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A review on dynamics of mass variable systems

(Dedicated to the 70th birthday of Prof. Milos Kojic)

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Abstract

In this review the results of dynamics of the systems with time-variable mass are presented. After the theoretical consideration the application of the theory is shown. Special attention is paid to mechanisms and machines and also to rotors with variable mass. The systems with both: discontinual and continual mass variation is analyzed. The influence of the reactive force on the motion of the system is investigated. Numerous analytical solving procedures are developed for solving the systems. The intention of the paper is to give the directions of further investigation.

Keywords: mass variable system, reactive force, rotor with variable mass, mechanism with time variable mass.

1. Introduction

The intention of this paper is to give the review of the results in dynamics of variable mass system and their application in machines and mechanisms. The expression ’variable mass system’ as used in the context of this paper refers to mechanical systems that lose and/or gain mass while in motion.

The problem of dynamics of the systems with variable mass was appointed in early XVII century. Namely, the variation of the secular acceleration of Moon's longitude has to be explained. Edmund Galileo compared the periods of motion of the Moon around the Earth and concluded that it decreases due to increase of the averaged velocity around the orbit and tangential acceleration of moon. Oppolzer (1884) stated that the reason of secundary acceleration of Moon is the increase of the eccentricity of the Earth’s orbit and also mass variation of the Moon and the Earth due to meteorite falls. Due to mass variation the radius of the Earth increases for half of millimeter during a century. In the same year Gil’den, 1884, formulated the differential equation of relative motion of two particles with time variable masses which attract each other in accordance with Newton’s law. If the relation

$$\ddot{\vec{r}} = -\frac{m(t)\vec{r}}{r^3}, \tag{1}$$

where $\vec{r}$ is the position vector between two considered particles or bodies, $m(t)$ is mass variation and r is distance between two particles. The same formulation was given by Meshchersky (see Meshchersky, 1893) for the motion of two variable mass particles in gravitational field. Nowadays, the formulation (1) is called non-stationary Gil'den-Meshchersky. Meshchersky obtained the certain mathematical laws of mass change which have to be fulfilled to reduce the equation (1) to its stationary form. The equation has the first integral of quadratic type for certain mass variations. It is found that the real mass variation is neither periodic, nor oscillatory. The aforementioned publications can be treated as the first one dealing with the problem of dynamics of variable mass systems. Nowadays, the first, second and general Meshchersky laws are generalized by using the autonomization method (Berkovich, 1980) and the extended version of the Gil'den-Meshchersky problem of two bodies in order to find all possible mathematical laws of mass change which give the solution of the problem in quadratures is formulated. The solution of the motion of a material point of variable mass in a central force field in the presence of a perturbing force is also obtained in quadratures (Grudtsyn, 1972). Using the Noether's theorem (Vujanovic and Jones, 1989) the conditions for existence of the conservation law for the system with variable mass under influence of central force and damping is determined (Cveticanin, 1994_1). In spite of the fact that the system presents a nonconservative system it has a Lagrangian and the first integrals for certain mass variation.

The further investigation done by Meshchersky was directed toward dynamics of the systems with discontinual and continual mass variation in time (Meshchersky, 1897). These are two basic directions for past and future researches in the matter.

In his master work (Meshchersky, 1952), Meshchersky was dealing with the problem of particle separation/adding in the system. He concluded that, in general, velocity and direction of the separated/added particle differs from the velocity of the remainder particle from which the particle was separated or was added. The separated or added particles produce the impact. For continual mass variation the continual impact is transformed into a force called 'reactive force'. The impact does not exist if the velocity of the added or sparated particle is equal to the velocity of mass variable particle. Then the differential equation of motion has the same form as for constant mass systems. However, more often these two velocities differ and the reactive force, which is the result of the impact phenomena, has to be included into consideration. Mathematical model of mass separation for one degree of freedom system is

$$m(t)\ddot{x} = F + \Phi, \tag{2}$$

where x is the generalized coordinate, $m(t)$ is the variable mass, t is time, F is the external force and Φ is the reactive force

$$\Phi = \frac{dm}{dt}(u - \dot{x}), \tag{3}$$

with relative velocity u of mass separation or adding. This study was followed by a long period of only sporadic activity in this field, and it was not until the 20th century that a resurgence of activity in this area occurred, mostly in connection with rocketry. Namely, the reactive force was of crucial importance for further investigations and the mentioned model (3) was the basic result for many researches, specially in rocketry (Eke and Wang, 1995).

The investigation the problems which are identified in techniques and engineering due to mass variation and up to those in the celestial mechanics needed long time research: from modeling of a mass variable particle to a rigid body and the system of particles. Examples of mass variable devices abound in the engineering literature. They include complex systems such as aircraft, rockets, automobiles, and moving robots picking up or letting go of objects, as well as simpler systems such as water splinkler systems or an inflated ballon with air loss through one or more holes (Eke and Wang, 1994). In this paper a restriction is introduced: the review of dynamics of rigid bodies and systems which can be considered to be inside the rigid frame will be presented. Besides, only the systems where the mass variation is a time function will be presented. After the section where the theoretical results are shown the sections about applications in the rotors with variable mass and mechanisms with variable mass follow.

2. Review on theoretical results

Variable mass systems can be divided into two classes: those with continuous mass variation and those with discrete mass variation. Rockets, for example, fall in the continuous variable mass class, and robots picking up or releasing objects, or a moving vehicle droping off some of its payload in discrete chunks, belong to the discrete variable mass system class.

2.1. Discontinual mass variation

The motion in the systems with discontinual mass variation can be divided into two separate parts: before and after mass variation. It requires the separate analysis of the motion as the constant mass system before and after separation. For the known velocity and angular velocity of the mass variable body, the velocity and the angular velocity of the remainder body after separation has to be calculated. The general laws and basic principles of classical Newton's dynamics are usually applied (Cveticanin and Djukic, 2008). Due to the discontinual mass variation, the jump-like change of the velocity and the angular velocity of the body are evident. Depending on the type of motion of the separated body various dynamic properties of the remainder body are obtained. Meshchersky (1952) investigated the motion of a rigid body which moves vertical straightforward whose mass varies due to dropping of the load. If the separation from a plane four-particle system is done, the problem of three-particle-system appears (Cveticanin, 2007). Velocity and angular velocity after particle separation depend on the initial conditions. The attraction and resistance forces between particles depend on viscous damping or Columb friction. The final position of the particles and the geometrical configuration directly depend on the separated particle velocity.

The most general motion of the mass variable system is the free motion of rigid mass variable body . For the mass variable rigid body the following constraints are introduced:

- form and dimensions of the body change permanently and it causes the continual variation of position of center of inertia, axial and centrifugal moment of inertia,
- velocities of particles before and after mass separation change continuously with time,
- resultant force and momentum vary in time and depend on time, position, velocity and angular velocity.

Dynamic parameters of a remainder body after mass separation may be obtained applying an analytical procedure, too. This method is also based on the general principles of momentum and angular momentum of a body and system of bodies. The kinetic energy of motion of the whole body and also of the separated and remainder body is considered. The derivatives of kinetic energies with respect to the generalized velocity determine the velocity and angular velocity of the remainder body (Cveticanin, 2009). Due to mass variation a theorem about increase of kinetic energies of the separated and remainder bodies for perfectly plastic separation is proved. The increase of the kinetic energies corresponds to the relative velocities and angular velocities of the separated and remainder bodies.

2.2. Continual mass variation

Early studies of systems with continual mass variation are focused on the translational motion. Such motion is described with equation (2). The same form of equation was used for describing the rotational motion of a rigid body around a fixed axes (Bessonov, 1967_1): mass is substitute with moment of inertia, displacement with angle of rotation and force with torque. In the mid 20th century, researchers started to grapple with rotational equations of motion for variable mass systems (Cornelisse, Schoyer and Wakker, 1979). The equations derived have forms similar to Euler's equations for rigid bodies, with extra terms accounting for mass variability. Recently, Wang and Eke (1995) presented closed-form solutions of the equations of attitude motion of a variable mass symmetrical cylinder and axisymmetric system that has two equal central principal moments of inertia at all times, without necessarily being a body of revolution cylinder. Their study assumed symmetric internal fluid flow with negligible whirling motion; its main conclusion was that the time history of the angular velocity of a variable mass cylinder depends on the initial dimensions of the cylinder, as well as on the manner in which its geometry changes as mass is lost. The motion of a mechanical system of coaxial axisimetrical bodies of variable mass is a translating system of coordinates (Aslanov and Doroshin, 2004). A theorem on change in the angular momentum of a system of coaxial bodies. of variable mass with respect to translating axis is presented. The dynamic equations of motion are constructed using the example of two coaxial bodies. Assuming that the relative displacements of the centre of mass, due to change in the mass of the system are small, approximate solutions are found for the spatial orientation angles and the condition for reducing the amplitude of nutational oscillations. The results obtained can be used to describe the motion of spacekraft, constructed in coaxial form, when performing active manoeuvres with a change in mass. The consideration is extended on modeling of motion of a rigid body with variable mass. For the non-linear non-holonomic variable mass systems the universal D'Alambert-Lagrange's principle of variable mass is formulated (see Ge, 1984 and Brankovic, 1987). The generalization of the previously obtained solutions is given for a rigid body with variable mass (Cveticanin and Kovacic, 2007). As the special case the planar motion is considered.

2.3. Conservation laws and adiabatic invariants

As it is previously shown, the motion of the system with variable mass is described with second order differential equations with time variable parameter. To find the closed form solution of the equation is, usually, impossible. In spite of that the conservation laws of dynamic systems with variable mass exist.

An extended Lagrangian formalism for the rheonomic systems with the nonstationary constraints is formulated with the aim to examine more completely the energy relations for such systems in any generalized coordinates, which in this case always refer to some moving frame of reference. Introducing new quantities, it is demonstrated that these quantities determine the position of this moving reference frame with respect to an immobile one. In the transition to the generalized coordinate they are taken as the additional generalized coordinates whose dependence on time is given a priori. In this way the position of the considered mechanical system relative to this immobile frame of reference is determined completely. Based on this and using the corresponding d'Alambert-Lagrange's principle, an extended system of the Lagrangian equations is obtained. It is demonstrated that they give the same equations of motion as in the usual Lagranian formulation, but substantially different energy relations. In this formalism two different types of energy change law and the coresponding conservation law exist. The energy relations are in full accordance with the corresponding ones in the usual vector formulation, when they are expressed in terms of rheonomic potential. The so called rheonomic potential expresses the influence of the nonstationary constraint (Musicki, 2004).

If for the one degree of freedom system two independent conservation laws are known, the exact solution of motion of the system is obtained. Due to the type of the system various methods for obtainin conservation laws for differential equations with time variable parameters is developed. In the paper of Cveticanin (1993_1), a method for obtaining conservation laws of dynamic systems with variable mass based on Noether's theorem and D'Alembert's variational principle are developed. In general, a dynamic system with variable mass is purely nonconservative. Noether's identity for such a case is expanded by the terms that describe the mass variation. If Noether's identity is satisfied, a conservation law exists. Using the suggested procedure the conservation laws for a non-linear vibrating machine and for a rotor with variable mass are determined.

Starting from the Lagrange equations for mechanical systems with variable mass the general energy change low is formulated by Musicki, 1999. The law is expressed also in terms of the metric tensor and connected with the corresponding generalized Noether's theorem (Vujanovic and Jones, 1989), from where it is concluded under which conditions the energy conservation is valied.

To form the invariants for the one degree of freedom mass variable oscillators, which have the Lagrangian (Cveticanin, 2000), the Noether's approach is also applied. The conservation laws of the rheo-linear (Cveticanin, 1996_1), pure-cubic oscillator and a pendulum with variable mass and length are determined. The obtained conservation law of energy type gives the analytic criterion for dynamic buckling (Cveticanin, 2001). The suggested method allows the determination of dynamic buckling load without solving the corresponding non-linear differential equation of motion. For this value of dynamic load the motion of the system becomes unbounded.

For the case when the conservation laws is impossible to be obtained, the adiabatic invariants have to be considered. The procedure for obtaining adiabatic invriants is as follows: The method is based on Noether's theory (Vujanovic and Jones, 1989) and the use of an asymptotic solving technique. Noether's theory requires the study of the invariant properties of the Langrangian function with respect to infinitesimal transformation of the generalized coordinates and time leaves the Langarangian function invariant. The necessary condition for the existence of first integrals, i.e. exact invariants, is determined. Any approximate solution of this necessary condition yields the corresponding adiabatic invariant.

For dynamical systems with one degree of freedom and small non-linearity the necessary condition for the existence of first integrals is transformed in the Krylov-Bogolubov-Mitropolski (KBM) variables (Djukic, 1981). Two independent invariants give the approximate solution of the vibrations of the oscillator with variable mass. The previously mentioned procedure developed for the system with small non-linearity is extended for the system with strong non-linearity (Cveticanin, 1995_1). The method is based on Noether's theorem for invariance and the elliptic Krylov-Bogolubov (EKB) method. A set of abiabatic invariants is applied to find the asymptotic solution for the motion. Two examples are considered: the pure cubic oscillator and the linear damped quasi-pure-cubic oscillator with variable mass. The suggested procedure is modified for solving of motion for systems with small non-linearity but with two degrees of freedom (Cveticanin, 1994_2). If four independent adiabatic invariants are known, the motion of the system is approximately known. The same statement is evident when the non-linearity is strong (Cveticanin, 1996_2). The elliptic-Krylov-Bogolubov (EKB) asymptotic technique is applied for constructing adiabatic invariants for the systems described with complex functions.

2.4. Qualitative analysis, stability and control

The well known procedure for qualitative analysis of the systems with constant parameters is extended and adopted for analyzing the systems with non-periodic time variable parameters and small non-linearity (Cveticanin, 2004). The advantage of the developed method is that the behavior of the system may be discussed without solving the differential equations of motion. As the special case the quasi-linear one-degree-of-freedom system with slow time variable parameter is considered. The effects of mass variation and reactive force are discussed. It is obvious that they have a significant influence on stability of motion.

Various methods for stability analysis for linear systems with time variable parameters are given by Shrivastava (1981) and Ahmadian (1986). As the special case the condition of instability of the position of equilibrium of a linear oscillator with variable parameters is discussed (Ignat'yev, 1991) and illustrated for the case of a rocket with fins about its centre of mass. The main disadvantage of these methods is that they are applicable only for linear systems. The most general procedures for stability analyses of systems with mass variation are Lyapunov's direct theorems. In the paper of Cveticanin (1995_2) a new type of Lyapunov function is formed which allows to follow the classical Lyapunov reults on asymptotic stability and instability of systems with variable mass. A stability theorem for a special type of second order differential equation with complex function z is also defined. The advantage of all the mentioned methods is that need not the differential equations of motion to be solved. The main conclusion for stability analysis is that the reactive forces are the basic factors for stability limits. This is the reason that the reactive forces are used as control parameters.

A theory is developed for the solution of an optimal motion control problem for mechanical system with variable mass and superimposed constraints, whose responses are reactive forces (Apykhtin and Iakovlev, 1980) which are taken as controls. As an example a problem on contact in minimal time is solved by the method of parallel approach of some target and a system of point of variable mass.

In general the control procedure requires to construct the equations of motion of a control system with constraints whose response are reactive forces (Azizov, 1986). The equations are constructed using the theory of motion of a system with non-ideal constraints which is applied to problems with friction. Two laws of variation of mass of the system ensuring the realization of the servoconstraints are determined, and the problem of stabilizing the motion with respect to a manifold defined by these constraints are studied. The method of investigation is based on the rules of combination of the constraints and the Chetayev's theory of parametric release. However, the systems in which the laws of variation of mass are known in advance, and all constraints effected by reactive forces are applied exactly over the whole period of motion, embrace only a narrow class of problems. A more general case is of interest, when only a part of the constrains realy on reactive forces, where the possible deviations of the motions from the servoconstraints are taken into account and the laws governing the variation of mass of the points are not known in advance and are found from the differential equations supplementing the equations of motion of the system.

2.5. Oscillators and oscillatory motion

Numerous machines and mechanisms are modeled as one, two or multi degree of freedom mass variable oscillators. It is the case for: cutting machines with produced object, connection between the engine and wagon, transportation sources, conveyers, etc. Usually, mass-time variation is assumed in a polynomial form. The motion is described with one or more second order differential equations. The simplest model of one degree of freedom oscillator is a linear differential equation which is solved analytically using the first and second order Bessel's functions (Abramowitz and Stegun, 1972). For the system with multiple degrees of freedom the linear differential equations with variable parameters are analytically solved applying the Chebychev matrix operator method (Hiegemann and Straub, 1994). Transforming the differential equations where the parameters are non-linear time functions into those with constant mass the non-stationary motion of the oscillator is possible to be discussed (Bessonov and Silvestrov, 1968). Unfortunately, only a small group of oscillators is the linear one. Usually, in oscillators some non-linearities exist.

In designing mechanical and structural systems it may be important to take into account small non-linearity and slow mass variation. Recently, some attention is paid to a simple non-linear oscillator with slowly varying parameters (Lamarque et al. 1993). Many classical methods can be used to study the case of weak non-linearity. Let us mention some of them: method of Mitropolski (1964), Krilov-Bogoljubov method (Krilov and Bogoljubov, 1937), Bogoljubov-Mitropolski method (Bogoljubov and Mitropolski, 1963), asymptotic method of Krylov-Bogolubov-Mitropolski (Arya and Bojadziev, 1981, and Bojadziev and Hung, 1984), method of projections (Merkin and Friedman, 1981), method of multiple scales (Nayfeh and Mook, 1979), etc. Much effort has been put into extending the methods to more general classes of oscillators. The first reason is that one hopes that any analytical method can give approximated solutions with bounded error. The main reason is the use of an analytical method in order to identify some physical parameters modelling a mechanical system from experimental dynamic tests. The normal form method is applied to compute an approximate solution of a cubic non-linear equation with slowly varying mass at short times and small enough amplitudes of the normal co-ordinates (Lamarque et al., 1993). If these amplitudes are very small, the analytical and accurate numerical results agree for large times. A new exact approach for analyzing free vibration of single degree of freedom systems with nonperiodically time varying mass is presented by Li (2000). The function for describing the variation of mass of a SDOF system with time is an arbitrary continuous real-valued function and the variation of stiffness with time is expressed as a functional relation with the variation of mass and vice versa. Using appropriate functional transformation, the governing differential equations for free vibration of SDOF systems with nonperiodically time varying mass are reduced to Bessel's equations or ordinary differential equations with constant coefficients for several cases and the corresponding exact analytical solution are thus obtained. A numerical example shows that the results obtained by the derived exact approach are in good agreement with those calculated by numerical methods, illustrating that the proposed approach is an efficient and exact method. The theoretical analysis and numerical results show that the effect of variation of mass with time on the free vibration of an SDOF system is equivalent to that of a viscous damping. The equivalent damping is positive for the case that the mass increase with time; otherwise, the damping is negative if the mass decreases as time increases. When the variation of mass is proportional to that of stiffness, the motion of the mass is pseudo-periodic. The vibration of oscillators with time variable mass depend also on internal friction and relaxation under the action of a harmonic force. The influence of reactive force on the vibration properties of the one-degree-of-freedom systems are analysed (Cveticanin, 1992). The amount of mass in the system is directly connected with the amplitude of vibrations. In the paper of Cveticanin (2012) the strong nonlinear oscillator with time variable parameter is considered. The influence of the order of the pure nonlinearity on the oscillatory motion of the system is analyzed.

The methods mentioned above are suitable for application for the system of two coupled differential equations which describe the oscillations of two degrees of freedom mechanical oscillator. One of the most suitable methods is the extended Bogolubov-Mitropolski method (Cveticanin, 1995_3) applied for a differential equation with complex function, small non-linearity and a slow variable parameter. If the non-linearity is strong the aforementioned methods are not suitable for application and new solving procedures have to be developed. The approximate solution procedure for the differential equation with function *z*, strong non-linearity and slow time variable parameter $\omega^2(\tau)$ is introduced into consideration (Cveticanin, 1993_2). The variation of parameter may be caused by mass which is the function of slow time $\tau=\varepsilon t$, where $\varepsilon<<1$ is a small parameter. The non-linearity in the system is of cubic order. The mathematical model is

$$\ddot{z}+\omega^2(\tau)z^3=\varepsilon f(z,\dot{z},\bar{z},\dot{\bar{z}}), \tag{4}$$

where $\bar{z}$ is a complex conjugate function of z and εf is a small non-linear function. The Bogolubov-Mitropolski procedure is adopted for solving strong non-linear differential equation with time variable parameter and complex function using the Jacobi elliptic functions (Byrd and Friedman, 1971) instead of circular ones. Two examples are considered: one, with small linear damping and the second, with small non-linear hidrodynamic force. The suggested analytical method makes possible for an engineer to define the influence of parameters on the real system.

3. Rotor dynamics

In many industrial machines rotors with variable mass are installed. Rotor's mass variation is in time. Rotors with variable mass are the fundamental working elements of many machines in textile, carpet, cable industry, process industry (centrifuges, separators...) etc. Very often, those rotors are modeled as shaft-disc systems. In the middle of a massless shaft a disc with variable mass is settled. Usually, the elastic force in the shaft is non-linear. The non-linearity is weak or strong. The mathematical model of the rotor is in general a second order differential equation with complex function z

$$m(\tau)\ddot{z}+z+F(z)=\varepsilon f(z,\dot{z},\tau), \tag{5}$$

where $m(\tau)$ is mass variation, $F(z)$ is the non-linear elastic force which may be small and εf is the additional small function dependent on deflection, velocity and slow time τ.

Modelling of mass variation is not an easy task. Different models are formed dependently on the system. Various models for winding up of the band on the cylindrical (Skopi, 1981) (see Fig.1) and conical drum (Ganlin, 1988) are given, where the mass distribution in one (Bessonov, 1967_1) and multi layers (Krutkin, 1982) is discussed.

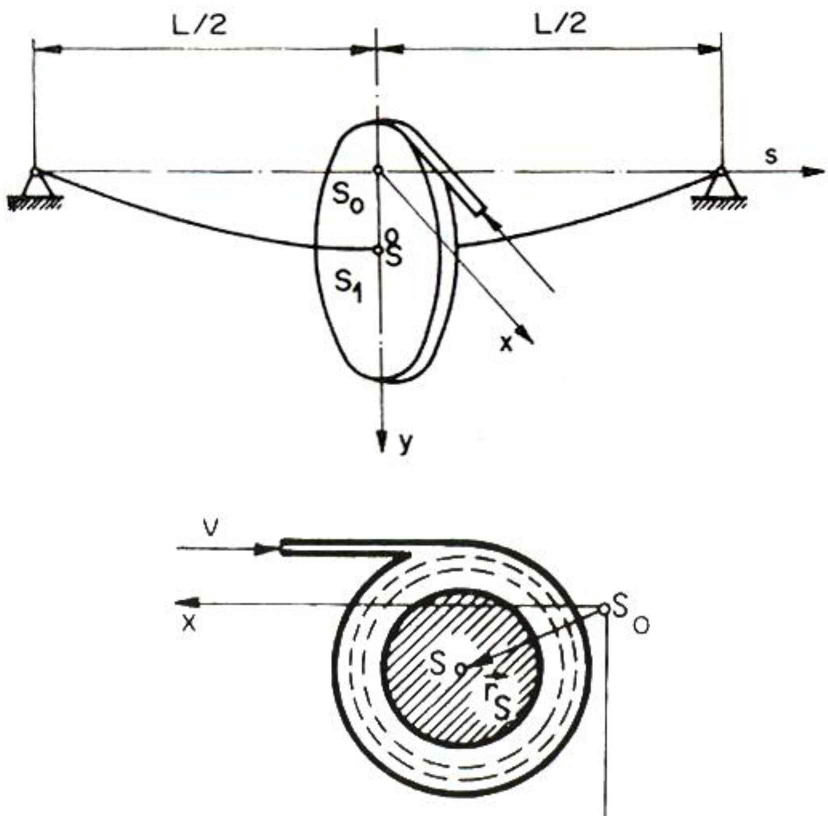

Fig.1. Model of the rotor with winding up band (Cveticanin, 1984_1)

Also for a rotating cylinder mass variation due to pouring of fluid or granular material (Fig.2) is presented (Afinasjev, 1977).

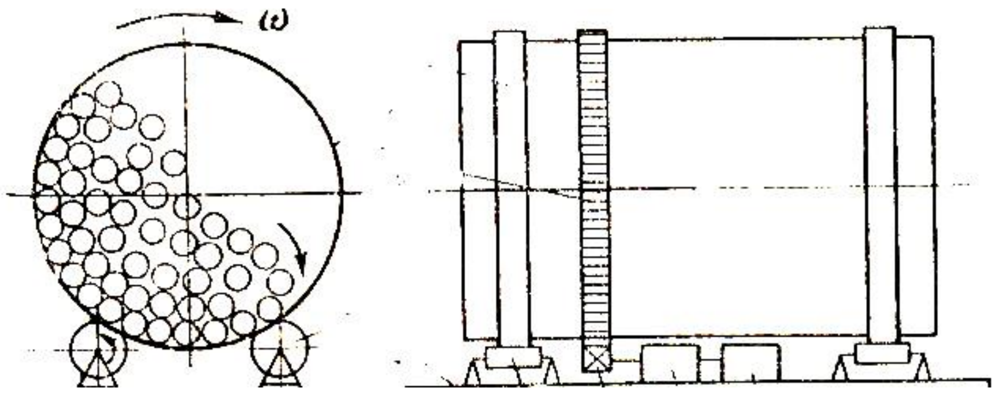

Fig.2. Model of the rotor with pouring granular material (Afinasjev, 1977)

Mass variation causes rotor vibrations. There are many papers dealing with the problem of vibration analysis. Based on the topic of interest all of these papers can be divided into following groups:

Determination of velocity of vibration

Stability of rotation

Deterministic chaos.

3.1. Vibrations of a mass variable rotors

In the papers of Cveticanin (1984_1) and (1986_1) the free vibrations of a textile machine rotor are considered. Mass of the disc is varying due to the winding up of the textile band with constant angular velocity. Severe vibrations occur produced by mass variation. In the case of small non-linearity and of “slow time” mass variation the mathematical model of rotor motion is a second order non-linear differential equation with time variable parameters which can be solved not only numerically but also analytically by use of the multiple scales method. The obtained approximate analytical and the exact numerical results are compared and show a good agreement. Extending the model taking into consideration the gyroscopic force (Cveticanin, 1995_4) the model of mass variable rotor becomes more complex. The solving procedure for differential equation with complex deflection function, time variable parameters and small non-linearity based on the Krylov-Bogolubov method is appropriate for application. Vibrations in non-resonant (Cveticanin, 1984_2) and resonant (Cveticanin, 1991) case are also analyzed applying the multiple scales method. The results for free vibrations are compared with those for non-resonant case, and the results for resonant case with stationary solutions. The special attention is paid to the influence of reactive force on the vibration properties of the textile machine rotor with variable mass (Cveticanin, 1993_3).

In the paper of Cveticanin (1993_4) an asymptotic solution for non-linear vibrations of the rotor with hydrodynamic force is obtained. It is assumed that the rotor system is under action of normal and tangential forces. The procedure for solving of the differential equation is based on the well-known method of linear vibrations and the asymptotic method of Bogolubov-Mitropolski. The Bogolubov-Mitropolski method is adopted for a differential equation with complex function, slow variable mass and small non-linearity. The following cases are considered: the hydrodynamic force is (a) a weak linear one, (b) strong linear one or (c) weak non-linear. The effect of hydrodynamic force on amplitude of vibration is discussed.

A method based on the invariant manifold approach to normal modes is generalized for obtaining normal modes of vibrations for rotors with continually distributed slow variable mass. The dynamics of modes is analyzed (Cveticanin, 1997). A rotor with slow time variable mass with cubic non-linearities is given as an example. The motion obtained by the mentioned method is compared with the results of the standard Galerkin procedure. The non-linear ordinary second order differential equations which govern individual modal dynamics are solved numerically. The influence of reactive force and mass variation on normal modes of vibrations is also considered.

Vibrations of strongly non-linear rotors with time variable parameters are also analyzed (Cveticanin, 1994_3). The mathematical model of the rotors is a strongly non-linear second order differential equation with complex function and time variable parameters. The Krylov-Bogolubov (KB) method and the elliptic-Krylov-Bogolubov (EKB) method are extended for solving the equation of motion. The solution is given in terms of Jacobi elliptic functions. The approximate solution is obtained by applying an averaging procedure. Two types of rotors with variable mass are considered: with cubic non-linearity and with quasi-pure-cubic non-linearity.

3.2. Stability and deterministic chaos

Two methods are usually used for analyzing of the stability of rotation of the mass variable rotor (Cveticanin, 1986_2): i) analysis of the solution of the differential equation of vibration which is the side effect in rotor motion and ii) application of direct Liapunov's method of stability, asymptotic stability and instability. The methods are compared for a mass variable rotor where the linear damping acts.

Stability of the rotating motion of the rotor with variable mass with zero deflection of the mass center is also analyzed (Cveticanin, 1988_1). It is concluded that mass variation has a great influence on the character of the rotor rotation stability. As a special case the stability of rotation of the rotor of a textile machine is analyzed. The stability is analyzed by the use of the methods of Lyapunov. The conditions and parameters of stable and asymptotic stable rotation are determined for the mass variable rotor without reactive force and existing disbalance force and small non-linear elastic force (Cveticanin and Zlokolica, 1988), too.

Analyzing the stability properties of rotor motion caused by forces like hydrodynamic and aerodynamic force, internal and external damping forces in the interaction with mass variation it is shown that instabilities may occur (Cveticanin, 1995_2). The influence of these forces and mass variation on the stability of motion is extremely complex.

Among the factors leading to instability of rotors on which the band is winding up is the rubbing between rotating and stationary parts. Stability of motion of a rubbing rotor on which the band is winding up is considered by Cveticanin (1989). The rotor is modeled as a clamped-free one. The limits of the regions of stable and unstable rotation of the rotor, with zero deflection of mass center, are defined by applying the direct Lyapunov method. These results are compared with those obtained for rotors with constant mass. It can be concluded that the rotation for no-rotor-stator contact and rub initation is asymptotically stable. For stator-rotor interaction the rubbing force appears which cases unstable rotation. The limits of unstable rotation are functions of the mass winding up on the rotor but also on the geometrical and physical characteristics of the band and the shaft, and also of clearance value.

A special type of rotating system is the rotor/fluid one (Cveticanin, 1998_1). The model of the rotor is a massless elastic shaft on which end is a fluid filled cylinder is fixed. The fluid is leaking and mass of the rotor is varying. The fluid force and the reactive force, which is the result of mass variation, act. Special attention is given to the effect of interactive influence of the inertial fluid force and the reactive force on the stability of rotation of the rotor. The stability of rotation is investigated applying the direct Lyapunov theorems but also analyzing the solution of the second order differential equation with a complex deflection function, small non-linearity and time variable parameters and a significant damping term which describes the vibration of the system. The conditions for stable rotation are determined. Analyzing the amplitude of self-excited vibrations the conditions for which unstable motion appears, are defined.

In the structural unstable rotor systems with variable mass deterministic chaos appears (Cveticanin, 1995_5). The steady-state chaotic motion of rotor center depends on the parameters of slowly varying mass. The motion of rotor center is bounded and strongly dependent on initial conditions.

3.3. Rotor balancing

Nowadays, there are many rotors which experience mass changes during their operation. Such a change of mass causes unbalance. The unbalance causes vibrations. Vibrations must be eliminated by balancing of the rotor. Bessonov (1967_2) gives a method for approximate balancing of the rotor. Mass of the counterweights is calculated using the principle of the minimal quadratic difference between the exact and approximate static moment in two balancing planes. The improvement of balancing is achieved by controlled movement of counterweights which have to satisfy the conditions of dynamic balance (Cveticanin, 1981).

4. Mechanisms and machines

The theory of mass variable systems is applied for dynamic analysis of machines and mechanisms with time variable mass (Cveticanin, 1998_2). Various dosing devices (Fig.3),

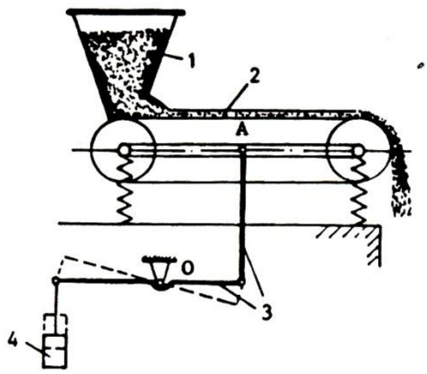

Fig. 3. Automatic dosing device. (Bessonov, 1967_1)

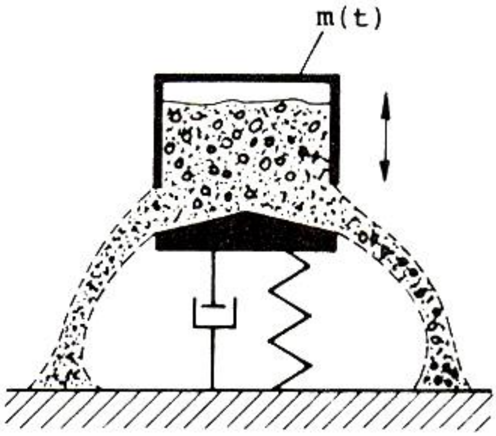

Fig. 4. Mechanism for material spreading (Cveticanin, 1998_2)

mechanisms for spreading material (Fig.4), excavators (Fig.5), cranes (Fig.6), etc. represent those machines and mechanisms. Numerous models for mass distribution in the mechanisms are developed. The most papers deals with the problem of mealy material distribution on the transport track (Entus, 1981). An improvement to the model is given by Saeki and Takano (1997) who analytically studied the flow of granular materials on vibratory conveyors. Using the suggested theoretical consideration vibrations (Strzalko and Grabski, 1995) and stability of motion (Cveticanin, 1995_6) of mechanisms with variable mass are investigated. It is concluded that the load variation and also the reactive force have a significant influence on dynamics of excavator and the planar crane (Strzalko and Grabski, 1995).

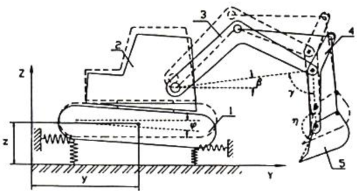

Fig. 5. Model of excavator. Strzalko and Grabski, 1995)

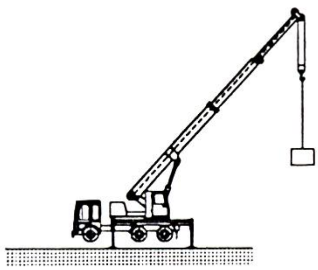

Fig. 6. Model of planar crane (Ross, 1979)

Loading and unloading of the lifting mechanism of a crane (Fig. 6) causes mass variation (Ross, 1979). The mechanical model of the mechanism is a simple pendulum with variable mass and length. Due to mass variation a reactive force appears. The influence of the reactive force on the motion of the system is investigated. Some special cases are of interest: (i) the relative mass variation rate is constant; (ii) the damping is varying and the relative length variation rate is constant and the wind force is present. The values of mechanism parameters, for which beside the regular also nonregular chaotic motion appears, are determined. To obtain the criteria of chaotic motion the method of Melnikov is applied (Guckenheimer and Holmes, 1983).

5. Conclusions and further investigation

Recently, one of the most important requirements in machining engineering is to increase the producing capacity of machinery. It requires the enlargement of the velocity of machines. As a side effect to working process the vibrations appear. To eliminate the vibrations the most accurate desinging of the mechines have to be done. It requires the model of the system to be more realistic. It means that the mass variation of the system is necessary to be taken into consideration. The model has to include besides mass variation also the non-linear effects. It needs the modification and extending the previous known theory.

The following is suggestion for further investigation:

1. The most of investigation are done in systems with ’slow mass’ variation due to ’slow time’. In real systems mass variation need not to be the function of ’slow time’. The extension of the consideration on the systems with mass variation which depend not only on ’slow time’ is necessary. There are no valied analytic methods which involve the solving of the differential equations with strong time variable parameters.

2. Usually, the real systems with time variable mass are non-linear. Unfortunately, the simplification is done and the systems are considered as linear one, or with small non-linearitirs. Only the strong cubic non-linearity is investigated. The future investigation would be directed to non-linear mass variable systems.

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Извод

Преглед истраживања у области динамике система са промељивом масом

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Резиме

У раду је дат приказ резултата у области динамике система са променљивом масом. Након теоретских разматрања, приказана је примена те теорије. Посебна пажња посвећена је машинама и механизмима као и роторима са променљивом масом. Анализирани су система са непрекидном и коначном променом масе. Посебно је разматран утицај реактивне силе на кретање система. Наведене су бројне аналитичке процедуре за решавање диференцијалних једначина које описују кретање система. Циљ рада је да да смернице за даља истраживања у овој области.

Кључне речи: систем са промељивом масом, реактивна сила, ротор са променљивом масом, механиsам са временски променљивом масом.

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