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Computation methods in residual fatigue life estimations of structural components using strain energy density method

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Abstract.

This work is focused on computational methods and software of aircraft structural components with respect to fatigue and fracture mechanics. Computational method is based on combining singular finite elements to determine stress intensity factors for cracked structural components, with the corresponding crack growth laws that include the effect of load spectra on number of cycles or blocks up to failure. The procedure is applied to aircraft structural components. In this investigation Strain Energy Density (SED) method is used in the domain of residual fatigue life of structural components under general load spectrum up to a crack initiation and crack growth. The SED method is based on using low cycle fatigue (LCF) properties for crack initiation and crack growth analyses. To determine analytic expressions for stress intensity factors (SIF), that are necesary in crack growth analysis for residual life estimation, 6-node singular finite elements are used. Computation results are compared with the analytic ones and they agree with the experiments.

Keywords: fatigue, residual life, singular finite elements, strain energy density, load spectrum

1. Introduction

Many failures of structural components occur due to cracks initiation due to local stress concentrations. Attachment lugs are commonly used for aircraft structural applications as a connection between components of the structure. In a lug-type joint the lug is connected to a fork by a single bolt or pin. Generally the structural design with difficulty in applying the fail-safe design there is a need for the damage tolerance design. Methods for design against fatigue failure are under constant improvement. In order to optimize constructions, the designer is often forced to use properties of materials as efficiently as possible. One way to improve the fatigue life predictions may be to use relations between crack growth rate and the stress intensity factor range. To determine residual life of damaged structural components two crack growth methods are used: (1) conventional Forman`s crack growth method and (2) crack growth model based on the strain energy density method [Sehitoglu et al 1996, Maksimovic et al 2011]. The last method employs the low cycle fatigue properties in the crack growth model.

The objective of this paper is to develop an adequate and efficient numerical approach which enables residual life prediction of structural elements during the crack propagation stage. Besides that, the formulated model is based on energy criterion. Within the scope of the suggested model/procedure, the same parameters, required for the stage which lasts until the occurrence of initial damage, are being used.

2. Crack growth models

To study the residual life conventional Forman`s crack growth model is defined in the form [Forman et al 1967]

$$\frac{da}{dN} = \frac{C\left(\Delta K\right)^n}{\left(1-R\right)K_C - \Delta K} \tag{2.1}$$

where K_C is the fracture toughness *C*, *n* – are experimentally derived material parameters. The strain energy density method can be written as [Maksimovic et al 2011],

$$\frac{da}{dN} = \frac{(1-n')\psi}{4EI_{n'}\sigma_f'\varepsilon_f'}\left(\Delta K_I - \Delta K_{th0}\left(\frac{1-R}{1+R}\right)^{1/2}\right)^2 \tag{2.2}$$

where: $\sigma_f^{/}$ is cyclic yield strength and $\varepsilon_f^{/}$ - fatigue ductility coefficient, ΔK_I is the range of stress intensity factor, ψ - constant depending on the strain hardening exponent $n^{/}$, $I_n^{/}$ - the non-dimensional parameter depending on $n^{/}$. ΔK_{th} is the range of threshold stress intensity factor and is a function of stress ratio, i.e.

$$\Delta K_{th} = \Delta K_{th0}(1-R)^{\gamma}, \tag{2.3}$$

ΔK_{th0} is the range of threshold stress intensity factor for the stress ratio R = 0, and γ is a coefficient (usually, $\gamma = 0.71$).

3.Numerical validation

To validate computation of the residual fatigue life estimation, here are considered cracked structural components of light training aircraft, Fig. 3.0. Attention in this investigation is focused on cracked wing skin and an attachment lug structural components.

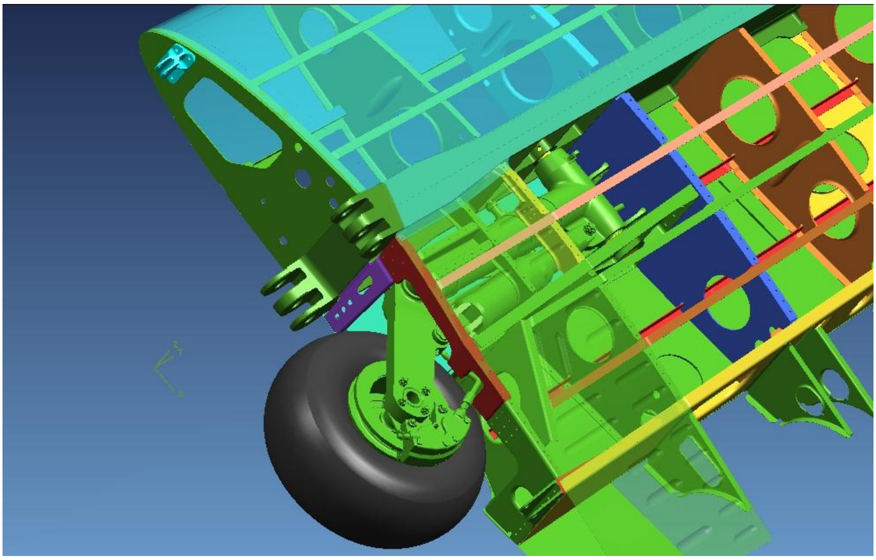
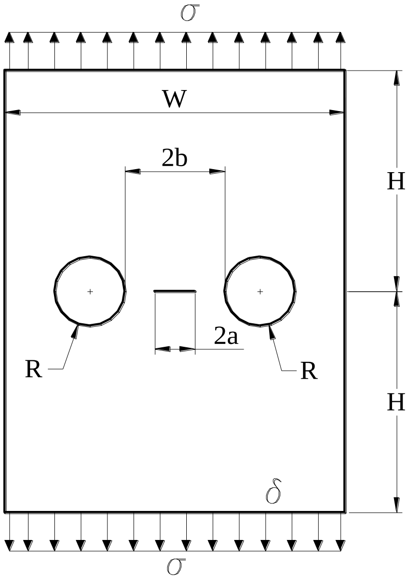


Fig. 3.0. Wing Structure of Light Training Aircraft

3.1 Residual life estimation of cracked wing skin

Residual fatigue life is considered for a skin containing a crack of length 2a symmetrically between two circular holes of radius R; skin is subjected, remote from the crack, to a uniform uniaxial tensile stress S in a direction perpendicular to the crack, Fig. 3.1.1.



R=2 mm
b=6 mm
W=26 mm
H=26 mm
δ=1 mm
Material properties:
(duraluminum)

E = $7.1\ 10^4$ MPa
S_y= 334 MПa
$\sigma_f^{/}$ = 613 MPa
$\varepsilon_f^{/}$ = 0.35
$n^{/}$ = 0.121
$k^{/}$ = 710 MPa
K_{IC}=120 MPa $m^{1/2}$
$I_n^{/}$ = 3.067
ψ = 0.95152
$\underline{\Delta K_{th0} = 8}$ MPa $m^{1/2}$

Fig. 3.1.1. Geometric Properties of a Skin with a Crack between two Circular Holes

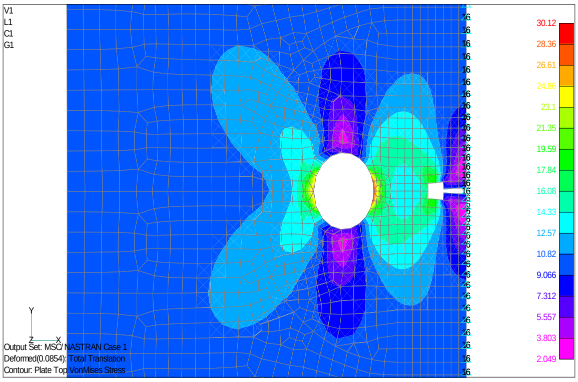


Fig. 3.1.2. Finite Element Model of Plate with Crack Between two Circular Holes

To develop analytic expressions for stress intensity factors (SIF`s), by using finite element discretization, it is necessary to determine SIF`s for various crack lengths. In Fig. 3.1.2 is shown a FE model for initial crack length a_0 =2 mm. In Table 3.1.1 are given values of the SIF determined by FE model for various values of the crack lengths.

a (mm)	**2**	**2.5**	**3**	**3.5**	**4**
K_I (daN/mm^2)	26.077	29.426	32.289	36.225	40.196
a/b	0.333	0.417	0.500	0.583	0.667
$K_O = \sigma\sqrt{\pi a}$	25.060	28.018	30.692	33.151	35.440
$Y=K_I/K_{O\ (FEM)}$	1.041	1.047	1.059	1.090	1.134
$Y=K_I/K_{O\ (ANAL)}$	1.09	1.1	1.12	1.145	1.19
Difference between Analytic and FEM (%)	4.5	4.8	5.5	4.8	4.7

Table 3.1.1. Comparison of SIF obtained by FEM with the Analytic Solutions

To determine the corrective function Y, we started with analytic expression of stress intensity factor in the next form:

$$K_I = Y\,S\sqrt{\pi\,a} \tag{3.1.1}$$

where **Y** is the corrective function, **S** is nominal stress, and **a** is the crack length. Here, the corrective function **Y** is unknown and will be determined by use of the finite element method. For that purpose special 6-node singular finite elements are employed [Barsoum 1977] around crack tip. The stress intensity factors for various crack length are obtained from the FE models, Table 3.1.1. With these SIF`s for various crack lengths, corrective functions are defined in polynomial forms of 4th and 5th order:

$$Y_4^{FEM} = 0.152 + 1.2883\,a - 0.68483\,a^2 + 0.15667\,a^3 - 0.01267\,a^4 \tag{3.1.2}$$

$$Y_5^{FEM} = 0.90288 + 0.09774\,a - 0.0058\,a^2 - 0.0053\,a^3 - 0.0007\,a^4 + 0.00047\,a^5 \tag{3.1.3}$$

Analytic form of the corrective function has the next form:

$$Y^{ANAL}{}_{I} = \left(1.08899 + 0.04369\left(\frac{a}{b}\right) - 1.77302\left(\frac{a}{b}\right)^{2} + 9.21212\left(\frac{a}{b}\right)^{5} - 15.8683\left(\frac{a}{b}\right)^{4} + 9.48718\left(\frac{a}{b}\right)^{5} \right) \quad (3.1.4)$$

Difference between present FE results and the analytic results for the corrective function is within 5%, Table 3.1.1.

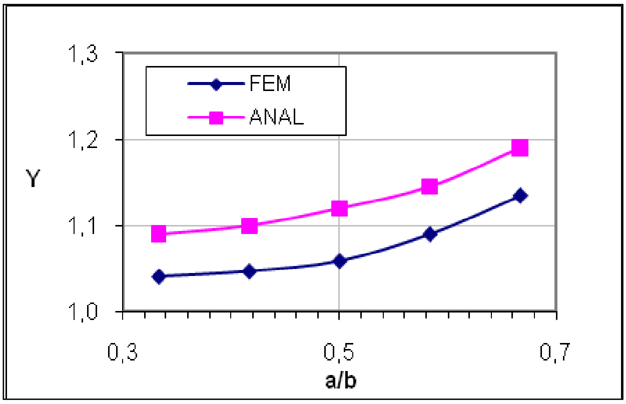


Fig. 3.1.3. Comparison of Corrective Functions Determined by FEM and Analytic Method

Using expressions for stress intensity factors obtained by use of the FE method, for skin with a crack between two circular holes, according to expressions (3.1.3) and (3.1.4) and the expression for crack growth based on Strain Energy Density (2.2), a relation **a-N** is defined**,** Fig. 3.1.4.

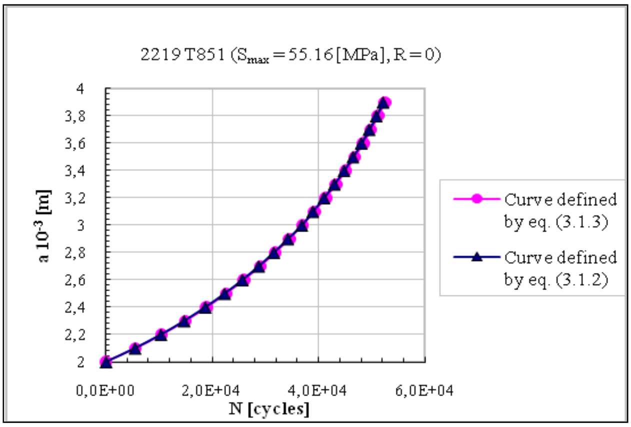


Fig. 3.1.4. Crack Growth Analysis of Plate with Crack Between two Circular Holes, using SED and derived analytic expressions for SIF using singular finite elements

It follows that, using relations **a-N,** Fig. 3.1.4**,** or residual life estimation of a wing skin/plate with a crack between two circular holes, we have that for cyclic loads S_{max}=55.16 MPa (R=0), determined by SED and analytic expressions of stress intensity factors defined by FEM. Presented computation procedure, which combines the finite element method to establish analytic expressions for SIF`s and SED in which are used cyclic material properties, represent a general approach for residual life estimation of cracked structural components.

3.2 Residual life estimation of cracked lugs

Here are considered cracked aircraft attachment lugs, Fig 3.2.1. Once a finite element solution has been obtained, Fig. 3.2.2, the values of the stress intensity factor can be extracted from it. To determine Stress Intensity Factors of cracked aircraft attachment lugs, a method based on extrapolation of displacements around tip of crack is used here.

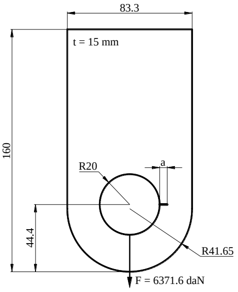


Fig. 3.2.1. Geometry of cracked lug No. 2

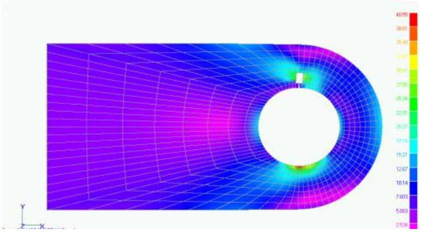


Fig. 3.2.2. Finite Element Model of cracked lug with stress distribution

Subject of this analyses are cracked aircraft lugs under cyclic load of constant amplitude and load spectra. For that purpose, a conventional Forman crack growth model and the crack growth model based on strain energy density method, are used. Material of lugs is Aluminum alloy 7075 T7351 with the next material properties: σ_m=432 N/mm^2 ⇔ Tensile strength of material, σ_{02}=334 N/mm^2, K_{IC}=2225 [N/mm$^{3/2}$], Dynamic material properties (Forman`s constants): C=3* 10$^{-7}$, n=2.39, Cyclic material properties: $\sigma_f^{/}$=613 MPa, $\varepsilon_f^{/}$=0.35, $n^{/}$=0.121**.** The stress intensity factors (SIF`s) of cracked lugs are determined for nominal stress levels: σ_g = σ_{max}=98.1 N/mm^2 and σ_{min}=9.81 N/mm^2. These stresses are determined in net cross-section of lug. The corresponding forces of lugs are defined as: F_{max}= σ_g (w-2R) t = 63716 N and F_{min}= 6371.16 N, that are loading the lugs. For stress analyses, a contact pin/lug finite element model is used. For cracked lug Fig 3.2.1, with initial crack a_0, SIF is determined using finite element model, Fig. 3.2.2.

For cracked lug No.2, Figure 3.2.1, with crack through the thickness, the crack growth behavior under two-level load spectra is considered.

The first level of load spectra is defined as: σ_{max}=142,8 N/mm^2, σ_{min}=14,28 N/mm^2 for the first 1000 cycles. The second level of load spectra is defined as: σ_{max}=38,1 N/mm^2 and σ_{min}=14,28 N/mm^2. The crack growth numerical simulation of cracked lug is carried out using SED method and conventional Forman`s method, Figure 3.2.3.

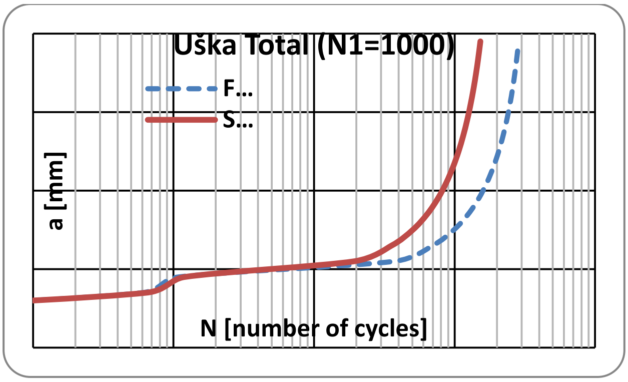


Fig. 3.2.3. Comparisons crack growth behavior using SED and Forman`s methods

In Figure 3.2.3 are shown results of crack growth results for cracked lug using two methods: (1) conventional Forman`s method [Forman et al 1967], and (2) strain energy density method [Maksimović et al 2006, Boljanović et al 2011, Maksimović et al 2011, Boljanovic and Maksimovic 2011] (SED).

4.Conclusions

This investigation is focused on developing efficient and reliable computation methods for residual fatigue life estimation of damaged structural components. Special attention has been directed on determination of fracture mechanics parameters of structural components, such as stress intensity factors of aircraft cracked structural elements. Computation fatigue life predictions of an attachment lug under load spectrum are presented. From this investigation, the following are conclusions can be derived: A model for the fatigue crack growth should incorporate the low cycle fatigue properties of the material. Comparisons of the predicted crack growth rate using strain energy density method with conventional Forman`s model, leads to the conclusion that this SED model could be effectively used for residual life estimations.

Извод

Рачунске методе у процени преосталог заморног века елемената конструкција применом методе густине енергије деформације

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Резиме

Предмет рада је усмерен на развој рачунских метода и софтвера за анализе чврстоће структуралних елемената авионских конструкција са аспекта замора и механике лома. Прорачунски метод је заснован на комбинованој примени сингуларних коначних елемената за одређивање фактора интензитета напона са одговарајућим законима ширења прскотине укључивши и спектре оптерећења за одређивање броја циклуса или блокова до лома. Процедура је примењена на структуралне елементе са иницијалним пркотинама код авионских конструкција. У овом истраживању примењен је метод Густине Енергије Деформације (ГЕД) за анализу ширења прскотине. Метод ГЕД је заснован на бази коришћења малоциклусних карактеристика материјала које се користе и при процени заморног века до појаве иницијалног оштећења. Да би се успоставили аналитички изрази за факторе интензитеа напона (ФИН), који су неопходни при процени преосталог заморног века, коришћени су 6-чворни сингуларни коначни елементи. Презентовани резултати су упоређени са аналитичким и одговарајућим експерименталним резултатима.

Кључне речи: Замор, Преостали век, Коначни елементи, Густина Енергије Деформације, Спектар оптерећења

References

Barsoum RS., Triangular quarter-point elements as elastic and perfectly-plastic crack tip elements. *International Journal for Numerical Methods in Engineering*, 1977, 11, 85-98.

Boljanovic S., Maksimovic S., Analysis of the crack growth propagation process under mixed-mode loading, *Engineering Fracture Mechanics*, Vol. **78**, Issue 8, May 2011, pp. 1565-1576.

Boljanović S., Maksimović S., Carpinteri A., Numerical Modeling of Semi-Elliptic Crack Growth under Cyclic Loading, *Third Serbian Congress Theoretical and Applied Mechanics*, Vlasina Lake, 5-8 July 2011, pp. 391-396, Organized: Serbian Society of Mechanics, *Editors*: S. Maksimović and T. Igić.

Forman, R.G., Kearney V.E. and Engle R. M., Numerical analysis of crack propagation in cyclic loaded structures, *J. Bas. Engng. Trans. ASME,1967*. 89, 459, 1967.

Maksimović K., Đurić M., Janković M., Fatigue Life Estimation of Damaged Structural Components Under Load Spectrum, *Scientific Technical Review*, 2011, Vol. 61, No. 2, pp. 16-23

Maksimovic S., Posavljak S., Maksimovic K., Nikolic V. and Djurkovic V., Total Fatigue Life Estimation of Notched Structural Components Using Low-Cycle Fatigue Properties, *Journal Strain*, 2011, 47 (suppl.2), pp 341-349,

Maksimović, S., Boljanović, S., Fatigue Life Prediction of Structural Components Based on Local Strain and an Energy Crack Growth Models, *WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS*, Issue 2, Vol. 1, 2006, pp 196-205.

Sehitoglu, H., Gall, K., Garcia, A.M., Recent Advances in Fatigue Crack Growth Modeling, *Int. J. Fract*.,1996, 80 , pp. 165-192.