Performance of magnetic fluid based squeeze film between a curved porous circular plate and a flat circular plate and effect of surface roughness

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Abstract

Efforts have been directed to study and analyze the squeeze film behavior between a curved rough porous circular plate and a flat rough porous circular plate under the presence of a magnetic fluid lubricant. The curved film thickness is described by a secant function. It is taken into consideration that the external magnetic field is oblique to the lower plate. The bearing surfaces are assumed to be transversely rough. The random roughness of the bearing surfaces is characterized by a stochastic random variable with non-zero mean, variance and skewness. The concerned Reynolds equation governing the film pressure is averaged with respect to the random roughness parameter. The associated non-dimensional partial differential equation is solved with appropriate boundary conditions in dimensionless form to obtain the pressure distribution, in turn, which leads to the calculation of load carrying capacity. The computed results show that the performance of bearing system enhances considerably as compared to that of a bearing system working with a conventional lubricant as the magnetization increases the effective viscosity of the lubricant. The results tend to indicate that the bearing suffers due to transverse surface roughness, in general. Probably this may be due to the fact that the transverse surface roughness offers resistance to the motion of the lubricant. The effect of variance (negative) is considerably positive at least in the case of negatively skewed roughness as the load carrying capacity arises sharply. The combined effect of porosity and standard deviation is relatively adverse, in the sense that the already decreased load carrying capacity due to porosity gets further decreased owing to standard deviation. However, this investigation suggests some ways to mitigate this adverse effect.

Keywords: Squeeze film, Magnetic fluid, Rough surface, Reynolds equation, Load carrying capacity, Circular plates.

1. Introduction

The squeeze film behavior between circular disks when one of them has a porous facing, press–fitted in solid wall; was discussed by [Murti 1975]. [Prakash et al. 1973] simplified the analysis of Murti by incorporating the Morgan Cameron approximation, when the porous facing thickness is assumed small enough. Several studies have appeared concerning the problem of

The squeeze film behavior between a curved upper plate and a flat lower plate was studied by [Murti 1975] and it was shown that the load carrying capacity rose sharply with curvature in the case of concave pads. [Gupta et al. 1980] discussed the corresponding problem in the case of annular plates. In the above analysis, the lower plate was considered to be flat one. [Ajwaliya 1984] extended these analyses by assuming the lower plate also to be curved. According to his investigations such situation could be found useful in the design of machine elements like clutch plates and collar bearings.

In all the above investigations conventional lubricants were used. Use of magnetic fluid as a lubricant modifying the performance of the bearing system, has been very well recognized. [Verma 1986] discussed the squeeze film performance under the presence of a magnetic fluid lubricant. [Bhat and Deheri 1991] analyzed the behavior of the squeeze film between porous annular disks. Here it was concluded that the application of magnetic fluid lubricant enhanced the performance of the squeeze film. However, they assumed that the plates were flat. But in actual practice the flatness of the plate does not endure owing to elastic, thermal and uneven wear effects. With this end in view, [Bhat et al. 1993] studied the effect of magnetic fluid lubricant on the configuration of [Ajwaliya 1984] considering the surface of the plate approximated by an exponential function. [Patel et al. 2002] deliberated on the squeeze film behavior between curved circular plates using a magnetic fluid lubricant. They found that the magnetic fluid lubricant improved the performance of bearing system. [Deheri and Patel 2006] studied the effect of magnetic fluid lubricant on the behaviour of a squeeze film between porous circular disks with sealed boundary. Here it was proved that the performance of the bearing system enhance considerably by sealing the boundary and choosing suitable magnetic strength. [Hsu et al. 2008] investigated magnetic hydrodynamic squeeze film characteristic between circular disks incorporating the effects of rotational inertia. From the results obtain it was clear that the rotational inertia was resulted in a reduced load carrying capacity while the applied magnetic fields provided an increase in load carrying capacity and response time. This study established that the squeeze film characteristics were found to be improved by the use of an electrically conducted fluid in the presence of a transverse magnetic field. [Hsu et al. 2009] presented a study dealing with the combined effects of surface roughness and rotating inertia up on squeeze film characteristic between circular disks while the surface structure with circumferential roughness increase the load carrying capacity and lengthened the response time, the radial structure reverse these trends. [Shimpi and Deheri 2010] analyzed the surface roughness and elastic deformation effects on the performance of a magnetic fluid based squeeze film between rotating porous circular plates with concentric circular pockets. Here it was concluded that the negative effect of porosity roughness and deformation could be minimized by the positive effect of magnetization parameters at least in the case of negatively skewed roughness by choosing suitably the pocket radius. [Davim 2011] investigated the performance of a hydrodynamics rough film. Here the Tribology aspects was examined with special emphases on surface topography, wear of materials and lubrications. [Huang et al. 2011] discussed the effect of Ferrofluid lubricant with an external magnetic field. It was concluded that the Ferrofluid registered a relatively good friction reduction performance in the presence of an external magnetic fluid compared with carrier liquid. Also, the life period was found to be improved.

After having some run–in and wear the bearing surfaces develop roughness. The roughness appears to be random in character. Several investigations were devoted to study and analyze the effect of surface roughness [Davies 1963, Burton 1963, Michell 1950, Tonder 1972, Tzeng et al.

Here it has been sought to investigate the effect of surface roughness on the behavior of magnetic fluid based squeeze film between a curved porous circular plate and a flat porous circular plate where in, the curved film thickness is described by a secant function.

2. Analysis

The configuration of the bearing system is displayed in Fig. 1.

![Fig. 1.](image)

It is assumed that the surface of the upper plate determined by

\[ h = h_0 \sec(Br^2), \quad 0 \leq r \leq a \]

approaches the lower plate with the normal velocity \( h_0 \), where \( h_0 \) is the central distance between the plates and \( B \) is the curvature parameter.

The bearing surfaces are assumed to be transversely rough. Following [Tzeng and Seibel 1967] the film thickness \( h(x) \) of the lubricant film is

\[ h(x) = \bar{h}(x) + h_s \]

where \( \bar{h}(x) \) is the mean film thickness and \( h_s \) is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. \( h_s \) is considered to be stochastic in nature and governed by the probability density function.
\[ f(h_s) = \begin{cases} 
  \frac{35}{32c^2} \left(1 - \frac{h_s^2}{c^2}\right)^2, & -c \leq h_s \leq c \\
  0, & \text{Otherwise}
\end{cases} \] (3)

where \( c \) is the maximum deviation from the mean film thickness. The mean \( \alpha \), the standard deviation \( \sigma \) and the parameter \( \varepsilon \), which is the measure of symmetry of random variable \( h_s \) are defined by the relationships

\[ \alpha = E(h_s) \] (4)

\[ \sigma^2 = E[(h_s - \alpha)^2] \] (5)

and

\[ \varepsilon = E[(h_s - \alpha)^3] \] (6)

where \( E \) denotes the expected value defined by

\[ E(R) = \int_{-c}^{c} Rf(h_s)dS \] (7)

Assuming axially symmetric flow of the magnetic fluid between the plates under an oblique magnetic field

\[ \bar{H} = (H(r)\cos \phi(r,z), 0, H(r)\sin \phi(r,z)) \] (8)

whose magnitude \( H \) vanishes at \( r = a \), the modified Reynolds equation governing the film pressure \( p \) becomes [Bhat, Patel et al., Gupta et al.]

\[ \frac{1}{r} \frac{d}{dr} \left[ rg(h) \frac{d}{dr} \left( p - \frac{\mu_0 \mu H^2}{2} \right) \right] = 12\eta \dot{h} \] (9)

where

\[ g(h) = h^3 + 3\alpha h^2 + 3\alpha^2 h + 3\sigma^2 h + \alpha^3 + \varepsilon + 12\phi H_0 \] (10)

Also, \( \mu_0 \) is permeability of free space, \( \mu \) is the magnetic susceptibility of particles and \( \eta \) is the viscosity of the lubricant, \( \phi \) is the permeability of porous facing, \( H_0 \) is the thickness of porous medium. Taking, for instance,

\[ H^2 = ka(a-r), \ 0 \leq r \leq a \] (11)

where \( k = 10^{14} A^2 m^{-4} \) chosen so as to have a magnetic field of strength over \( 10^5 \) [Bhat 2003] and remembering that the magnetic field arises out of a potential, it can be shown that the inclination \( \varphi \) satisfies the equation
\[
\cot \varphi \frac{\partial \varphi}{\partial r} + \frac{\partial \varphi}{\partial z} = \frac{3r-2a}{2r(a-r)}
\]

whose solutions are

\[
C^2 \cos e c^2 \varphi = \frac{1.5r-a}{a-r}
\]

and

\[
Z = -2C \sqrt{\frac{a-r}{1.5r-a}}
\]

where \( C \) is a constant of integration. Introducing the non-dimensional quantities

\[
P = -\frac{h_0^3 p}{\eta a^2 h_0}, \quad R = \frac{r}{a}, \quad \beta = \beta a^2, \quad \psi = \frac{\phi H_0}{h_0^3},
\]

\[
\mu^* = -\frac{h_0^3 \mu_0 b k}{\eta a^2 h_0}, \quad \sigma = \frac{\sigma}{h_0}, \quad \alpha = \frac{\alpha}{h_0}, \quad \varepsilon = \frac{\varepsilon}{h_0^3}.
\]

with the usual assumptions of hydromagnetic lubrication. The modified Reynolds equation governing the film pressure \( p \) turns out to be [Bhat 2003, Deheri et al. 2005]

\[
\frac{d}{dR}\left[RG(h) \frac{d}{dR} \left\{-\frac{h_0^3 p}{\eta a^2 h_0} + \frac{h_0^3 \mu_0 b k}{\eta a^2 h_0} (1-R)\right\}\right] = -12R
\]

where

\[
G(h) = \sec^3 \left(\beta R^2\right) + 3\alpha \sec^2 \left(\beta R^2\right) + 3\left(\alpha^2 + \sigma^2\right) \sec \left(\beta R^2\right) + 3\alpha \sigma^2 + \alpha + \varepsilon + 12\psi
\]

In view of the non-dimensional quantities introduced above, the associated non-dimensional Reynolds equation governing the dimensionless pressure \( P \) is obtained from

\[
\frac{d}{dR}\left[g\left(h\right) + A\beta R^4\right] \frac{d}{dR} \left\{P - \frac{\mu^*}{2} (1-R)\right\} = -12R
\]

where

\[
g\left(h\right) = 1 + 3\alpha + 3\alpha^2 + 3\sigma^2 + 3\alpha \sigma^2 + \alpha + \varepsilon + 12\psi
\]

and

\[
A = 0.5 \left(3 + 6\alpha + 3\alpha^2 + 3\sigma^2\right)
\]

while \( \psi \) is the porosity parameter. Solving equation (18) under the boundary conditions,
$$P(1) = 0, \quad \frac{dP}{dR} = -\frac{\mu^*}{2} \quad \text{when} \quad R = 0$$  \hspace{1cm} (21)

One obtains the non-dimensional pressure distribution:

$$P = \frac{\mu^*}{2} (1 - R) + \frac{3(1 - R^2)}{g\bar{h} + A\bar{\beta}^2 R^2}$$  \hspace{1cm} (22)

Lastly, the load carrying capacity $W$ is determined in dimensionless form as

$$W = -\frac{h_0^3 w}{2\pi\eta h_0} \int_0^1 RP dR$$  \hspace{1cm} (23)

and hence

$$W = \frac{\mu^*}{12} \frac{1.5}{A\bar{\beta}^3} + \frac{3\left(A\bar{\beta}^3 + g\bar{h}\right)}{2\left(A\bar{\beta}^3\right)^2} \ln \left[\frac{A\bar{\beta}^3 + g\bar{h}}{g\bar{h}}\right]$$  \hspace{1cm} (24)

### 3. Results and discussion

It is clearly seen from Equation (22) and Equation (24) that the non-dimensional pressure distribution is determined from Equation (22) while Equation (24) presents the distribution of dimensionless load carrying capacity. These two expressions depend on various parameters such as $\mu^*, \bar{\sigma}, \bar{e}, \bar{\alpha}, \bar{\beta}$ and $\psi$. These parameters describe respectively, the effect of magnetization, roughness, curvature of the plate and porosity. Furthermore, a close glance at the Equation (22) and Equation (24) reveals that the increase in pressure is

$$\frac{\mu^*}{2} (1 - R)$$  \hspace{1cm} (25)

while the load carrying capacity enhances by

$$\frac{\mu^*}{12}$$  \hspace{1cm} (26)

due to the magnetic fluid lubricant. Furthermore, it is seen that taking roughness parameters to be zero, this study reduces to the performance of a squeeze film in curved porous circular plates under the presence of a magnetic fluid lubricant. In the absence of magnetization, this investigation gives the squeeze film performance in curved porous circular plates. Further, taking the curvature parameter to be zero, one obtains the performance of a squeeze film in porous circular plates [Prakash et al. 1973]. This reduces to the performance of a squeeze film in circular plates in the absence of porosity.

In order to analyze the quantitative effect of the computed results the following fixed values of different parameters are considered $\mu^* = 0.5, \bar{\sigma} = 0.05, \bar{\alpha} = -0.05, \bar{e} = -0.05, \psi = 0.001, \bar{\beta} = 0.45$. 
The variation of load carrying capacity with respect to the magnetization parameter for different values of $\overline{\sigma}$, $\bar{\varepsilon}$, $\overline{\alpha}$, $\overline{\beta}$ and $\psi$ presented in Figures 1–5, suggest that the load carrying capacity rises sharply due to magnetic fluid lubricant. The effect of curvature parameter is to decrease the load carrying capacity.

**Fig. 1.** Variation of Load carrying capacity with respect to $\mu^*$ and $\overline{\sigma}$

**Fig. 2.** Variation of Load carrying capacity with respect to $\mu^*$ and $\overline{\alpha}$
Fig. 3 Variation of Load carrying capacity with respect to $\mu^*$ and $\bar{\varepsilon}$

Fig. 4. Variation of Load carrying capacity with respect to $\mu^*$ and $\bar{\beta}$

Fig. 5. Variation of Load carrying capacity with respect to $\mu^*$ and $\psi$
Figures 6-9, describe the variation of load carrying capacity with respect to standard deviation for various values of variance $\alpha$, skewness $\varepsilon$, $\beta$ and $\psi$ respectively. It is clearly observed that $\beta$ has a significantly negative effect, which means that the already decreased load carrying capacity due to $\sigma$ further decreases owing to $\beta$. Therefore, the combined effect of $\sigma$ and $\beta$ is considerably negative in the sense that the load carrying capacity decreases sharply. But the effect of porosity with respect to standard deviation is almost negligible for the values of $\psi$ less than 0.001.

![Graph](image1)

**Fig. 6.** Variation of Load carrying capacity with respect to $\sigma$ and $\alpha$

![Graph](image2)

**Fig. 7.** Variation of Load carrying capacity with respect to $\sigma$ and $\varepsilon$
Fig. 8. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\beta}$

Fig. 9. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\psi$

Fig. 10. Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $\bar{\varepsilon}$
In Figures 10-12, one can see the profile for the load distribution with respect to the variance for different values of \( \varepsilon \), \( \beta \) and \( \psi \) respectively. From these three Figures, it is easily seen that variance (+ve) decreases the load carrying capacity while variance (-ve) increases the load carrying capacity. However, the effect of the curvature parameter \( \beta \) with respect to the variance \( \alpha \) is significant in the sense that the combined effect of the curvature parameter and the variance (+ve) is considerably adverse as indicated in Figure. 11.

Fig. 11. Variation of Load carrying capacity with respect to \( \alpha \) and \( \beta \)

The effect of skewness is depicted in Figures 13–14, so far as load carrying capacity is concerned. It is observed that skewness follows the path of the variance with regards to the trends. But the effect of curvature parameter on the distribution of the load carrying capacity
with respect to \( \bar{e} \) is significant. Further, the effect of skewness is sharper as compared to that of the variance.

\[ \beta = 0.25 \quad \beta = 0.35 \quad \beta = 0.45 \quad \beta = 0.55 \quad \beta = 0.65 \]

**Fig. 13.** Variation of Load carrying capacity with respect to \( \bar{e} \) and \( \bar{\beta} \)

\[ \phi = 0.0001 \quad \phi = 0.001 \quad \phi = 0.01 \quad \phi = 0.1 \quad \phi = 1 \]

**Fig. 14.** Variation of Load carrying capacity with respect to \( \bar{e} \) and \( \phi \)

Lastly, the effect of \( \phi \) with respect to the curvature parameter on the distribution of the load carrying capacity is negligible for the values of porosity less than 0.001. This can be observed in Figure. 15.
Some of these figures establish that the negative effect of porosity and the standard deviation can be compensated up to some extent by the positive effect of magnetic fluid lubricant in the case of negatively skewed roughness, particularly, when the negative variance is involved; by choosing a suitable value of the curvature parameter $\beta$. It is clearly noticed that the combined effect of negatively skewed roughness and negative variance is significantly positive.

4. Conclusion

This article offers ample scopes for reducing the adverse effect of the standard deviation and porosity by the positive effect of magnetization parameter in the case of negatively skewed roughness by suitably choosing the curvature parameter. The reduction becomes sharper particularly, when negative variance occurs. Thus, this analysis makes it mandatory that the roughness must be accorded due consideration while designing the bearing system especially, from bearing’s life period point of view. Further, this investigation establishes that the bearing can support a load even when there is no flow, which does not happen in the case of conventional lubricant.
Извод

Перформанс притиснутог филма магнетног флуида између закривљене порозне кружне плоче и равне кружне плоче и ефекат површинске храпавости

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Резиме

Рад је усмерен ка анализи понашања притиснутог филма између закривљене грубе порозне кружне плоче и равне грубе порозне кружне плоче у присуству магнетног подмазивача. Дебљина закривљеног филма је описана функцијом сечице. Узето је у обзир да је спољашње магнетно поље усмерено према доњој плочи. Претпостављају се да су попречне грубе површине лежајева. Насумична храпавост површина лежаја охарактерисана је стихастичком случајном променливо са ненултног средњег вредношћу, варијансом и нагибом. Одговарајућа Рејнолдсова једначина која одређује притисак филма је узета као просечна у односу на случајан параметар храпавости. Одговарајућа бездимензионална парцијална диференцијална једначина је решена са граничним условима у бездимензионалном облику да би се добила статистичка притиска, што заузврат води до анализа носивости. Добијени резултати показују да се перформанс лежајног система значајно побољшава у поређењу са лежајним системом који ради са уобичајеним подмазивачем пошто магнетизација повећава ефекте вискозности подмазивача. Резултати генерално показују да лежај трпи због попречне храпавости површине. Ово, вероватно, може бити због чињенице да попречна храпавост површине пружа отпор кретању подмазивача. Ефекат варијансе (негативне) је значајан, поготово као се носивост оптерећења нагло посече. Комбинација ефекта порозности и стандардне девијације је релативно неповољна у смислу да се већ смањена носивост услед порозности постоји што још смањује због стандардне девијације. Међутим, ово истраживање предлаже неке начине за ублажавање ових лоших ефеката.

Кључне речи: Танак филм, магнетни флуид, храпава површина, Рејнолдсова једначина, носивост, кружне плоче

References


