Materials Sciences*, 11(4), 338-342.
- Christensen H and Tonder KC (1969 a). Tribology of rough surfaces: Parametric study and comparison of lubrication models, SINTEF, Report No. 22/69-18.
- Christensen H and Tonder KC (1969 b). Tribology of rough surfaces: Stochastic models of hydrodynamic lubrication, SINTEF, Report No. 10/69-18.
- Christensen H and Tonder KC (1970). The hydrodynamic lubrication of rough bearing surfaces of finite width, ASME-ASLE lubrication conference, paper No.70-Lub-7.
- Gupta JL, Deheri GM (1996). Effect of roughness on the behavior of squeeze film in a spherical bearing, *Tribology Transactions*, 39(1), 99-102.
- Lin JR (2013). Inertia force effects in the non-Newtonian couple stress squeeze film between a sphere and a flat plate, *Tribology International*, 67, 81-89.
- Lin JR, Lu RF, Lin MC, Wang PY (2013). Squeeze film characteristics of parallel circular disks lubricated by ferrofluids with non-Newtonian couple stresses, *Tribology International*, 61, 56–61.
- Nanduvanamani NB, Hiremath PS, Gurubasavaraj G (2005). Effect of surface roughness on the couple-stress squeeze film between a sphere and a flat plate, *Tribology International*, 38(5), 451-458.
- Neuringer JL and Rosensweig RE (1964). Magnetic fluid, *Physics of Fluids*, 7(12), 19-27.
- Patel RM, Deheri GM (2003). A study of magnetic fluid based squeeze film behavior between porous circular plates with a concentric circular pocket and effect of surface roughness, *Industrial Lubrication and Tribology*, 55(1), 32-37.

- Patel RM, Deheri GM, Patel HC (2011). Effect of surface roughness on the behavior of a magnetic fluid- based squeeze film between circular plates with porous matrix of variable thickness, *Acta Polytechnica Hungarica*, 8(5), 171-190.
- Patel SJ, Deheri GM (2014). Ferrofluid lubrication of a squeeze film in rough porous parallel circular disks, considering slip velocity, *Journal of Mechanical and Industrial Engineering Research*, 3(2), 9-24.
- Prajapati BL (1992). Squeeze film behavior between rotating porous circular plates with a concentric circular pocket: Surface roughness and elastic deformation effects, *Wear*, 152(2), 301-307.
- Prajapati BL (1995). On certain Theoretical Studies in Hydrodynamic and Electro-magneto-hydrodynamic Lubrication, Ph.D. Thesis, Sardar Patel University, Gujarat.
- Tzeng ST, Saibel E (1967). Surface roughness effect on slider bearing lubrication, *Trans. ASLE* 10, 334-342.
- Zueco J, Beg AO (2010). Network numerical analysis of hydromagnetic squeeze film flow dynamics between two parallel rotating disks with induced magnetic field effects, *Tribology International*, 43(3), 532-543.