

## **Effect of Surface Roughness on the Performance of a Magnetic Fluid Based Parallel Plate Porous Slider Bearing with Slip Velocity**

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### **Abstract**

An endeavour has been made to investigate the performance of a transversely rough porous parallel plate slider bearing with slip velocity taking a magnetic fluid as the lubricant. The associated Reynolds equation is stochastically averaged with respect to the random roughness parameter characterizing the roughness. In view of suitable boundary conditions this equation is solved to obtain the pressure distribution resulting in the calculation of load carrying capacity. Besides, friction on the slider is also computed. The computed results presented in graphical form indicate that the magnetic fluid lubricant improves the performance of the bearing system. The negatively skewed roughness further increases the already increased load carrying capacity due to magnetization. This effect becomes more sharp when variance (-ve) is involved. Although, porosity, slip velocity and standard deviation decrease the load carrying capacity, this negative effect can be minimized by the magnetic fluid lubricant in the case of negatively skewed roughness. In addition, the friction remains unaltered.

**Keywords:** Parallel Plate Slider, Magnetic Fluid, Roughness, Slip Velocity, Porosity, Pressure.

### **1. Introduction**

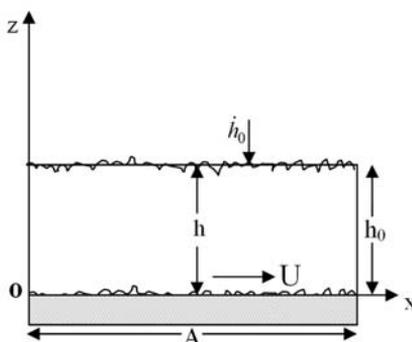
**Ramanaiah and Gundala** (1967) theoretically investigated the performance of a parallel plate slider bearing with a non uniform magnetic field parallel to the plates and perpendicular to the direction of the flow. Here, the magnetic field profile for the maximum load carrying capacity was determined to be a step function with the step location and step height ratio depending on maximum field strength and the electric potential difference between the plates. **Bhat** (1978) initiated theoretical investigations of a parallel plate porous slider under a non uniform applied transverse magnetic field. Here also, it was shown that the optimum magnetic field profile was a step function and the porosity slightly affected the performance characteristics, such as load carrying capacity, frictional force, friction factor and magnetic field state location. **Shah and Bhat** (2003) analyzed the ferrofluid lubrication of a parallel plate squeeze film between circular plates using Jenkins model. It was established that the load carrying capacity increased with increasing values of the axial permeability or material constant of Jenkins model and attained a maximum when the value of the material constant was nearer to unity. **Shah and Bhat** (2005)

theoretically analyzed the effects of slip velocity and a magnetic fluid lubricant characterized by material parameter on a parallel plate porous slider bearing. Here, it was shown that the increase in the slip parameter failed to alter the load capacity and the position of centre of pressure on the other hand, increase in the material parameter caused reduced friction. Recently, Patel and Deheri (2011) studied the Shliomis model based ferrofluid lubrication of a plane inclined rough slider bearing with slip velocity. It was concluded that although, the transverse surface roughness adversely affected the bearing system the magnetization sharply increased the load carrying capacity. Further, the slip parameter not only decreases the load but also decrease the friction on the slider.

Owing to elastic, thermal and uneven wear effects the configurations encountered in practice are usually not smooth. In fact, the bearing surface develops roughness after receiving some run in and wear. The roughness appears to be random in character. The stochastic model adopted by Tzeng and Saibel (1967) to analyze the effect of surface roughness was refined and modified further by Christensen and Tonder (1969 a., 1969 b., 1970) to present a comprehensive study on the effect of both transverse as well as longitudinal surface roughness. The roughness was characterized by a stochastic random variable with non zero mean, variance and skewness. This method of Christensen and Tonder found its application in a number of contributions (Gupta and Deheri (1996), Andharia, Gupta and Deheri (1997), Deheri, Patel and Patel (2011), Ting (1975), Prakash and Tiwari (1982), Prajapati (1995), Guha (1993).

Here, it has been sought to deal with the performance of a rough porous parallel plate squeeze film slider bearing under the presence of a magnetic fluid lubricant considering velocity slip. The analysis found here is based on Jenkins model.

## 2. Analysis



**Fig. 1.** Parallel Plate Slider Bearing

Jenkins (1972) proposed a model to describe the flow of a magnetic fluid. With Maugin's modification, equation of the model for steady flow is (Jenkins, 1972) and (Paras Ram and Verma, P.D.S, 1999)

$$\rho(\bar{q} \cdot \nabla)\bar{q} = -\nabla p + \eta \nabla^2 \bar{q} + \mu_0 (\bar{M} \cdot \nabla)\bar{H} + \frac{\rho \alpha^2}{2} \nabla \left[ \frac{\bar{M}}{M} \{ (\nabla \bar{q}) \bar{M} \} \right] \quad (1)$$

$$\rho(\bar{q} \cdot \nabla)\bar{q} = -\nabla p + \eta \nabla^2 \bar{q} + \mu_0 (\bar{M} \cdot \nabla)\bar{H} \quad (2)$$

together with

$$\nabla \cdot \bar{q} = 0, \nabla \times \bar{H} = 0, \bar{M} = \bar{\mu} \bar{H}, \nabla \cdot (\bar{H} + \bar{M}) = 0,$$

(Bhat, 2003),  $\alpha$  being a material constant. From Equations (1) and (2) we conclude that Jenkins model is a generalization of N-R model with an additional term

$$\frac{\rho \alpha^2}{2} \nabla \left[ \frac{\bar{M}}{M} \{ (\nabla \bar{q}) \bar{M} \} \right] = \frac{\rho \alpha^2 \mu}{2} \nabla \left[ \frac{\bar{H}}{H} \{ (\nabla \bar{q}) \bar{H} \} \right] \quad (3)$$

which modifies the velocity of the fluid. Thus, N-R model modifies the pressure while Jenkins model modifies both the pressure and velocity of the ferrofluid.

In a one dimensional flow, as in a slider bearing with

$$H = H(r) (\cos \varphi, 0, \sin \varphi), \text{ It follows that}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta \left( 1 - \frac{\rho \alpha^2 \bar{\mu} H}{2\eta} \right)} \frac{\partial}{\partial x} \left( p - \frac{\mu_0 \bar{\mu}}{2} H^2 \right) \quad (4)$$

Following the assumptions of Bhat (2003), solving this under the boundary conditions  $u=0$  at  $z=0$  and  $u=U$  at  $z=h$  one obtains

$$u = \frac{z^2 - hz}{2\eta \left( 1 - \frac{\rho \alpha^2 \bar{\mu} H}{2\eta} \right)} \frac{\partial}{\partial x} \left( p - \frac{\mu_0 \bar{\mu}}{2} H^2 \right) + \frac{Uz}{h} \quad (5)$$

Substituting this value of  $u$  in the integral form of the continuity equation

$$\frac{\partial}{\partial x} \int_0^h u dz + w_h - w_0 = 0 \quad (6)$$

one gets

$$\frac{d}{dx} \left[ \frac{h^3}{1 - \frac{\rho \alpha^2 \bar{\mu} H}{2\eta}} \frac{d}{dx} \left( p - \frac{\mu_0 \bar{\mu}}{2} H^2 \right) \right] = 6\eta U \frac{dh}{dx} - 12\eta (w_0 - w_h) \quad (7)$$

So incorporating the effect of porosity and slip, the associated Reynolds type equation comes out to be

$$\frac{d}{dx} \left[ \frac{g(h)}{1 - \frac{\rho\alpha^2\bar{\mu}H}{2\eta}} \frac{d}{dx} \left( p - \frac{\mu_0\bar{\mu}}{2} H^2 \right) \right] = 6\eta U \frac{dh}{dx} + 12\eta \dot{h}_0 \quad (8)$$

where

$$g(h) = \left\{ h^3 + 3\alpha h^2 + 3(\alpha^2 + \sigma^2)h + 3\sigma^2\alpha + \alpha^3 + \varepsilon + 12\phi h \right\} \frac{(4 + sh)}{(2 + sh)} \quad (9)$$

Taking  $H^2 = Kx(A - x)$  and introducing the dimensionless quantities

$$X = \frac{x}{A}, \quad \bar{h} = \frac{h}{h_0}, \quad \mu^* = \frac{K\mu_0\bar{\mu}h_0^2A}{\eta U}, \quad \bar{\alpha}^2 = \frac{\rho\alpha^2\bar{\mu}A\sqrt{K}}{2\eta}, \quad P = \frac{h_0^3 p}{\eta A^2 \dot{h}_0}, \quad \beta_1 = \frac{h_0^3}{2\dot{h}_0 A}$$

$$W = \frac{h_0^3 w}{\eta A^4 \dot{h}_0}, \quad F = \frac{-h_0 f}{\eta L^2 \dot{h}_0}, \quad \bar{\alpha} = \frac{\alpha}{h_0}, \quad \bar{\sigma} = \frac{\sigma}{h_0}, \quad \bar{\varepsilon} = \frac{\varepsilon}{h_0^3}, \quad \bar{s} = sh_0, \quad \psi = \frac{\phi h}{h_0^3}$$

Equation (8) yields on integration.

$$\frac{d}{dx} \left[ P - \frac{1}{2} \mu^* X(1 - X) \right] = \frac{6}{g(\bar{h})} \left( \bar{h} - \beta^{-1} X + Q \right) \left( 1 - \bar{\alpha}'^2 \sqrt{X(1 - X)} \right) \quad (10)$$

where

$$g(\bar{h}) = \left\{ \bar{h}^3 + 3\bar{\alpha}\bar{h}^2 + 3(\bar{\alpha}^2 + \bar{\sigma}^2)\bar{h} + 3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\varepsilon} + 12\bar{\psi}\bar{h} \right\} \frac{(4 + \bar{s}\bar{h})}{(2 + \bar{s}\bar{h})} \quad (11)$$

In a parallel plate slider bearing  $h = h_0$  then  $\bar{h} = 1$ . So taking  $\bar{h} = 1$  in Equation (10) the film pressure  $p$  is given by the equation

$$\frac{d}{dx} \left[ P - \frac{1}{2} \mu^* X(1 - X) \right] = \frac{6}{g(1)} \left( 1 - \beta^{-1} X + Q \right) \left( 1 - \bar{\alpha}'^2 \sqrt{X(1 - X)} \right) \quad (12)$$

where

$$g(1) = \left\{ 1 + 3\bar{\alpha} + 3(\bar{\alpha}^2 + \bar{\sigma}^2) + 3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\varepsilon} + 12\bar{\psi} \right\} \frac{(4 + \bar{s})}{(2 + \bar{s})} \quad (13)$$

Solving Equation (12) under

$$P(0) = P(1) = 0$$

yields,

$$P = \frac{1}{2} \mu^* X(1 - X) + \frac{6}{g(1)} \left\{ X - A_1 A_2 - \beta^{-1} \frac{X^2}{2} + \bar{\alpha}'^2 \beta^{-1} A_3 + A_4 X - A_5 \right\} \quad (14)$$

where

$$A_1 = \frac{\frac{\beta^{-1}}{2} - \frac{\bar{\alpha}'^2 \beta^{-1}}{8}}{1 - \frac{\pi}{8} \bar{\alpha}'^2}, \quad A_2 = \frac{(2X-1)}{4} \sqrt{X(1-X)} + \frac{1}{8} \sin^{-1}(2X-1),$$

$$A_3 = \frac{X}{8} + \frac{\sqrt{X(1-X)}}{8} - \frac{\sqrt[3]{X(1-X)}}{3} - \frac{\sqrt{X}}{4} \sqrt[3]{(1-X)}, \quad A_4 = \frac{\frac{\pi}{8} \bar{\alpha}'^2 + \frac{\beta^{-1}}{2} - \frac{\bar{\alpha}'^2 \beta^{-1}}{8} - 1}{1 - \frac{\pi}{8} \bar{\alpha}'^2}$$

and

$$A_5 = \frac{\frac{\pi}{16} \bar{\alpha}'^2 \left( \frac{\beta^{-1}}{2} - \frac{\bar{\alpha}'^2 \beta^{-1}}{8} \right)}{1 - \frac{\pi}{8} \bar{\alpha}'^2}$$

The load capacity  $W$ , frictional force  $F$  are respectively expressed in dimensionless form as

$$W = \frac{\mu^*}{24} + \frac{6}{g(1)} \left\{ \frac{1}{3} - \frac{5\pi}{128} \bar{\alpha}'^2 A_1 - \frac{\beta^{-1}}{6} + \frac{7}{128} \bar{\alpha}'^2 \beta^{-1} + \frac{1}{2} A_4 - A_5 \right\} \quad (15)$$

and

$$F = \int_0^1 \left[ \frac{\bar{h}}{2} \frac{dP}{dX} + \frac{1}{\bar{h}} \right] dX = 1 \quad (16)$$

### 3.Results and Discussion

It is clearly seen that the dimensionless pressure distribution is determined from Equation (14) while the non dimensional load carrying capacity is obtained from Equation (15). Besides, the variation of friction is given by Equation (16). For a non porous smooth bearing this study reduces to the investigation of Bhat (2003) in the absence of slip velocity. From equations (14) and (15) it is observed that the dimensionless pressure increases by

$$\frac{\mu^*}{2} X(1-X)$$

while the dimensionless load carrying capacity gets enhanced by

$$\frac{\mu^*}{12}$$

as compared to the case of conventional lubricant. Furthermore, Equation (16) tells that the friction remains constant.

The variation of load carrying capacity with respect to the magnetization parameter displayed in Figures 2-7 indicates that the magnetization increases sharply the load carrying capacity in case of  $\bar{s}$ ,  $\bar{\alpha}$ ,  $\bar{\sigma}$ ,  $\bar{\varepsilon}$  and  $\psi$  while in case of  $\bar{\beta}$  this increase is negligible.

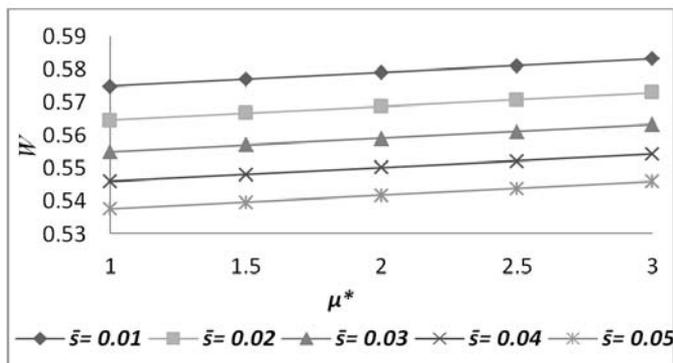


Figure 2 Variation of load carrying capacity with respect to  $\mu^*$  and  $\bar{s}$

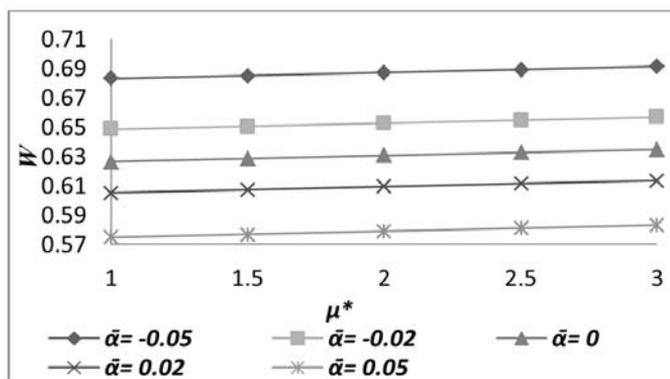


Figure 3 Variation of load carrying capacity with respect to  $\mu^*$  and  $\bar{\alpha}$

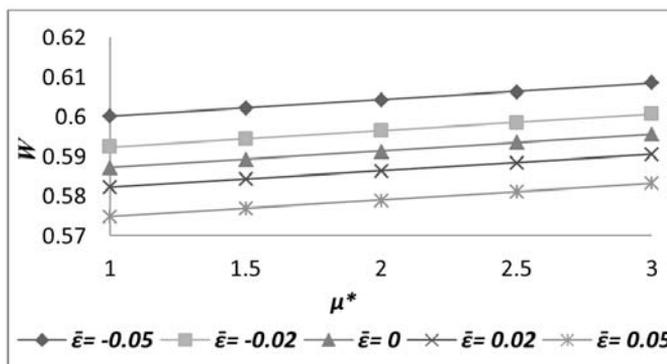


Figure 4 Variation of load carrying capacity with respect to  $\mu^*$  and  $\bar{\varepsilon}$

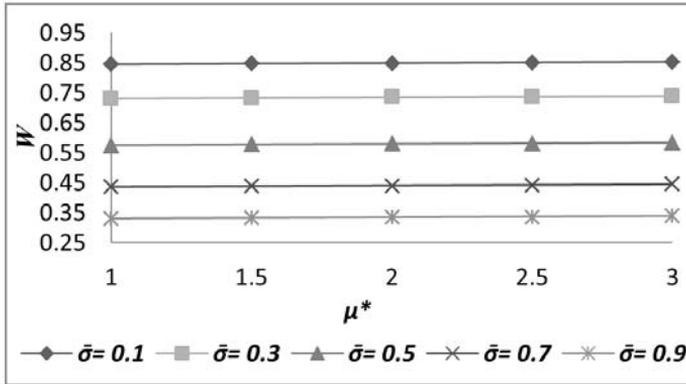


Figure 5 Variation of load carrying capacity with respect to  $\mu^*$  and  $\sigma^-$

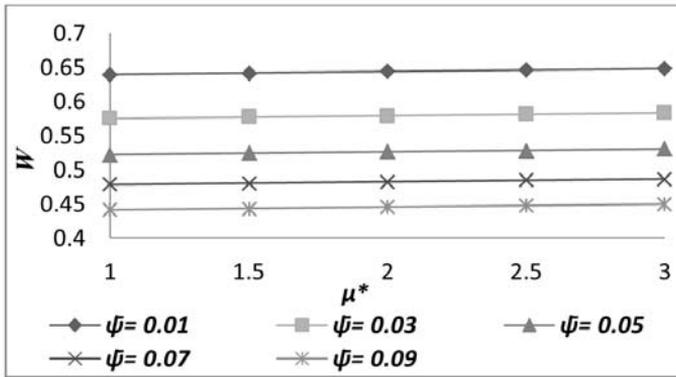


Figure 6 Variation of load carrying capacity with respect to  $\mu^*$  and  $\psi^-$

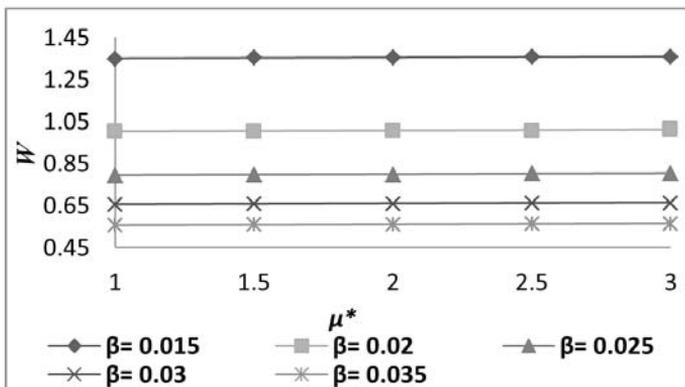


Figure 7 Variation of load carrying capacity with respect to  $\mu^*$  and  $\beta^-$

The variation of load carrying capacity with respect to the slip parameter is presented in Figures 8-10. It is found that the load carrying capacity decreases considerably with increasing

value of slip parameter. This decrease in the load carrying capacity is more in the case of skewness.

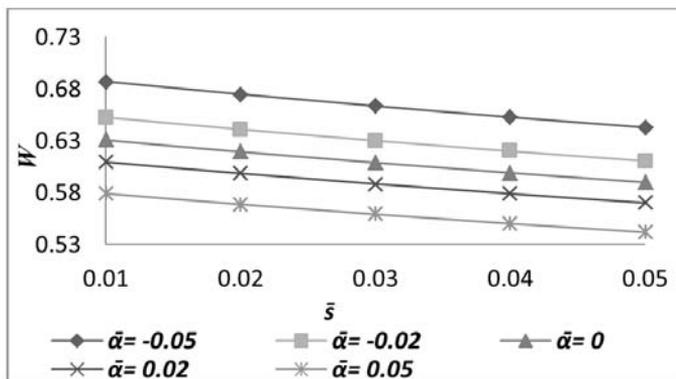


Figure 8 Variation of load carrying capacity with respect to  $s\bar{}$  and  $\alpha\bar{}$

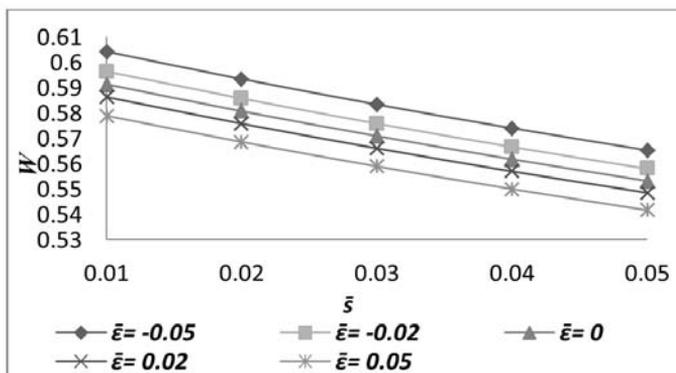


Figure 9 Variation of load carrying capacity with respect to  $s\bar{}$  and  $\epsilon\bar{}$

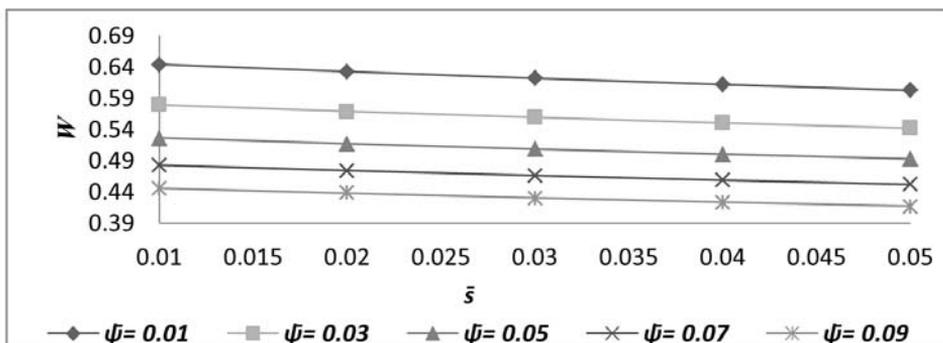


Figure 10 Variation of load carrying capacity with respect to  $s\bar{}$  and  $\psi\bar{}$

The effect of the variance  $\alpha^-$  on the distribution of load carrying capacity is depicted in Figures 11-14. It becomes clear that variance (+ve) decreases the load carrying capacity while variance (-ve) increase the load carrying

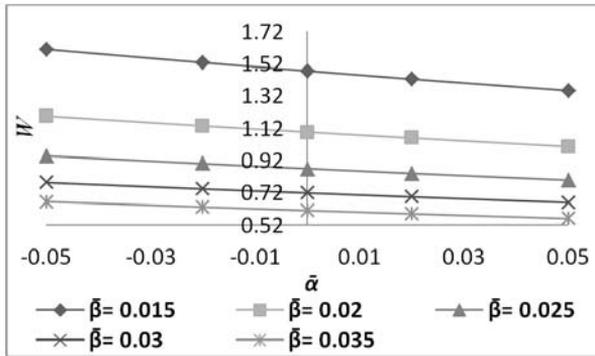


Figure 11 Variation of load carrying capacity with respect to  $\alpha^-$  and  $\epsilon^-$

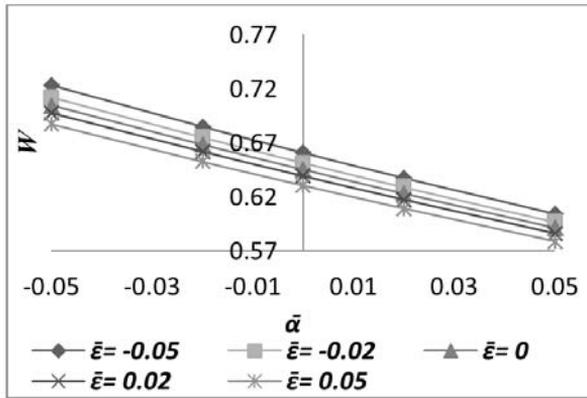


Figure 12 Variation of load carrying capacity with respect to  $\alpha^-$  and  $\sigma^-$

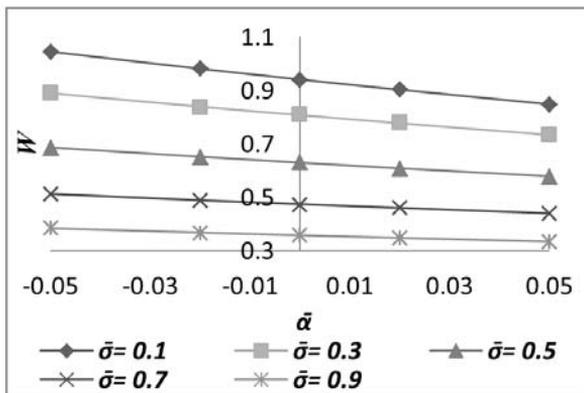


Figure 13 Variation of load carrying capacity with respect to  $\alpha^-$  and  $\psi^-$

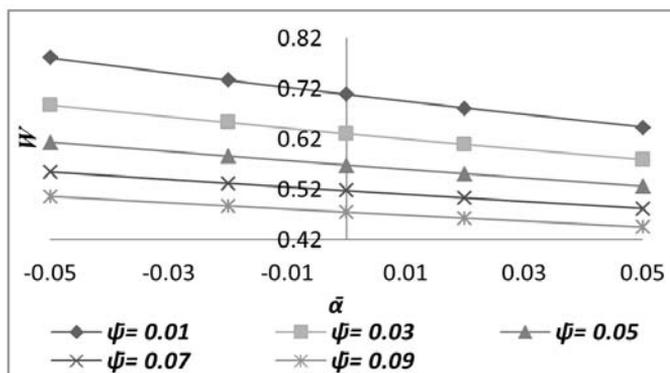


Figure 14 Variation of load carrying capacity with respect to  $\bar{\alpha}$  and  $\bar{\beta}$

The effect of standard deviation presented in Figures 15-16 suggests that the standard deviation has considerably adverse effects on the performance of the bearing system in the sense that it decreases the load carrying capacity considerably

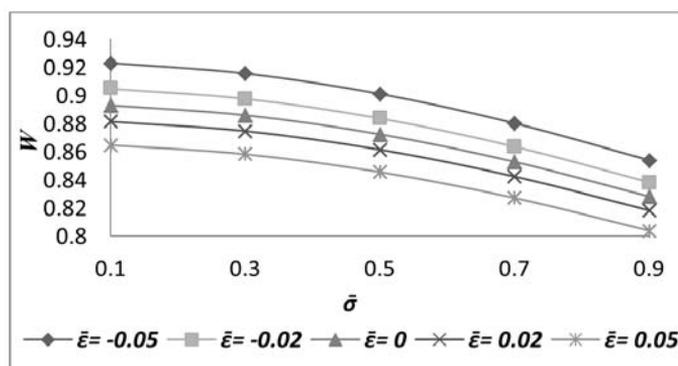


Figure 15 Variation of load carrying capacity with respect to  $\bar{\sigma}$  and  $\bar{\epsilon}$

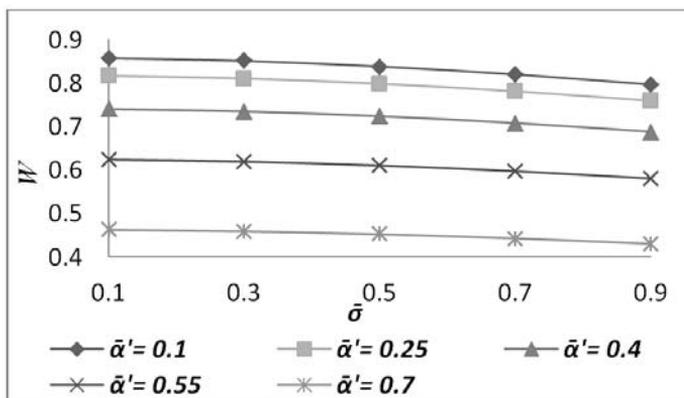


Figure 16 Variation of load carrying capacity with respect to  $\bar{\sigma}$  and  $\bar{\alpha}'$

Figures 17-18 representing the effect of skewness indicate that the skewness follows the path of the variance regarding the trends of load carrying capacity.

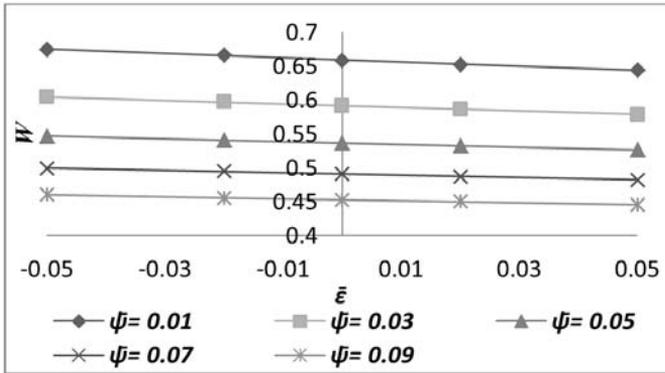


Figure 17 Variation of load carrying capacity with respect to  $\bar{\epsilon}$  and  $\bar{\psi}$

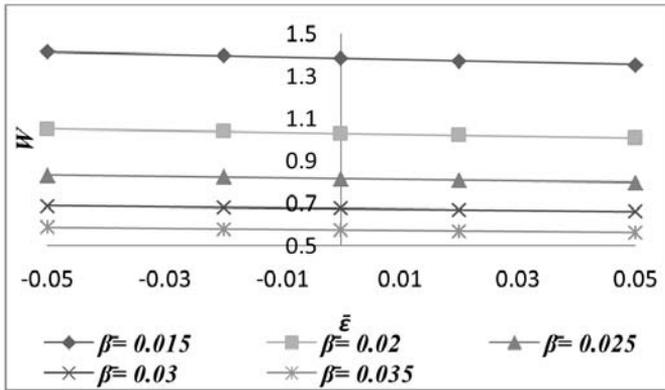


Figure 18 Variation of load carrying capacity with respect to  $\bar{\epsilon}$  and  $\bar{\beta}$

The fact that the material parameter  $\bar{\beta}$  decreases the load carrying capacity significantly can be found in Figures 19-20.

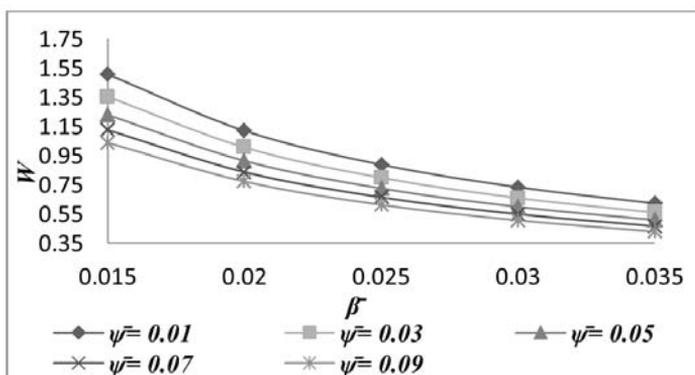


Figure 19 Variation of load carrying capacity with respect to  $\bar{\beta}$  and  $\bar{\psi}$

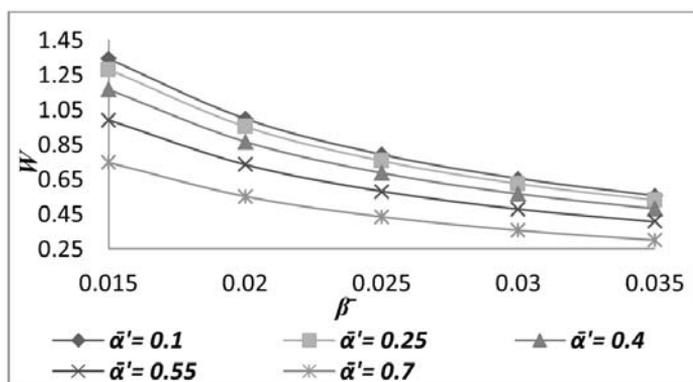


Figure 20 Variation of load carrying capacity with respect to  $\bar{\beta}$  and  $\bar{\alpha}'$

A close scrutiny of some of the graphs presented here makes it clear that the negative effect of porosity, standard deviation and material constant can be compensated up to some extent by the positive effect of magnetization in the case of negatively skewed roughness, particularly, when negative variance occurs.

A comparison of this study with the investigation of Shah and Bhat (2005) reveals that the overall performance is relatively better here in spite of the fact that the transverse roughness affects the performance adversely.

#### 4. Conclusion

From bearings life period point of view this investigation suggests that the roughness must be given due consideration while designing the bearing system even if the magnetic field strength is suitably chosen. Further, when a parallel plate slider bearing cannot support a load with conventional lubricant, this article makes it clear that the bearing can support a load with a magnetic fluid lubricant with constant frictional force on the slider.

## 5.Acknowledgements

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## Nomenclature

$f$	Frictional force
$h$	Fluid film thickness at any point
$k$	Permeability of porous matrix
$p$	Lubricant pressure
$s$	Slip parameter
$u$	X component of film fluid velocity
$w$	Load carrying capacity
$F$	Dimensionless frictional force
$H$	Magnitude of the magnetic field
$A$	Length of the bearing
$P$	Dimensionless pressure
$U$	Velocity of slider
$W$	Dimensionless load carrying capacity
$X$	X coordinate of the centre of pressure
$h_0$	Fluid film thickness at $x=0$
$\bar{H}$	External magnetic field
$q$	( $u, v, w$ ) is the fluid viscosity in the film region
$\bar{s}$	Dimensionless slip parameter
$\beta$	Material Parameter
$\eta$	The fluid viscosity
$\sigma$	Standard deviation
$\alpha$	Variance
$\varepsilon$	Skewness
$\psi$	Porosity
$\phi$	Inclination of $M$ with the $x$ -axes
$\bar{\sigma}$	Dimensionless standard deviation
$\bar{\alpha}$	Dimensionless variance
$\bar{\varepsilon}$	Dimensionless skewness
$\mu_0$	The permeability of the free space
$\mu^*$	Magnetization parameter in non dimensional form
$\bar{\mu}$	Magnetic susceptibility

## ИЗВОД

**Ефекат храпавости површине на перформансе клизећег порозног лежаја са две паралелне стране, магнетним флуидом и брзином клизања****N. D Patel<sup>1\*</sup>, G. Deheri<sup>2</sup>**

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**Резиме**

Учињен је напор да се истраже перформансе трансверзално храпавог порозног клизног лежаја са две паралелне стране и брзином клизања, коришћењем магнетног флуида као подмазивача. Одговарајућа Рејнолдсова једначина је стохастички усредњена у односу на случајни параметер храпавости који карактерише храпавост. Са гледишта погодних граничних услова ова једначина је решена да би се добио распоред притисака који долази из рачунања носивости. Поред тога, трење на клизачу је такође рачунато. Срачунати резултати представљени графички указују да магнетни флуидни подмазивач побољшава перформансе носећег система. Негативно закошена храпавост даље повећава ионако увећану носивост због магнетизације. Овај ефекат постаје израженији када је варијанса (-ve) укључена. Мада, порозност, брзина клизања и стандардна девијација умањују носивост, овај негативан ефекат може бити минимизован магнетним флуидом као подмазивачем у случају негативно закошених неравнина. Поред тога, трење остаје непромењено.

**Кључне речи:** Клизач са паралелним странама, магнетни флуид, храпавост, брзина клизања, порозност, притисак

**References**

- Andharia P, Gupta J, Deheri G (1997). Effect of longitudinal surface roughness on hydrodynamic lubrication of slider bearings, Proc. Tenth International Conference on Surface Modification Technologies, the Institute of Materials, 872-880.
- Bhat M (2003). Lubrication with a magnetic fluid, Team Spirit (India) Pvt. Ltd.
- Bhat M (1978). Optimum profile for the magneto hydrodynamic parallel-plate porous slider, Wear, 51, Issue 1, 49-55.
- Christensen H, Tonder K (1969 a). Tribology of rough surfaces: Stochastic models of hydrodynamic lubrication, SINTEF Report No. 10/69-18.
- Christensen H, Tonder K (1969 b). Tribology of rough surfaces: parametric study and comparison of lubrication, SINTEF Report No. 22/69-18.
- Christensen H, Tonder K (1970). The Hydrodynamic lubrication of rough bearing surfaces of finite width, ASME-ASLE lubrication conference, Paper No. 70. Lub-7.

- Deheri G, Patel H, Patel R (2011). Load carrying capacity and time height relation for squeeze film between rough porous rectangular plates, *Annals of Faculty Engineering Hunedoara International J. of Eng*, Tome IX.
- Guha S (1993) Analysis of dynamic characteristics of hydrodynamic journal bearings with isotropic roughness effects, *Wear*, 167, Issue 2, 2,173-179.
- Gupta J, Deheri G (1996). Effect of roughness on the behavior of squeeze film in a spherical bearing, *Tribol Trans.* 39, 99-102.
- Jenkins J (1972). A magnetic fluid, *Arch. Ration., Mech. Anal*,46(1) 42.
- Paras Ram, Verma P (1999). Ferrofluid lubrication in porous inclined slider bearing, *Indian J.pure appl. Math.*, 30(12), 1273-1281.
- Patel N, Deheri G (2011). Shliomis model based ferrofluid lubrication of a plane inclined slider bearing with slip velocity, *International J. of fluids engineering*, 3, No 3, 311-324.
- Prajapati B (1995). On certain theoretical studies in hydrodynamics and electro magneto hydrodynamics lubrication, Dissertation, S.P.University, Vallabh Vidyanagar.
- Prakash J, Tiwari K (1982). Lubrication of a Porous Bearing with Surface Corrugations, *Jour. of Lubr. Tech.*104, 127.
- Ramanaiah, Gundala (1967). Optimum Load Capacity of a Parallel Plate Slider Bearing with Non uniform Magnetic Field, *Japanese Journal of Applied Physics*, Volume 6, Issue7, 797.
- Shah R, Bhat M (2003). Ferrofluid lubrication of a parallel plate squeeze film bearing, *Theoret. Appl. Mech.*, Vol.30, No.3, 221-240, Belgrade.
- Shah R, Bhat M (2005). Lubrication of porous parallel plate slider bearing with slip velocity, material parameter and magnetic fluid, *Industrial Lubrication and Tribology*, Vol. 57 Issue 3, 103 – 106.
- Ting L (1975). Engagement behavior of lubricated porous annular disks. Part I: Squeeze film phase -- surface roughness and elastic deformation effects, *Wear*, Volume 34, Issue 2, 159-172.
- Tzeng S, Saibel E (1967). Surface roughness effect on slider bearing lubrication, *Trans. ASME, J. Lub. Tech.*, 10, 334-338.