## Effect of slip velocity and roughness on the performance of a Jenkins model based on magnetic squeeze film in curved rough circular plates

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### Abstract

An endeavour has been made to study and analyze the performance of a Jenkins model based ferrofluid squeeze film in curved rough circular plates considering the effect of slip velocity. The slip model of Beavers and Joseph has been used and Christensen and Tonder's stochastic modeling of roughness has been employed. With the aid of suitable boundary conditions the associated Reynolds type equation has been solved to derive the expression for pressure distribution, paving the way for the calculation of load carrying capacity. The graphical representations make it clear that although the load carrying capacity increases sharply with increasing magnetization, the magnetization has a limited option for mitigating the adverse effect of slip velocity and standard deviation. However, it is desirable that the slip parameter must be kept at minimum even if curvature parameters are suitably chosen. Lastly, the Jenkins model moves ahead of Neuringer- Rosensweig model for reducing the adverse effect of roughness.

Keywords: Slip velocity, Roughness, Jenkins model, Magnetic Fluid, Curved Circular plates.

### 1. Introduction

Since the last decade squeeze film characteristics has played a crucial role in many applications, such as lubrication of machine elements, automatic transmissions and artificial joints. The squeeze film action comes from two lubricated surfaces approaching each other with a normal velocity. Analysis of squeeze film performance assumes that the lubricant behaves essentially as a Newtonian viscous fluid although, to establish the flow properties and to increase the lubricating quantities, the use of ferrofluid has been emphasized. In several applications, the flow pattern corresponds to a slip flow, the fluid presents a loss of adhesion at the welted wall making the fluid slide along the wall. Flow with slip becomes useful for problems in chemical engineering for example, flow through pipes in which chemical reactions occur at the walls, two phase flows in bearing system.

Ferrofluids are suspensions of small magnetic particles with a mean diameter of about 10 nm in appropriate carrier liquids. The particles contain only a single magnetic domain and can thus be treated as small thermally agitated permanent magnets in a carrier liquid. The special feature of magnetic fluid is the combination of normal liquid behaviour with super paramagnetic properties. Moreover, some properties like the viscosity, the phase behaviour, or their optical birefringence properties, can be changed by applying an external magnetic field. Magnetic fluid has a wide range of potential technical and biomedical applications (Odenbach 2002). Ferrofluid and its applications is treated by various researcher ((Rosensweig 1985), (Shukla and Kumar 1987), (Raj et al. 1995), (Scherer and Neto 2005), (Odenbach 2009)). Later on, (Agrawal 1991), (Ram and Verma 1999), (Shah and Bhat 2002) and (Ahmed and Singh 2007) discussed the steady-state performance of bearings with Jenkins model based magnetic fluids and concluded that the load carrying capacity of the bearing system increased with increasing magnetization of the magnetic fluid.

The hydrodynamic lubrication theory for rough surfaces has been studied with considerable interest in recent years, because, all bearing surfaces are rough to some extent. All bearing surfaces develop roughness after having some run-in and wear. In some cases, contamination of lubricant is also one of the reasons to generate surfaces roughness through chemical degradation. In literature, many approaches have been proposed to study the effect of surface roughness on the bearing surfaces. (Burton 1963) discussed a model of roughness by a Fourier series type approximation. The surface roughness distribution is random in nature and hence a stochastic approach has to be adopted. (Christensen and Tonder 1969a, 1969b, 1970) developed a stochastic theory for the study of rough surfaces in hydrodynamic lubrication theory. Many researchers ((Ting1975), (Prakash and Tiwari 1982), (Guha 1993), (Gupta and Deheri 1996), (Chiang et al. 2004), (Deheri et al. 2007), (Bujurke et al. 2008), (Patel et al. 2009), (Shimpi and Deheri 2010), (Patel et al. 2011), (Abhangi and Deheri 2012)) have adopted broadly this model to study roughness effect on bearing surfaces. All above researchers found that roughness played an important role to improve the performance of bearing system. (Patel and Deheri 2012) investigated the performance of ferrofluid lubricated rough porous inclined slider bearing considering slip velocity. It was manifest that the performance of the bearing system could be made to improve by suitably choosing the magnetization parameter and slip coefficient in the case of negatively skewed roughness. (Rao et al. 2013) investigated the effects of velocity slip and viscosity variation in squeeze film lubrication of two circular plates. The effects of various porous structures on the performance of a Shliomis model based ferrofluid lubrication of a squeeze film in rotating rough porous curved circular plates was investigated by (Patel and Deheri 2013). It was noticed that the adverse effect of transverse roughness could be overcome by the positive effect of ferrofluid lubrication in the case of negative skewed roughness by suitably choosing curvature parameters and rotational inertia. Recently, (Patel and Deheri 2014) theoretically analyzed the effect of Shliomis model based ferrofluid lubrication on the squeeze film between curved rough annular plates with comparison between two different porous structures. It was noticed that the effect of morphology parameter and volume concentration parameter increases the load carrying capacity.

In this paper, it has been proposed to discuss the effect of slip velocity and roughness on the performance of a Jenkins model based magnetic squeeze film in curved rough circular plates.

#### 2. Analysis

Fig. 1 shows the geometry of the bearing system which consists of two circular plates, each of radius a .



Fig. 1. Configuration of the bearing system

Assuming the bearing surfaces transversely rough and following the stochastic model of (Christensen and Tonder 1969a, 1969b, 1970), the thickness h of the lubricant film is considered as

$$\mathbf{h} = \mathbf{h} + \mathbf{h}_{\mathrm{s}} \tag{1}$$

where h is the mean film thickness and  $h_s$  is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces.  $h_s$  is governed by the probability density function

$$f(h_s) = \begin{cases} \frac{35}{32c^7} (c^2 - h_s^2)^3, -c \le h_s \le c\\ 0, \text{elsewhere} \end{cases}$$

wherein c is the maximum deviation from the mean film thickness. The mean  $\alpha$ , the standard deviation  $\sigma$  and the parameter  $\epsilon$  which is the measure of symmetry of the random variable  $h_s$  are discussed in (Christensen and Tonder 1969a, 1969b, 1970).

According to the discussions of (Bhat 2003), (Abhangi and Deheri 2012) and (Patel and Deheri 2013), it is considered that the upper plate lying along the surface determined by the relation

$$z_u = h_0 [\sec(\beta r^2)]; 0 \le r \le a$$

approaches with normal velocity  $h_0$  to the lower plate lying along the surface given by

$$\mathbf{z}_1 = \mathbf{h}_0 \left[ \exp\left(-\gamma \mathbf{r}^2\right) - I \right]; 0 \le \mathbf{r} \le \mathbf{a}$$

where  $\beta$  and  $\gamma$  are the curvature parameters of the corresponding plates and  $h_0$  is the central film thickness. The film thickness h(r) then, is defined by ((Bhat 2003), (Abhangi and Deheri 2012))

$$\mathbf{h}(\mathbf{r}) = \mathbf{h}_0 [\sec(\beta \mathbf{r}^2) - \exp(-\gamma \mathbf{r}^2) + I] \mathbf{0} \le \mathbf{r} \le \mathbf{a}$$
(2)

In 1972, the flow model of a ferrofluid was discussed by Jenkins. In this paper the magnetisable liquid was regarded as an anisotropic fluid and added to the motion and the temperature the vector magnetization density to complete the description of the material. The use of local magnetization as an independent variable allowed Jenkins to treat static and dynamic situation in a uniform fashion and to make a natural distinction between paramagnetic and ferromagnetic fluids. A uniqueness theorem was established for incompressible paramagnetic fluids and determined that in these materials the magnetization vanishes with the applied magnetic field.

In view of Maugin's suggestions, equations of the model for steady flow are ((Jenkins 1972) and (Ram and Verma 1999))

$$\rho(\overline{\mathbf{q}}.\nabla)\overline{\mathbf{q}} = -\nabla_{\mathbf{p}} + \eta\nabla^{2}\overline{\mathbf{q}} + \mu_{0}(\overline{\mathbf{M}}.\nabla)\overline{\mathbf{H}} + \frac{\rho\mathbf{A}^{2}}{2}\nabla\times\left[\frac{\overline{\mathbf{M}}}{\mathbf{M}}\times\left\{\!\!\left(\nabla\times\overline{\mathbf{q}}\right)\!\!\times\overline{\mathbf{M}}\right\}\!\!\right]$$
(3)

together with

$$\nabla \cdot \overline{\mathbf{q}} = 0, \nabla \times \overline{\mathbf{H}} = 0, \overline{\mathbf{M}} = \overline{\mu}\overline{\mathbf{H}}, \nabla \cdot \left(\overline{\mathbf{H}} + \overline{\mathbf{M}}\right) = 0$$

((Bhat 2003)),  $\rho$  is the fluid density,  $\overline{q}$  represents the fluid velocity in the film region,  $\overline{H}$  is external magnetic field,  $\mu$  denotes magnetic susceptibility of the magnetic field, p is the film pressure,  $\eta$  represents the fluid viscosity,  $\mu_0$  denotes the permeability of the free space,  $\overline{M}$  is magnetization vector, M is magnitude of  $\overline{M}$ , H represents the magnitude of  $\overline{H}$  and A being a material constant. From the above equation it is noticed that Jenkins model is a generalization of Neuringer-Rosensweig model with an additional term

$$\frac{\rho A^2}{2} \nabla \times \left[ \frac{\overline{M}}{M} \times \left( \nabla \times \overline{q} \right) \times \overline{M} \right] = \frac{\rho A^2 \overline{\mu}}{2} \nabla \times \left[ \frac{\overline{H}}{H} \times \left( \nabla \times \overline{q} \right) \times \overline{H} \right]$$
(4)

which modifies the velocity of the fluid. Neuringer- Rosensweig modifies the pressure while Jenkins model modifies both the pressure and velocity of the ferrofluid.

Let (u, v, w) be the velocity of the fluid at any point  $(r, \theta, z)$  between two solid surfaces, with Z axis. Using the assumptions of hydrodynamic lubrication and recalling that the flow is steady and axially symmetric, the equations of motion can be written in a form

$$\left(I - \frac{\rho A^2 \bar{\mu} H}{2\eta}\right) \frac{\partial^2 u}{\partial z^2} = \frac{I}{\eta} \frac{d}{dr} \left(p - \frac{\mu_0 \bar{\mu}}{2} H^2\right)$$
(5)

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0$$
(6)

Using the boundary conditions, u = 0 when z = 0, h, solution of above equation (5) can be expressed as,

$$u = \frac{z(z-h)}{2\eta \left(1 - \frac{\rho A^2 \overline{\mu} H}{2\eta}\right)} \frac{d}{dr} \left(p - \frac{\mu_0 \overline{\mu}}{2} H^2\right)$$
(7)

By substituting the expression of u in equation (6) and integrating it with respect to z over the interval (0,h) one can obtain Reynolds type equation for film pressure

$$\frac{l}{r}\frac{d}{dr}\left(\frac{h^3}{\left(l-\frac{\rho A^2\bar{\mu}H}{2\eta}\right)}r\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}H^2\right)\right) = l2\eta\dot{h}_0$$

(Christensen and Tonder 1969a, 1969b, 1970) proposed a method for the stochastic averaging theory of this differential equation. Here an attempt has been made to modify this technique, which on certain simplifications yields, under the usual assumptions of hydro-magnetic lubrication ((Bhat 2003),(Prajapati 1995), (Deheri et al. 2005)) the modified Reynolds equation,

$$\frac{I}{r}\frac{d}{dr}\left(\frac{g(h)}{\left(I-\frac{\rho A^{2}\bar{\mu}H}{2\eta}\right)}r\frac{d}{dr}\left(p-\frac{\mu_{0}\bar{\mu}}{2}H^{2}\right)\right) = I2\eta \dot{h_{0}}$$
(8)

where

$$g(h) = (h^3 + 3h^2\alpha + 3(\sigma^2 + \alpha^2)h + 3\sigma^2\alpha + \alpha^3 + \varepsilon)\left(\frac{4 + sh}{2 + sh}\right)$$

The following non-dimensional quantities are introduced

1

$$\overline{\mathbf{h}} = \frac{\mathbf{h}}{\mathbf{h}_0}, \mathbf{R} = \frac{\mathbf{r}}{\mathbf{a}}, \mathbf{P} = -\frac{\mathbf{h}_0^3 \mathbf{p}}{\eta \mathbf{a}^2 \mathbf{h}_0}, \mathbf{B} = \beta \mathbf{a}^2, \mathbf{C} = \gamma \mathbf{a}^2, \mathbf{H}^2 = \mathbf{k} \mathbf{r}^2 \frac{(\mathbf{a} - \mathbf{r})}{\mathbf{a}},$$
$$\mu^* = -\frac{\mathbf{k} \mu_0 \overline{\mu} \mathbf{h}_0^3}{\eta \mathbf{h}_0}, \overline{\mathbf{A}}^2 = \frac{\rho \mathbf{A}^2 \overline{\mu} \sqrt{\mathbf{k}} \mathbf{a}}{2\eta}, \overline{\mathbf{\sigma}} = \frac{\mathbf{\sigma}}{\mathbf{h}_0}, \overline{\mathbf{\alpha}} = \frac{\alpha}{\mathbf{h}_0}, \overline{\mathbf{c}} = \frac{\varepsilon}{\mathbf{h}_0^3}, \overline{\mathbf{s}} = \mathbf{s} \mathbf{h}_0$$
(9)

where k is chosen to suit the dimension of both sides and the strength of the magnetic field. Making use of the equation (9), equation (8) takes the form

$$\frac{1}{R}\frac{d}{dR}\left(\frac{g(\bar{h})}{\left(I-\bar{A}^{2}R\sqrt{I-R}\right)}R\frac{d}{dR}\left(P-\frac{\mu^{*}}{2}R^{2}(I-R)\right)\right) = -12$$
(10)

wherein,

$$g(\overline{h}) = \left(\overline{h}^{3} + 3\overline{h}^{2}\overline{\alpha} + 3\left(\overline{\sigma}^{2} + \overline{\alpha}^{2}\right)\overline{h} + 3\overline{\sigma}^{2}\overline{\alpha} + \overline{\alpha}^{3} + \overline{\epsilon}\right)\left(\frac{4 + sh}{2 + s\overline{h}}\right)$$

Solving equation (10) under the boundary conditions

$$P(I) = 0, \left(\frac{dP}{dR}\right)_{R=0} = 0$$
(11)

one can obtain the expression of non dimensional pressure distribution as

$$\mathbf{P} = \frac{\mu^*}{2} \mathbf{R}^2 (I - \mathbf{R}) - 6 \int_{I}^{\mathbf{R}} \frac{\mathbf{R}}{g(\mathbf{\bar{h}})} \left( I - \overline{\mathbf{A}}^2 \mathbf{R} \sqrt{I - \mathbf{R}} \right) d\mathbf{R}$$
(12)

The non-dimensional load carrying capacity of the bearing system then, is obtained as

$$W = -\frac{h_0^3}{2\pi a^4 h_0} W = \int_0^I RP dR = \frac{\mu^*}{40} + 3\int_0^I \frac{R^3}{g(h)} \left(I - \overline{A}^2 R \sqrt{I - R}\right) dR$$
(13)

#### 3. Results and Discussions

It is noticed that the non dimensional pressure distribution is determined by equation (12) while equation (13) presents the variation of load carrying capacity with respect to various parameters. It is clearly observed from equation (12) that the non dimensional pressure enhances by

$$\frac{\mu^*}{2}R^2(l-R)$$

while the load carrying capacity gets increased by

$$\frac{\mu^*}{40}$$

as can be seen from equation (13) in comparison with the conventional lubricant based bearing system.

Equation (13) suggests that the expression involved is linear with respect to the magnetization parameter and therefore the load carrying capacity gets augmented with increase in the magnetization parameter. This is so because the magnetization increases the viscosity of the lubricant.

Further, it is found that the load carrying capacity gets elevated by at least 3 to 5 % approximately as compared to the case of Neuringer-Rosensweig model based identical bearing system (Bhat 2003).

In the absence of slip velocity this study reduces to the performance of a Jenkins model based magnetic squeeze film in curved rough circular plates. Further, for a bearing with smooth surfaces this leads to the behavior of Jenkins model based magnetic squeeze film in circular plates when the curvature parameters are neglected.

The fact that the dimensionless load carrying capacity rises sharply with increase in the magnetization is reflected in Fig. 2-8.



Fig. 2. Variation of Load carrying capacity with respect to  $\mu^*\,$  and A .



Fig. 3. Variation of Load carrying capacity with respect to  $\mu^*$  and B.



Fig. 4. Variation of Load carrying capacity with respect to  $\mu^*$  and C .



Fig. 5. Variation of Load carrying capacity with respect to  $\mu^*$  and  $\sigma$ .



Fig. 6. Variation of Load carrying capacity with respect to  $\mu^*$  and  $\overline{\epsilon}$ .



**Fig. 7.** Variation of Load carrying capacity with respect to  $\mu^*$  and  $\overline{\alpha}$ .



**Fig. 8.** Variation of Load carrying capacity with respect to  $\mu^*$  and l/s.

The material constant causes severe load reduction which can be seen from Fig. 9-14. Probably, this may be due to the fact that the material constant leads to the dilution of the viscosity of the lubricant. The effect of skewness on load carrying capacity with respect to material constant is not that significant. Here it is noticed that the effect of variance is comparatively more than that of skewness.



Fig. 9. Variation of Load carrying capacity with respect to A and B.



**Fig. 10.** Variation of Load carrying capacity with respect to  $\overline{A}$  and C.



Fig. 11. Variation of Load carrying capacity with respect to  $\overline{A}$  and  $\overline{\sigma}$ .



**Fig. 12.** Variation of Load carrying capacity with respect to  $\overline{A}$  and  $\overline{\epsilon}$ .



Fig. 13. Variation of Load carrying capacity with respect to A and  $\alpha$ .



**Fig. 14.** Variation of Load carrying capacity with respect to  $\overline{A}$  and l/s.

The combined effect of upper plate's curvature parameter and lower plate's curvature parameter is to significantly decrease the load carrying capacity (Fig. 15-23). It is interesting to note that the effect of skewness on the load carrying capacity with respect to the upper plate's curvature parameter is almost negligible.



Fig. 15. Variation of Load carrying capacity with respect to B and C.



**Fig. 16.** Variation of Load carrying capacity with respect to B and  $\overline{\sigma}$ .



Fig. 17. Variation of Load carrying capacity with respect to B and  $\varepsilon$ .



**Fig. 18.** Variation of Load carrying capacity with respect to B and  $\overline{\alpha}$ .



Fig. 19. Variation of Load carrying capacity with respect to B and l/s.



Fig. 20. Variation of Load carrying capacity with respect to C and  $\overline{\sigma}$ .



Fig. 21. Variation of Load carrying capacity with respect to C and  $\bar{\epsilon}$ .



**Fig. 22.** Variation of Load carrying capacity with respect to C and  $\overline{\alpha}$ .



**Fig. 23.** Variation of Load carrying capacity with respect to C and 1/s.

The effect of standard deviation causes reduced load carrying capacity which is exhibited in Fig. 24-26. This is because the roughness retards the motion of the lubricant.



Fig. 24. Variation of Load carrying capacity with respect to  $\sigma$  and  $\epsilon$ .



Fig. 25. Variation of Load carrying capacity with respect to  $\sigma$  and  $\alpha$ .



**Fig. 26.** Variation of Load carrying capacity with respect to  $\sigma$  and l/s.

The effect of skewness presented in Fig. 27-28 indicates that negatively skewed roughness increases the load carrying capacity while the load decreases when positive skewness increases. Further, the trends of load carrying capacity with respect to variance are quite similar to that of skewness (Fig. 29). Roughness of the bearing surfaces has an adverse effect in general because the roughness retards the motion of the lubricant resulting in decreased pressure and consequently the load carrying capacity. However, the combined effect of negatively skewed roughness and variance (-ve) goes a long way in mitigating the adverse effect of the standard deviation when lower value of slip parameter is considered.

The slip effect is not that predominant for Jenkins model as compared to that of Neuringer-Rosensweig model.



**Fig. 27.** Variation of Load carrying capacity with respect to  $\overline{\varepsilon}$  and  $\overline{\alpha}$ .



**Fig. 28.** Variation of Load carrying capacity with respect to  $\bar{\varepsilon}$  and  $l/\bar{s}$ .



Fig. 29. Variation of Load carrying capacity with respect to  $\overline{\alpha}$  and l/s.

The effect of slip can be reduced to some extent by the positive effect of Jenkins model based magnetic fluid flow by choosing the curvature parameters, at least in the case of negatively skewed roughness.

Many figures are there to suggest that for an effective performance of the bearing system the slip parameter is required to be kept at minimum.

### 4. Conclusions

Although, there are a number of parameters bringing down the load carrying capacity, this article offers the suggestion that there exist some scopes for obtaining a better performance. This makes all the more necessary to account for roughness while designing the bearing system because the magnetization has a limited option to compensate the adverse effect of slip and standard deviation even if the curvature parameters are chosen suitably. However, it becomes interesting to note that this type of bearing system sustains certain amount of load even in the absence of flow unlike the case of conventional lubricant based bearing system. It is note worthy that Jenkins model scores over Neuringer- Rosensweig model for this type of bearing system as Jenkins model modifies the velocity of the lubricant as well.

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Извод

## Утицај брзине клизања и храпавости на перформансе Jenkins-овог модела заснованог на магнетном филму код закривљених плоча

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### Резиме

Уложен је труд да се испитају и анализирају перформансе Jenkins-овог модела заснованог на ферофлуидном филму у закривљеним хрпавим кружним плочама узимајући о убзир утицај брзине клизања. Коришћен је Beavers-ов и Joseph-ов модел клизања као и Christensen-ово и Tonder-ово стохастичко моделирање. Уз помоћ одговарајућих граничних услова, повезана Reynold-ова једначина је решена како би се добио израз за дистрибуцију притиска, што је отворило пут ка израчунавању капацитета носивости. Графички прикази јасно показују да иако капацитет носивости знатно расте са повећањем магнетизације, магнетизација има ограничене могућности за ублажавање негативних ефеката брзине клизања и стандардне девијације. Међутим, пожељно је да параметар клизања буде на минимуму иако су одабрани одговарајући параметри кривине. На крају, Jenkins-ов модел иде даље од Neuringer- Rosensweig модела у смањивању негативних ефеката храпавости.

Кључне речи: брзина клизања, Jenkins-ов модел, магнетна течност, закривљене кружне плоче

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