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Damping identification for building structures subjected to earthquakes: A review

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Abstract

This paper reviews existing methods employed for the identification of damping in building structures and the properties, practical application and problems commonly associated with the most important of those methods. Some characteristic examples are presented to demonstrate the capabilities of these methods. Emphasis is given to the application of these methods to identify damping when a seismically excited building structure vibrates in the non-linear region. Further needs on damping identification are also addressed.

Keywords: damping, identification, buildings, seismic motions, linear response, non-linear response.

1. Introduction

During the last five decades or so, the application of system identification methods in building structures has significantly contributed to the successful investigation of several problems in earthquake engineering. The crucial point for these methods was to treat correctly two major issues: a) the determination of a mathematical model having a finite set of parameters. This model should be able to represent the behavior of the structure within an acceptable tolerance; b) the identification of these parameters based on the observed behavior of the structure. In general, system identification methods can be separated into two categories according to the kind of parameters identified: a) methods for identifying modal parameters and b) methods for identifying physical parameters. The methods belonging to the former category are computationally simple, make use of a small number of measured data and aim to identify the general dynamic properties of the structure, i.e., modes, natural frequencies, modal damping ratios. On the other hand, the methods of the latter category are computationally more complex, need a great number of measured data and aim at the identification of the mass, stiffness and damping matrices of the structure.

Among the parameters that should be identified, those related to damping are considered to be the most difficult ones for identification purposes simply because the modeling of damping in a structure is a very difficult process as one should take into account various mechanisms like friction, heat, plasticity, damage etc. Due to the complexity inherent in these mechanisms, the representation of damping in a building structure has been commonly assumed to be that of a linear viscous type or of a Rayleigh type. Under the condition that the damping matrix of the structure is a linear combination of its mass and stiffness matrices, one has the widely known classical damping model (Clough and Penzien 1975). On the other hand, in cases where the damping matrix does not obey the aforementioned requirement, e.g., a soil-structure system, then one has to adopt the non-classical damping model (Clough and Penzien 1975). It should be noted that the viscous damping consideration although convenient may be a cause of error when estimating the response of a structure even in the elastic range. This error has been examined by Val and Segal (2005). On the other hand, the equivalent viscous damping approach (Jacobsen 1930) has been used by several researchers to model the work performed due to inelastic deformations of seismically excited structures (Jennings 1968, Gülkan and Sozen 1974, Shibata and Sozen 1976, Takewaki 1997, Blandon and Priestley 2005).

The purpose of this review paper is to present the up-to-date efforts to the problem of damping identification in building structures subjected to earthquakes. Firstly, a brief description of the modeling and estimation of damping in structures is given. Then, the most important methods are mentioned and the properties, practical application and problems commonly associated with them are discussed. Some characteristic examples are presented to demonstrate the capabilities of these methods. The damping identification methods presented herein cover both the linear and non-linear seismic response of structures. However, emphasis is given to the application of these methods to identify damping when a seismically excited building structure vibrates in the non-linear region. Finally, further needs on damping identification are also addressed.

2. Modeling and estimation of damping in building structures

Finite element procedures for constructing mass and stiffness matrices for the individual members of a structure and the global mass and stiffness matrices of that structure through appropriate assemblage are well established. However, similar procedures for damping representation are very few (e.g., Feriani and Perotti 1996) as one would have to construct the damping matrix of the structure using the damping properties of its individual members (Hart et al. 1974). To find the seismic response of an elastic structure in the context of modal synthesis, damping is prescribed at the level of modes. Alternatively, if a classical damping matrix is assumed, then by application of the orthogonality conditions (Clough and Penzien 1975) one can find the initially prescribed modal damping ratios. The selection of this diagonal damping matrix can be done following Caughey and O'Kelly (1965), Wilson and Penzien (1972), Luco (2008a and 2008b) and Kausel (2008). On the other hand, to find the seismic response of an inelastic structure, one considers a damping matrix of the classical form (Rayleigh damping proportional to mass and stiffness matrices) in order to provide realistic levels of damping at small vibration amplitudes or to guarantee numerical stability (Kausel 2008). In this case, one selects damping ratios for two modes (the first and a higher one) and determines the constants of proportionality in the Rayleigh damping expression as well as the damping ratios for the rest of the modes (Clough and Penzien 1975). In passing, it should be noted that Caughey series (Caughey and O'Kelly 1965), that can be also used to assemble a classical damping matrix, may lead to several drawbacks (Luco 2008b).

The aforementioned damping considerations for both elastic and inelastic responses of a structure are valid when this damping property is uniformly distributed along its various members. However, in many practical cases damping in a structure is not uniformly distributed in its various members, e.g., the cases of a soil-structure system, a base-isolated structure or a structure having added dampers. In these cases the damping matrix is considered to be of non-classical form although the individual damping matrices of the elements of the structure or its substructures can be of a classical form. The response of a seismically excited elastic or inelastic structure having non-classical damping can be exactly found by time integration of its equations of motion. Nevertheless, classical damping can be approximately considered for these structures instead of a non-classical one (e.g. Thomson et al 1974), hoping that the errors introduced by this approximation are small (e.g. Warburton and Soni 1978). Alternatively, the response of a seismically excited elastic or inelastic structure having non-classical damping can be obtained by an iterative procedure as in Udwadia and Esfandiari (1990). More advanced damping models capable of describing the dynamic response of non-classically damped structures need to be developed (Wang 2009).

At this point an important deficiency that several researchers have pointed out regarding the use of classical damping matrix to obtain the inelastic response of a structure should be mentioned. This deficiency mainly has to do with the consideration of the initial stiffness of the structure and not its tangent stiffness for the construction of the classical damping matrix (Mohraz et al. 1991, Léger and Dussault 1992, Bernal 1994, Priestley and Grant 2005, Hall 2006, Charney 2008, Zareian and Medina 2010, Erduran 2012). However, the use of tangent stiffness in modeling the classical damping matrix lacks sufficient experimental evidence (Petrini et al. 2008, Smyrou et al. 2011) and can be avoided if a more physical approach is adopted, i.e., controlling the damping forces of the members of a structure against excessive and unjustifiable values (Bernal 1994, Hall 2006, Charney 2008).

Damping properties of a building structure are not well-established and cannot take into account various important mechanisms like friction in steel connections, opening and closing of micro-cracks in reinforced concrete and friction among structural and non-structural elements. The reason is that each one of these mechanisms cannot be effectively isolated and quantified. Thus, one uses either empirical damping estimates coming from full-scale experimental measurements on buildings or, as described in the next section, employs damping identification techniques.

Engineers often assign a constant value for the damping ratio of the first mode or even of all significant modes using judgment, e.g., 2% for steel and 5% for reinforced concrete structures. These values as well as other ones for different materials can be used in spectral analysis in conjunction with modal synthesis or in linear time history analysis with the aid of Rayleigh type of damping (Newmark and Hall 1982). More advanced empirical modal damping expressions for the linear range of dynamic response based on relatively small data sets have been given by Tanaka et al. (1969), Haviland (1976), Davenport and Hill-Carroll (1986), Jeary (1986), Lagomarsino (1993) and Satake et al. (2003). These empirical expressions are attractive because of their simplicity but present several limitations e.g., the few data considered and the inherent variability of the estimates.

In an effort to address these limitations, Fritz et al. (2009) compiled 1572 damping measurements and they associated with each measurement ten parameters describing the characteristics of building and excitation. They observed a decrease in damping as building height increases and, for steel and reinforced concrete structures, a twofold increase in damping for earthquake excitations as opposed to other lower amplitude excitations. Similar trends have been observed by Satake et al. (2003). The results of Satake et al. (2003) are considered to be more exact in comparison to those of Newmark and Hall (1982) and Fritz et al. (2009) regarding steel and reinforced concrete structures because different damping values for each mode are provided. Nevertheless, the effect of coupling of modes, i.e., two modes having almost the same frequency, to the value of damping (Kareem and Gurley 1996) has not been considered in any of the aforementioned empirical models.

3. Identification of damping in building structures

One of the main problems in structural analysis for design is the availability of reliable dynamic structural properties (e.g. mass, stiffness, damping) for a successful modeling and response determination. Of particular interest is the identification of damping which has also originally received attention under the so-called dynamic testing. Estimates of damping ratio from dynamic testing have been originally given by Nielsen (1968), Rea et al. (1969), Jennings et al. (1972), Wood (1976) etc. However, most of these damping estimates should be viewed with caution because of the frequency resolution of the data used and the signal processing techniques conducted on those data (Hamming 1998).

Damping identification constitutes part of structure (system) identification for which detailed and comprehensive reviews can be found in the works of Hart and Yao (1977), Kozin and Natke (1986), Imai et al. (1989), Ghanem and Shinozuka (1995) and Shinozuka and Ghanem (1995) and Kerschen et al. (2006). Taking into account the complexity of the damping mechanism in structures, its identification usually leads to a classical or a non-classical viscous damping matrix. Although objections have been raised regarding the use of viscous damping, it appears doubtful that the use of more refined and sophisticated damping models can be justified yet in view of the considerable more information needed to define their parameters (Adhikari and Woodhouse 2001 and Adhikari 2002).

The first complete works on damping identification came from the field of mechanical and aerospace engineering. More specifically, identification of damping in a general dynamic system involved perturbational (Hasselman 1972), sub-structuring (Hasselman 1976, Jezequel 1980) and optimization techniques (Caravani and Thomson 1974). Up-to-date they are many methods to identify damping (in a matrix form or in terms of modal damping ratios) both in time and frequency domains (Huang et al. 2007, Phani and Woodhouse 2007, Li and Law 2009 and references therein). But, the most promising methods seem to be the recently developed ones that make use of the wavelet transform (Ruzzene et al. 1997, Staszewski 1998, Hans et al. 2000, Slavič et al. (2003), Boltežar and Slavič (2004), Ceravolo 2004, Yin et al. (2004), Chen et al. 2009, Slavič and Boltežar 2011). In passing, it should be noted that many researchers have been dealt with the identification of the damping matrix but not in conjunction with a building structure. Therefore, these identification methods are not considered herein.

The first efforts of civil engineers to identify damping in building structure (shear type ones) were conducted by Béliveau (1976) and Udwadia et al. (1978). These two damping identification approaches were applied to a linear structure. Hart and Vasudevan (1975) were the first who attempted to identify modal damping ratios from measurements of seismically excited real buildings. However, in spite of the importance of damping in seismic design of buildings, its identification techniques have not been sufficiently developed, especially when damping in the non-linear range of seismic response is sought. In the following, the most important damping identification methods are briefly described taking into account the linearity or non-linearity of the structure, the domain where identification is employed as well as torsional coupling and soil-structure interaction effects.

Damping identification for linear building structures in the time domain has been performed by Udwadia and Marmarelis (1976), Caravani et al. (1977), Beck and Jennings (1980), Şafak (1989), Toki et al. (1989), Koh et al. (1991), Lee and Yun (1991), Şafak (1991), Tan and Cheng (1993), Wang and Haldar (1997), Yun et al. (1997), Luş et al. (1999), Quek et al. (1999), Luş et al. (2002), Yang et al. (2003a & 2003b), Takewaki and Nakamura (2005) and Ulusoy et al. (2011). Identification of time-varying damping ratios from the linear seismic responses of structures has been studied by Loh and Tou (1995), Shinozuka and Ghanem (1995), Loh and Lin (1996), Yang and Lin (2005) and Wang and Chen (2012). Frequency domain identification resulting in time-invariant damping ratios for linear structures has been proposed by McVerry (1980), Zhao et al. (1995), Brincker et al. (2001), Kim et al. (2005), Papagiannopoulos and Beskos (2006), Mahmoudabadi et al. (2007) and Papagiannopoulos and Beskos (2009). Identification of time-varying damping ratios from the non-linear seismic responses of buildings has been studied by Marmarelis and Udwadia (1976), Beck and Jennings (1980), Loh and Tsaur (1988), Şafak (1989), Loh and Tou (1995), Shinozuka and Ghanem (1995), Loh and Lin (1996), Loh et al. (2000),Yang and Lin (2005), Moaveni and Asgarieh (2012) and Shi and Chang (2012). The majority of the latter works belong to the so-called damage identification (detection) methods. Finally, Papagiannopoulos and Beskos (2010) developed a seismic design method on the basis of the identification of time-invariant damping ratios from the non-linear seismic responses of structures.

Damping identification for torsionally coupled buildings has been performed by Lin and Papageorgiou (1989), Li and Mau (1991), Ueng et al. (2000), Lin et al. (2005). The effect of soil-structure interaction to the damping identification of symmetrical and un-symmetrical buildings has been taken into account in the works of Şafak (1989), Papageorgiou and Lin (1991), Şafak (1991), Şafak and Çelebi (1991), Stewart and Fenves (1998), Stewart et al. (1999a & 1999b), Lin et al. (2008) and Hong et al. (2009). Finally, one should mention the works of Nagarajaiah and Li (2004) and Kampas and Makris (2012) on damping identification of base isolated buildings and Ji et al. (2012) on damping identification of buildings equipped with dampers.

4. Discussion on some damping identification methods for building structures

In this section the properties, practical application and problems commonly associated with the most important damping identification methods are mentioned. Some characteristic examples are presented to demonstrate the capabilities of these methods. The ability of these methods to identify damping when a seismically excited building structure vibrates in the non-linear region is also discussed.

4.1 Damping identification using wavelets

Damping identification by using wavelets essentially operates in the joint time-frequency domain. Thus, instantaneous estimates for damping can be obtained. The accuracy of these estimates depends on the time segments where the mode, and, thus, the modal damping, to be identified are predominant and it is not affected by the presence of other modes. Consequently, one has to either work with time weighted averages or to select time segments to improve estimation of the other modes. On the other hand, damping estimates depend on the type of window to be used in the transforms, the time length of the window as well as on the damping level of the structure. Figure 1 shows the variation of damping in time of the third mode of the Luciani Hospital of Caracas, Venezuela, after applying time-frequency instantaneous estimators (Ceravolo 2004).

The application of wavelet transforms, despite their intrinsic limitations, addresses in a direct manner the problem of damping identification offering computational advantages in signal analysis. The ability of multi-resolution inherent in the wavelet analysis can filter out the noise from the signal that constitutes a problem in damping identification. The frequency resolution of a wavelet transform can be tuned sufficiently small to separate two signals with close frequency contents. This property is very useful for damping identification of structures having closely spaced frequencies (modes). Furthermore, very light and very heavy damping can be identified using the wavelet transform. However, more work needs to be done towards the identification of damping using the wavelet transform for buildings subjected to severe strong ground motion and, thus, exhibit strong non-linear behavior. The effects of closely spaced modes in unsymmetrical buildings in conjunction with their non-linear seismic response may constitute a challenge to the performance of wavelet-based damping identification methods, which up-to-now have been involved with only simple structures and mild levels of non-linearity.

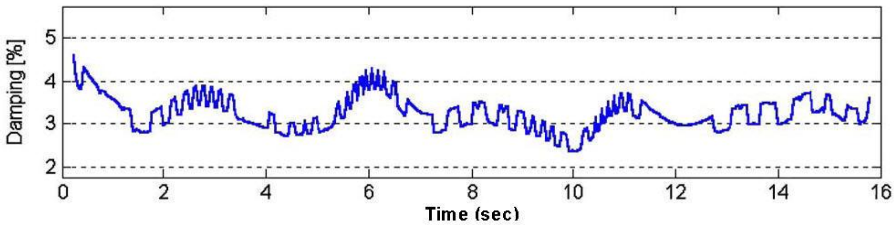

Fig. 1. Instantaneous damping ratio (after Ceravolo 2004)

4.2 Damping identification using the modal minimization method

In the context of identifying a linear dynamic model from seismic response recordings, Beck and Jennings (1980) used input (seismic motion) – output (seismic response) relationships to create a minimal realization capable of reproducing the recorded input-output relationships. Their method is known as the 'modal minimization method', performs in the time domain and is suitable for problems that require high-frequency resolution and non-linear identification. To obtain reasonable accuracy, only the parameters of the dominant modes are estimated performing a series of identifications in which modes are successively added to the models until a measure-of-fit is no longer significantly decreased. This way, optimal linear models were determined. Moreover, the variation of these optimal estimates with time using overlapping time windows shows how the linear parameters change during an earthquake due to non-linear structural response. Figure 2 presents the optimal estimates for different segments of the records from the Union Bank Building studied by Beck and Jennings (1980) using one-mode and two-mode models. In that figure T_1, T_2 are the periods, ζ_1, ζ_2, are the damping ratios and p_1, p_2 are participation factors of the first two modes, respectively, while $J^{1/2}$ is the measure-of-fit. Optimal models were determined by matching displacements (one mode) and by matching velocities (one and two modes) whereas overlapped time windows of 10sec were used.

Time interval (s)	Record used	$\hat{T}_1$	$\hat{\zeta}_1$	$\hat{p}_1$	$\hat{T}_2$	$\hat{\zeta}_2$	$\hat{p}_2$	$J^{\frac{1}{2}}$ (%)
5–15	Displacement	4·41	2·2	0·73				5·1
	Velocity	4·46	4·2	0·79				11·0
	Velocity	4·42	4·1	0·80	1·50	4·6	0·42	6·1
10–20	Displacement	4·58	4·0	0·83				3·8
	Velocity	4·55	3·9	0·70				8·2
	Velocity	4·57	4·3	0·83	1·47	4·8	0·41	2·8
15–25	Displacement	4·64	3·7	0·72				1·9
	Velocity	4·65	3·7	0·72				5·0
	Velocity	4·64	4·0	0·80	1·49	5·0	0·29	2·5
20–30	Displacement	4·72	2·8	0·58				1·6
	Velocity	4·74	2·9	0·68				4·9
	Velocity	4·75	2·8	0·72	1·60	2·9	0·33	2·1

Fig. 2. Optimal estimates from Union Bank Building (after Beck and Jennings 1980)

The modal minimization method is simple and practical and has been used by Lin and Papageorgiou (1989), Li and Mau (1991) and Papageorgiou and Lin (1991) to the identification of torsional modes, a topic that was not included in its original version by Beck and Jennings (1980). According to the opinion of the authors, this method can adequately reproduce the seismic response of buildings that exhibit linear or mildly non-linear behavior. However, the identification of damping using the modal minimization method depends on the level of noise in the recorded responses, the sensitivity between the individual values of damping ratios and participation factors and the presence of coupled modes.

4.3 Damping identification using recursive least-squares method

The least-squares recursive method, originally employed by Caravani et al. (1977), is developed from one time point and updated to the next one. Since measurements equal to the number of floors are required and the presence of noise leads to biased damping estimates, this method seems to be impractical for buildings of many stories. However, the efficiency of the method can be substantially improved by implementing a process by which less weight is given to older data. This can be done using exponential-window or rectangular-window algorithms (Ghanem and Shinozuka 1995). Figure 3 shows damping ratio results corresponding to the least-squares estimation for the fifth floor of a five-story steel building using an exponential window. According to Shinozuka and Ghanem (1995), the least-squares recursive method and its variations always yield results, the significance of which is intimately related to the concept of least-squares interpolation. In general, this method requires only minimal expertise, offers always numerical convergence and by using windows enhances reliability of results. It can be used to track the variation of dynamic properties of non-linear structures but demands engineering judgment when it is applied to windows of seismic response instead of the entire duration seismic response (Nagarajaiah and Li 2004).

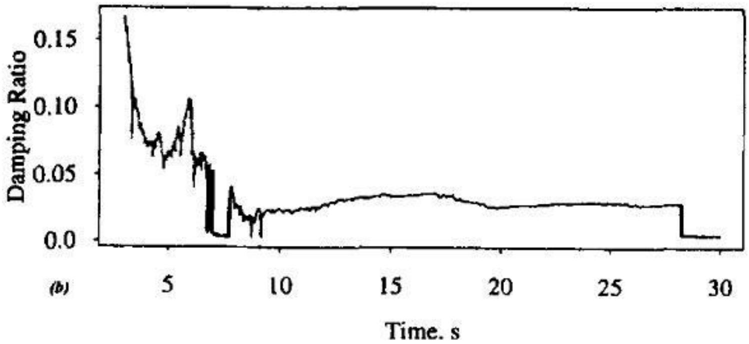

Fig. 3. Damping ratio of a five-story steel building using the recursive least-squares method (after Shinozuka and Ghanem 1995)

4.4 Damping identification using the extended Kalman filter method

The basic algorithm of the extended Kalman filter is a recursive process for estimating the optimal state of linear or non-linear structures based on observed data for the input (seismic motion) and output (seismic response). The algorithm is summarized in Imai et al. (1989) and Ghanem and Shinozuka (1995). Estimates obtained by this method may be biased or divergent and, thus, some modified algorithms have been proposed to improve convergence and reduce the error in estimation (Imai et al. 1989, Ghanem and Shinozuka 1995). Damping identification results using the extended Kalman filter to seismic responses of a five-story steel building are shown in Fig.4. In that figure a dash denotes that results were not obtained using this method. It should be noted that although the extended Kalman filter has been broadly applied to non-linear system identification, it requires long-length data and appropriate initial values of the unknown model parameters. These parameters are many in typical building structures and the number of measurements is usually small, rendering the use of this method impractical without mentioning the substantial expertise needed in conjunction with the initial guess of these parameters. A possible alternative method for structures with a large number of degrees of freedom and with measurement data for a limited number of degrees of freedom can be the one developed by Yun et al. (1997). However, the latter method performs well only for linear and equivalent linear systems.

Mode (1)	Frequency (Hz) (2)	Damping ratio (3)
	(*a*) First Floor	
1	3.17	0.375
2	10.2	0.059
3	18.7	0.129
4	28.1	4.88
5	30.1	5.81
	(*b*) Second Floor	
1	3.17	0.377
2	10.2	0.0480
3	18.6	0.600
4	25.6	2.08
5	30.3	2.70
	(*c*) Third Floor	
1	3.17	0.382
2	10.2	0.058
3	18.7	0.160
4	—	—
5	—	—
	(*d*) Fourth Floor	
1	3.17	0.382
2	10.2	0.036
3	18.7	0.125
4	28.1	0.946
5	—	—
	(*e*) Fifth Floor	
1	3.17	0.380
2	10.2	0.055
3	18.7	0.680
4	25.9	10.2
5	—	—

Fig. 4. Damping ratio of a five-story steel building using the extended Kalman filter method (after Shinozuka and Ghanem 1995)

4.5 Damping identification using a frequency domain transfer function

This damping identification method has been originally applied by Hart and Vasudevan (1975) to seismic recordings of some real reinforced concrete and steel buildings that responded linearly or non-linearly to the 1971 San Fernando earthquake, with rather inconclusive results with respect to its validity and range of applicability. Papagiannopoulos and Beskos (2006, 2009) theoretically generalized this method and established its range of applicability on the basis of numerical studies involving seismically excited linear plane steel frames having classical and non-classical damping distribution. Papagiannopoulos and Beskos (2010) extended the method by identifying equivalent modal damping ratios from the numerical non-linear seismic response of a large number of plane steel frames excited by various seismic motions. They also implemented these equivalent modal damping ratios in a seismic design framework by providing them as function of seismic deformation and damage. Figure 5 gives design equations for equivalent modal damping ratios ξ as functions of period T for specific seismic performance levels in terms of interstorey drift ratio (IDR) and plastic hinge rotation θ_p and for three cases of seismic motions mentioned in detail in Papagiannopoulos and Beskos (2010). In this figure, a dash is used to denote that the damping ratio of these modes cannot be found by this method but has to be considered for accurate response purposes (using, e.g., a value of 100%). More details are given in Papagiannopoulos and Beskos (2010). The method of Papagiannopoulos and Beskos (2010) cannot be easily verified experimentally. However, it is believed that this method can be extended to cover the case of un-symmetrical structures although the effect of closely spaced modes in conjunction with very high damping might render its reliability questionable.

Seismic motions	Mode	IDR=0.6%	IDR=1.5% and $\theta_p=\theta_y$	IDR=2.0% and $\theta_p=3.5\theta_y$	IDR=2.5% and $\theta_p=6\theta_y$
$M_w \leq 6.8$	1	$\xi=0.015$ for $0.5 \leq T \leq 2.5$	$\xi=0.08$ for $0.5 \leq T \leq 2.5$	-	$\xi=0.26-0.19\cdot(T-1.5)$ for $0.5 \leq T \leq 1.5$ and $\xi=0.26$ for $1.5 \leq T \leq 2.5$
	2	$\xi=0.006$ for $0.15 \leq T \leq 0.85$	$\xi=0.045$ for $0.15 \leq T \leq 0.85$	-	-
	3	$\xi=0.006$ for $0.11 \leq T \leq 0.48$	$\xi\xi=0.275\cdot(T-0.3)+0.025$ for $0.3 \leq T \leq 0.5$	-	-
	4	$\xi=0.006$ for $0.11 \leq T \leq 0.32$	-	-	-
	5	$\xi=0.005$ for $0.10 \leq T \leq 0.24$	-	-	-
$M_w > 6.8$	1	$\xi=0.01$ for $0.5 \leq T \leq 2.5$	$\xi=0.08$ for $0.5 \leq T \leq 2.5$	-	$\xi=0.38$ for $0.5 \leq T \leq 2.5$
	2	$\xi=0.004$ for $0.15 \leq T \leq 0.85$	$\xi=0.06$ for $0.15 \leq T \leq 0.85$	-	$\xi=0.10$ for $0.15 \leq T \leq 0.85$
	3	$\xi=0.003$ for $0.11 \leq T \leq 0.48$	$\xi=0.05$ for $0.3 \leq T \leq 0.35$ and $\xi=0.615\cdot(T-0.35)+0.05$ for $0.35 \leq T \leq 0.48$	-	-
	4	$\xi=0.006$ for $0.11 \leq T \leq 0.32$	-	-	-
	5	$\xi=0.007$ for $0.10 \leq T \leq 0.24$	-	-	-
Long duration	1	$\xi=0.015$ for $0.5 \leq T \leq 2.5$	$\xi=0.025\cdot(T-0.5)+0.10$ for $0.5 \leq T \leq 2.5$	$\xi=0.47$ for $0.5 \leq T \leq 2.5$	-
	2	$\xi=0.005$ for $0.15 \leq T \leq 0.85$	$\xi=0.055$ for $0.15 \leq T \leq 0.85$	$\xi=0.11$ for $0.15 \leq T \leq 0.85$	-
	3	$\xi=0.004$ for $0.11 \leq T \leq 0.48$	$\xi=0.035$ for $0.11 \leq T \leq 0.48$	$\xi=0.10$ for $0.32 \leq T \leq 0.47$	-
	4	$\xi=0.004$ for $0.11 \leq T \leq 0.32$	$\xi=0.035$ for $0.11 \leq T \leq 0.27$ and $\xi=0.8\cdot(T-0.27)+0.035$ for $0.27 \leq T \leq 0.32$	-	-
	5	$\xi=0.004$ for $0.10 \leq T \leq 0.24$	$\xi=0.929\cdot(T-0.17)+0.035$ for $0.17 \leq T \leq 0.24$	-	-

Fig. 5. Design equations for equivalent modal damping ratios as function of seismic deformation and damage (after Papagiannopoulos and Beskos 2010)

Data set	Peak input acc. (gal)	Frequency (Hz)						Damping ratio (%)					
		Bx1	By1	T1	Bx2	By2	T2	Bx1	By1	T1	Bx2	By2	T2
1	3.58	2.218	2.436	3.252	6.673	7.807	9.430	3.98	2.53	3.30	2.42	2.79	1.56
2	12.88	2.010	2.039	2.945	6.615	7.816	9.423	8.55	6.65	6.72	4.07	3.56	4.49
3	3.95	2.357	2.419	3.269	6.863	8.031	9.566	4.16	2.10	2.18	3.03	1.71	2.79
4	15.05	2.240	2.414	3.154	6.656	7.835	9.156	5.87	4.06	3.95	2.93	2.63	4.41
5	3.60	2.345	2.469	3.220	6.813	8.008	9.724	2.88	2.85	2.19	3.12	2.36	2.50
6	4.60	2.394	2.512	3.289	6.996	8.026	9.770	2.77	2.19	1.53	2.69	2.23	2.86
7	12.91	2.246	2.492	3.163	6.766	7.930	9.411	3.55	3.79	2.64	3.74	2.20	3.39
8	20.05	1.981	2.260	2.923	6.299	7.347	8.867	5.03	9.17	3.97	4.53	4.57	4.05
9	7.08	2.252	2.508	3.262	6.866	8.008	9.592	4.72	2.82	2.74	2.81	2.59	3.13
Mean		2.227	2.394	3.164	6.727	7.868	9.438	4.61	4.02	3.25	3.26	2.74	3.24
Std. deviation		0.14	0.15	0.14	0.20	0.22	0.28	1.78	2.39	1.54	0.70	0.85	0.96

Fig. 6. Modal damping ratios for a seven-story reinforced concrete structure having a flexible base (after Hong et al. 2009)

4.6 Damping identification of a soil-structure system

Hong et al. (2009) obtained damping ratios from a soil-structure system using a linear system identification technique called Eigensystem Realization Algorithm (ERA) together with an Observer Kalman Filter Identification (OKID). This identification technique usually leads to overestimated damping values unless the bias introduced in the least-squares estimates in

ERA/OKID is minimized. This bias has been successfully treated by the authors who in their study obtained modal damping ratios of a seven-story reinforced concrete structure using acceleration records of ground motion and floors response induced by ten aftershocks of the 1999, Chi-Chi Taiwan earthquake. The record of the main-shock of that earthquake was not included in the linear identification process employed by the authors because the response of the building under study during the main-shock was highly non-linear. Figure 6 provides modal damping ratios for the six mode shapes shown in Figure 7 for nine data sets, i.e., seismic motions considering a flexible base.

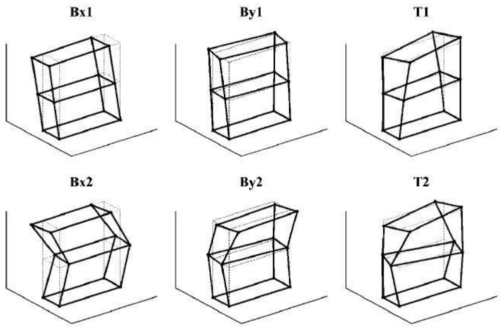

Fig. 7. Identified mode shapes for a seven-story reinforced concrete structure having a flexible base (after Hong et al. 2009)

The method of Hong et al. (2009) requires some computational effort but highlights the importance of including soil-structure interaction to the determination of damping ratios for an actual structure. However, it seems that even one of the most refined identification techniques as the one used by Hong et al. (2009) cannot account for the effect of missing modes during the identification process of a complex system such as a soil-structure one. The problem will be even more pronounced if severe non-linear behavior in the soil-structure system is exhibited where this method falls short off the mark. The simple linear identification method presented by Stewart et al. (1999a), with proper considerations for time delay between input and output, number of modes used and non-linearity, is believed to be more reliable for evaluating modal damping ratios in soil-structure systems. Figure 8 shows the time variation of the damping ratio for the first mode of a six-story office building in Los Angeles, considering a fixed and a flexible base, using its seismic response induced by the 1994 Northridge earthquake.

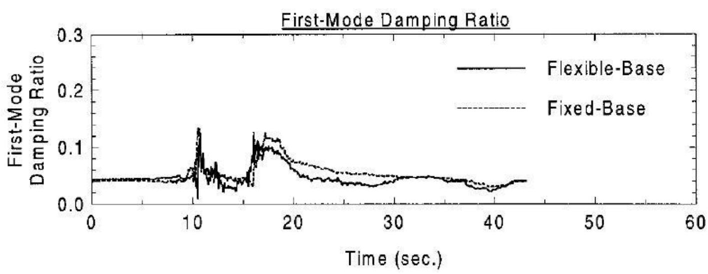

Fig. 8. Time variation of the damping ratio for the first mode of a six-story office building (after Stewart et al. 1999a)

5. Conclusions - Further needs on damping identification

The last section of this review paper attempts as conclusions to address future needs in damping identification for building structures, an issue that according to the opinion of the authors has not gain up to now the importance it deserves and has not been systematically studied. What has to be stressed is the need for robust and simple damping identification methods although the following paragraphs may imply the opposite.

Identification in building structures is by definition an ill-conditioned inverse problem. It requires substantial knowledge from the theory of system identification, which is the inverse procedure of structural dynamics, and makes use of a small number of available measurements, e.g., response recordings at specific floors of a building, while aims at reproducing as accurate as possible the response of all floors of a building. In this sense, all damping identification methods presented above are approximate and a unique solution cannot be provided excluding the idealized case where a building structure can be considered as a single-degree-of freedom system. On the other hand, the presence of noise in measurements, that essentially has to be removed using signal processing techniques, complicates the identification of damping as the sensitivity of the aforementioned noise-removal procedure with respect to damping seems to be case-dependent.

Due to the complexity of the damping mechanism, simplified models have been developed with the modal damping one to be the most widely used during the identification process. This consideration leads to the inevitable step of accounting damping at a global scale whereas under specific conditions the damping ratio of an element could be found. However, this requires a sub-structuring technique to be applied to the structure under identification. This obviously increases the computational effort and does not guarantee more reliable global damping results. Damping ratios of various elements need to be combined somehow in order to be expressed in terms of modes; otherwise one should use the element damping ratios and construct a damping matrix in the context of the finite element method.

The sub-structuring identification techniques may reveal the time-varying behavior of damping ratios that could help towards the correlation between damping and restoring forces during the seismic response of a structure. On the other hand, most of the existing damping identification techniques provide estimates of damping ratios for the few lower modes. Although this is acceptable in seismic design since a few lower modes are usually needed to represent the seismic response of a structure, the identified results fail to detect structural damage. This is an additional reason of supporting sub-structuring identification methods as damping coefficients in local parts of the structure can be identified.

For the majority of the methods mentioned in the previous section, identification is performed on the basis of the assumption that the structure under study is linear and classically damped. When the outcome of the identification process indicates failure of this assumption, non-linear identification is adopted to get a better representation of the damping characteristics of the structure. Nevertheless, performing non-linear identification neglects the fact that the error in the identification process comes from the effect of non-classical damping. Thus, before the use of non-linear identification, one should use a linear non-classical damped identification model. This kind of identification can be performed either in time or in frequency domain depending on the structure under study. For example, if linear soil-structure interaction has to be taken into account, the frequency domain identification may be more useful due to the frequency dependency of the parameters of the soil in cases of simplified soil models.

Damping identification of torsional modes of vibration needs more development as from measured responses of nominally symmetrical or un-symmetrical buildings, a special identification problem arises, i.e., the presence of closely spaced translational and torsion modes (modal coupling) and the possible energy transfer between them. If this modal coupling is ignored, damping estimates are unreliable. For these building cases, a multiple input (translational and torsional components of ground motion) multiple output (translational and torsional responses) identification procedure has to be employed.

Finally, an important aspect regarding damping identification in building structures in direct connection with the aforementioned modal coupling problem is that of the proper placement of measurement devices. One can observe in the existing literature, that the more sophisticated identification methods yield reliable damping results for some of the measurements but fail for the remaining ones. This is due to the initial guess of the parameters of these methods and the noise inherent in the measurements. Additional criteria to those that exist in the literature have to be established for selecting those locations in a structure that would give the least uncertain damping estimates. This need is even more pronounced in the case of symmetrical or un-symmetrical structures including their interaction with soil.

Извод

Идентификација пригушења за грађевине изложене земљотресу: прегледни рад

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Извод

У овом прегледном раду су описани постојећи методи за идентификацију пригушења, карактеристика практичних примена и уобичајених проблема за грађевине изложене земљотресу. Приказани су неки карактеристични примери да демонстрирају могућности описаних метода. Нагласак је на идентификацији пригушења када се јављају сеисмичке вибрације у нелинеарним подручјима. Потребе за будућим разматрањем идентификације пригушења су такође описане.

Кључне речи: пригушење, идентификација, грађевине, сеизмичка кретања, линеаран одзив, нелинеаран одзив.

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