

Homogenization principle based multi-scale modeling of cellular structures

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Abstract

Cellular materials are of particular importance as energy absorbers in numerous engineering applications. Due to their advanced structure the numerical modeling of cellular materials represents a particular challenge. In the presented paper the influence of a filler material on the energy absorption characteristics of open-cell cellular specimen is observed. In order to deeply investigate and compare its effect, regular open-cell cellular specimens have been used. For such two phase materials, the appropriate numerical modeling approach is homogenization of representative volume elements. The highly nonlinear material behavior of open-cell cellular materials in compression includes several consequent collapse scenarios. This requires a particular attention when choosing an effective material model and special care for the homogenization procedure.

Key words: Multi-Scale, Cellular Structures, Homogenization

1. Introduction

Open-cell cellular materials are widely used as crash absorbers due to their high energy absorption performance. Energy absorbers are of particular importance in automotive, railway, ship and aerospace industry where the impact energy is absorbed and dissipated during the compaction of cell structure. In our research Vesenjak et al. (2007, 2006, 2009) the benefit of using second phase filler material, resulting in energy absorption increase, is addressed. Usually irregular cellular materials are used in engineering applications, however when developing numerical models for such materials, repeatability of testing results, used to formulate numerical models, cannot be ensured. The presented research focuses on regular open-cell cellular structures and the way how numerical models based on homogenization principles should be developed. Extensive testing has been performed on various regular open-cell cellular

structures made of polymer (manufactured by rapid prototyping techniques) and aluminum alloy material. Testing included quasi-static and dynamic compression tests, where the influence of filler material has been investigated as well Vesenjak et al (2007, 2009). The experimental results are used as a basis to formulate the finite element material model. At the meso-scale level (i.e. level of few unit cells) cellular structures are numerically complex to model. To model such open-cell materials on the macro-scale (i.e. level of an engineering structure), homogenization on the meso-scale, used to determine the non-linear material parameters for a classical FE model, is an appropriate approach (Loehnert and Wriggers 2008, Krstulovic-Opara et al. 2008). The homogenization of open-cell cellular structures involves a different approach compared to representative volume elements (RVE), as it is common for homogenization of multi-phase materials excluding voids, e.g. concrete. As buckling is commonly the collapse mechanism of such structures, homogenization of a RVE of an open-cell structure can be applied until instabilities or localization phenomena do not occur within the microstructure. Using the effective stress-strain behavior experimentally obtained for the RVE, the material parameters of a nonlinear effective material model for the open-cell cellular structure can be evaluated. The presented general multi-scale approach in modeling of complex structures enables a numerical simulation of real irregular open-cell structures under mechanical loading conditions.

The paper is structured as follows. Section 2 explains the physical effect of filler material within an open-cell cellular structure with the special application to shock absorbers. The physical effects can also be observed in experiments performed with small portions of the material. Section 3 deals with the numerical modeling of cellular structures. In section 4 a summary of general finite deformation homogenization theory with an emphasis on the effects that can be observed in cellular structures is given. Section 5 introduces the numerical model of a unit cell for the regular open-cell cellular structure with and without filler material. In section 6 the results of a representative volume element computation of the open-cell structure including filler material at finite elastic deformation is presented. Section 7 deals with the computation of a part of an irregular open-cell structure without filler material but including finite plastic deformations. Finally, section 8 concludes the paper and gives some outlook to future work.

2. The influence of filler material

Cellular materials are able to undergo large deformations when subjected to compressive loading. Under compressive loading cellular materials have a characteristic stress-strain relationship that can be divided into four main areas (Fig. 1): (i) quasi-linear elastic response, (ii) transition zone, where the cellular materials exhibit buckling, plastic deformation and collapse of intercellular walls, (iii) stress plateau, where the mechanisms of buckling and collapse becomes even more pronounced, resulting in large strains at almost constant stress and (iv) densification, where the cellular material stiffness increases and consequently converges towards the stiffness of the base material due to self contact Vesenjak et al. 2007, Surace et al 2009.

The benefit of such material response is the high efficiency of energy absorption that occurs within the stress plateau region. An ideal energy-absorbing material dissipates the kinetic energy of the impact while keeping the reacting force in certain limits in order to avoid any serious or even lethal injuries of passengers. In Vesenjak et al. (2007, 2006, 2009) it is shown that the introduction of filler materials enables an energy absorption increase by moving the densification zone towards larger deformation (depicted in red in Fig. 1), while keeping the stress plateau constant.

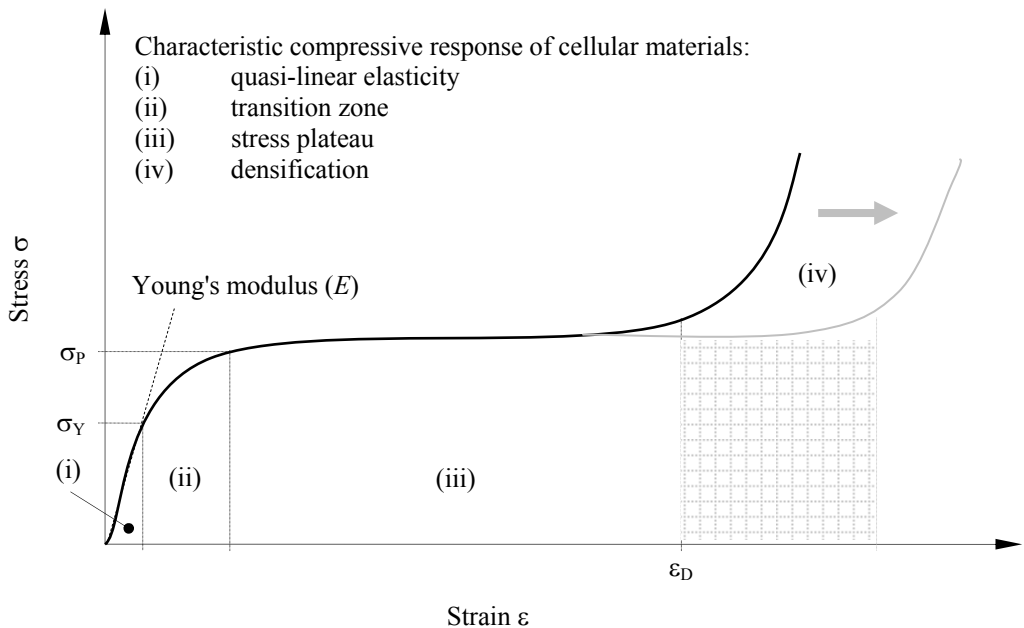


Fig. 1. Compressive behavior of cellular materials with energy absorption increase

To investigate the properties and to develop the numerical model, open-cell cellular structures made of aluminum alloy have been experimentally tested. Their behavior and mechanical properties have been determined under quasi-static ($\dot{\epsilon} \approx 0.01 \text{ s}^{-1}$) and dynamic ($\dot{\epsilon}_{\max} = 10 \text{ s}^{-1}$) compressive loading conditions.

When testing cellular materials (e.g. metal foams), the basic problem is usually the irregularity of the specimen structure. Even the properties of specimens from the same production cycle differ, resulting in a stochastic scatter of material parameters. Therefore, the geometry, topology and morphology of the cellular structures have been carefully selected to assure reproducibility and to minimize the effect of randomness due to the structure irregularity.

The cubic cellular specimens ($24 \times 24 \times 24 \text{ mm}$) were manufactured with unidirectional holes of 3 mm in diameter in all three directions, resulting in a structure of $6 \times 6 \times 6$ unit cells (Fig. 2). The cellular specimens made of aluminum alloy Al3003-H12 were manufactured by a computer controlled drilling machine.

The two component quasi-elastic molding silicone rubber (2K-Z010) was used as the second phase filler material (Fig. 3) to increase the capability of energy absorption and improve the unsteady behavior of studied brittle cellular structures. The vacuum treated silicon rubber is easily applicable and chemically stable with the advantage of a very low friction coefficient between the silicon filler and the cellular structure. Such low-friction behavior reduces the conservation of elastic energy that could be accumulated and released back to the structure upon the unloading phase, which would diminish the energy absorption capabilities.

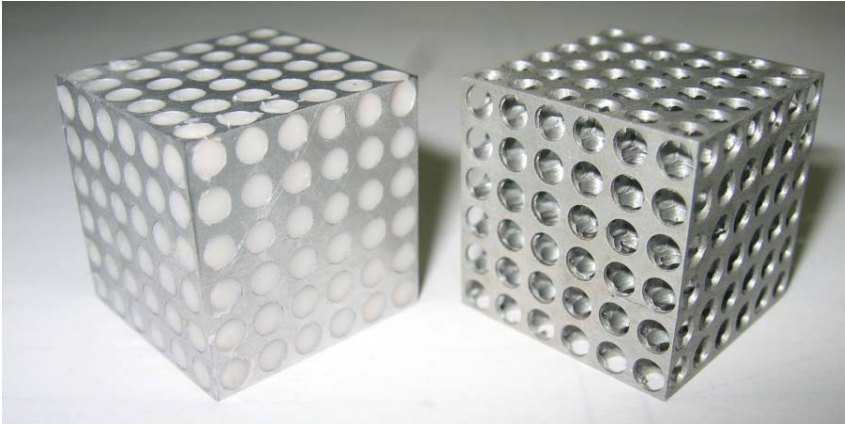


Fig. 2. Cubic aluminum alloy specimens with and without silicone filler.



Fig. 3. Compression and collapse of the specimen with the silicone filler.

The experimental results in form of the stress-strain curves are shown in Fig. 4. The advantageous influence of the silicon is higher densification strain. The stress plateau of the cellular structure with filler is comparable to the stress plateau of the cellular structure without the silicon filler. However, the main difference is observed at larger strains. It is obvious that the filler material affects the densification strain. The cellular structure accounting for pore filler has a much higher densification strain resulting in the plateau region increase. Since the plateau region is the most important region regarding the energy absorbing capabilities, the filler significantly improves the energy absorbing capabilities, without affecting the stresses of the plateau zone.

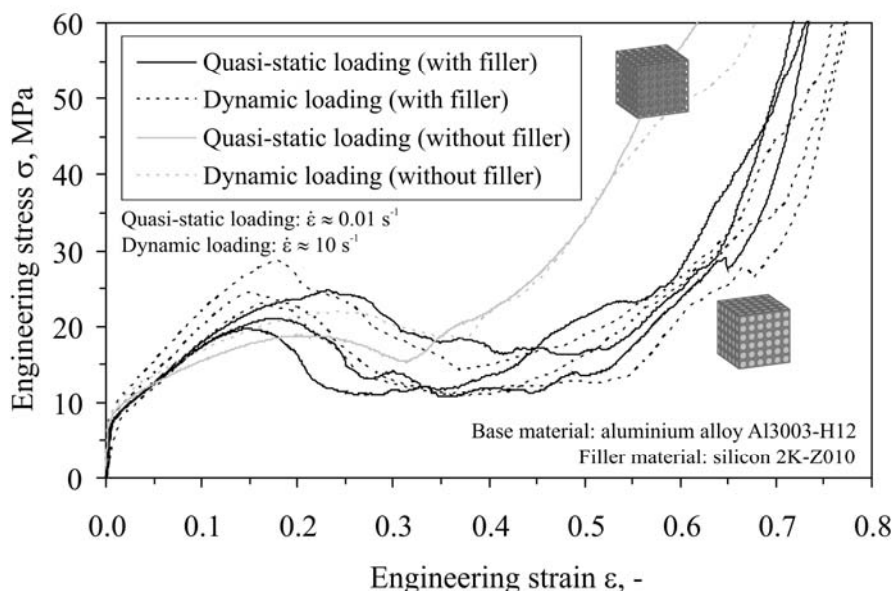


Fig. 4. Quasi-static and dynamic behavior of aluminum alloy with and without silicon pore filler under compressive loading

3. Numerical modeling of cellular structures

Numerical modeling of cellular structures represents a particular challenge in computational mechanics. Complex irregular structures can hardly be modeled on a macro-scale, i.e. the real engineering structure. There are several modeling approaches of irregular cellular structures, mostly based on homogenization of RVE or based on periodical boundary conditions.

The homogenization of a portion of open-cell cellular structure, based on Kelvin cell geometry, standard ABAQUS beam elements with constant cross section, and introduced structural irregularities, is presented in works of Demiray et. al. 2007, and Luxner et. al. 2007. To model the densification of the open-cell structure, in works of Gong and Kyrakides 2005 and Jang and Kyrakides 2009, the contact is approximated by spring elements introduced between beam element junctions of the Kelvin cell geometry.

The ligament nodes of struts in the Kelvin cell geometry are characterized by a larger cross section area, while the remaining part of a strut is characterized by a triangular cross section with a decrease of the cross section area towards the middle of the strut. The work of Jang et. al. 2008, Okomura et. al. 2008 and Gong et. al. 2005 is focused on modeling such strut geometries of a Kelvin cell by using 3D solid elements and periodic boundary conditions. In Jang et al. 2008 the influence of introducing local irregularities within a Kelvin cell has been reported as well.

There are approaches based on the RVE and structure geometry generated by random spherical voids, e.g. in the work of Guessama et.al. 2008 the Random Sequential Addition algorithm is used to generate open-cell models of a bred structure. The generated geometry was meshed by standard ABAQUS linear solid elements.

In the here presented case of open-cell cellular structure with filler material included, a two-phase material model is required. The numerical model is based on the homogenization of

a RVE, similar to approaches reported in work of Hain and Wriggers 2006 and Loehnert and Wriggers 2008, where two phase materials have been modeled as well. On the micro-scale (i.e. the unit cell), the cell lattice is modeled by eight-node solid elements describing a non-linear material behavior.

In this particular research, the unit cell ($1 \times 1 \times 1$ holes) is considered to be the micro-scale, few unit cells are the RVE on meso-scale, while the whole regular open-cell specimen ($6 \times 6 \times 6$ or larger) represents the macro-scale. The presented research is a base for future investigations of the influence of filler materials on macroscopic open-cell structure behavior. The whole structure will therefore represent a macro-scale model. In such a case the numerical modeling requires a homogenized nonlinear material model representing the open-cell structure. The material parameters of this material model are computed by means of RVE calculations and homogenization.

4. Homogenization of inhomogeneous materials

The homogenization of heterogeneous materials at finite deformations requires certain special prerequisites in addition to the assumptions one imposes for the small deformation case as well. Important requirements for homogenization using representative volume elements are imposed on the size of the RVE and on the applied boundary conditions. The size of the RVE has to be chosen large enough such that a sufficient number of inclusion particles or pores are present and the RVE really is representative. On the other hand, the RVE should be small enough that from the macroscopic point of view it can still be regarded as a material point. In addition to these prerequisites, for the finite deformation case, as well as for inelastic material behavior, it is important that any kind of local instability such as material softening, crack initiation, crack propagation, or buckling is excluded. In general, local instabilities lead to localization behavior and a loss of representativeness of the volume element. In such a case any multi-scale method based upon a simple homogenization step fails. Then either special multi-scale techniques that are capable of handling localizing phenomena need to be applied Loehnert and Belytschko 2007, Belytschko and Loehnert 2008, or the microstructure in the vicinity of the localization needs to be resolved explicitly. However, in this paper we only consider cases including finite deformations but excluding instabilities. That means, after the first buckling point is reached, we do not employ homogenization anymore.

4.1 Averaging theorems at finite deformations

The averaging theorems form a basis for multi-scale techniques based on the homogenization concept. The average strain theorem states that the volume average of the linearized strain tensor is equal to the effective / applied strain tensor used to prescribe uniform displacement boundary conditions. This is true in case there are no discontinuities of the displacement field inside the microstructure of the RVE. However, for porous material the displacement field inside the RVE exhibits many discontinuities in the displacement field that need to be considered in the average strain theorem. For finite deformations the average strain theorem cannot be applied to any of the nonlinear strain measures. It has to be applied to a measure that is linear in the displacement field such as the material deformation gradient or the material displacement gradient. For the deformation gradient the average strain theorem including the terms that arise in case of displacement discontinuities within the RVE reads

$$\langle \mathbf{F} \rangle_{\Omega_0} := \frac{1}{V} \int_{\Omega_0} \mathbf{F} \, dV = \mathcal{F} + \frac{1}{V} \int_{\partial\Omega_0^1 \cap \partial\Omega_0^2} \llbracket \mathbf{u} \otimes \mathbf{N} \rrbracket \, dA^{1,2} \quad (1)$$

The average stress theorem states that in the absence of body forces the volume average of the stress tensor is equal to the effective, constant stress tensor with which the applied tractions can be prescribed. For the finite deformation case, the average stress theorem can be applied to the Cauchy stress tensor as well as to the first Piola-Kirchhoff stress tensor. It cannot be applied to e.g. the second Piola-Kirchhoff stress. Applying the average stress theorem to the first Piola-Kirchhoff stress tensor and including the extra term that arises in case body forces are present reads

$$\langle \mathbf{P} \rangle_{\Omega_0} := \frac{1}{V} \int_{\Omega_0} \mathbf{P} \, dV = \mathcal{P} + \frac{1}{V} \int_{\Omega_0} \mathbf{f}_0 \otimes \mathbf{X} \, dV \quad (2)$$

An important issue when it comes to determining an appropriate size of the RVE is Hill's condition. For the small deformation case it is an energy criterion which states that the micro strain energy is equal to the macro strain energy. If this condition is approximately fulfilled, the choice of the RVE size is appropriate. For the finite deformation case the micro stress power needs to equal the macro stress power. For the general case that includes displacement discontinuities and body forces, using the first Piola-Kirchhoff stresses, as well as the power conjugate material velocity gradient, Hill's condition reads

$$\langle \mathbf{P} : \dot{\mathbf{F}} \rangle_{\Omega_0} = \frac{1}{V} \int_{\Omega_0} \mathbf{P} : \dot{\mathbf{F}} \, dV = \langle \mathbf{P} \rangle_{\Omega_0} : \langle \dot{\mathbf{F}} \rangle_{\Omega_0} + \frac{1}{V} \int_{\partial\Omega_0^1 \cap \partial\Omega_0^2} [[\dot{\mathbf{x}}]] \cdot \mathbf{t} \, dA^{1,2} + \frac{1}{V} \int_{\Omega_0} \dot{\mathbf{x}} \cdot \mathbf{f}_0 \, dV \quad (3)$$

4.2 Boundary conditions

The boundary conditions are essential when it comes to homogenization of RVEs. Since the homogenized RVE reflects the effective material at a specific point in space that takes up a specific stress and strain state, the boundary conditions for the RVE should either be uniform or periodic. For periodic boundary conditions often a periodicity of the microstructure is assumed. Uniform boundary conditions can be prescribed as pure traction boundary conditions where the tractions along the entire external boundary of the RVE are given by means of a spatially constant stress tensor and the outer normal vector to the boundary

$$\mathbf{t}|_{\partial\Omega_0} = \mathcal{P} \cdot \mathbf{N} \quad \mathcal{P} = \mathcal{P}(\mathbf{t}) \quad (4)$$

When applying the pure traction boundary conditions, one has to consider that rigid body motions of the RVE are possible and a priori undetermined. Similar to the Reuss assumption, for the RVE computations pure traction boundary conditions in general lead to a slightly too soft material response.

The alternative to applying pure traction boundary conditions is to apply pure displacement boundary conditions. Here, the displacements on the entire outer boundary of the RVE are prescribed by means of a constant displacement gradient and the position vector.

$$\mathbf{u}|_{\partial\Omega_0} = \mathcal{H} \cdot \mathbf{X} \quad \mathcal{H} = \mathcal{H}(\mathbf{t}) \quad (5)$$

Similar to the Voigt assumption and in contrast to pure traction boundary conditions in general, the effective material response of a RVE is slightly too stiff when applying pure displacement boundary conditions.

4.3 Effective material properties

In the case of isotropic linear elasticity and randomly distributed pores or filler material, the effective material will also reflect isotropic linear elastic behavior. This is due to the principle of superposition. In such a case the determination of the material constants of the effective

material can easily be achieved, e.g. by a least square fitting procedure for an isotropic linear elastic material.

In the case of finite deformations however, it is not obvious that the effective material model is similar to the material model chosen for the matrix and the filler material. Due to the nonlinearities and the fact that the principle of superposition does not hold anymore, the effective material can react quite differently even if one excludes instabilities within the microstructure, see e.g. Loehnert and Wriggers 2008.

For the case of small deformations and metal matrix material investigated in the numerical analysis, the material is characterized by elastoplastic behavior. Due to the strong path dependence it was not possible to find a general, nonlinear, anisotropic effective material model for an RVE with elastoplastic micro constituents.

After the unit cells start to buckle, localization occurs and homogenization is not possible anymore. Due to the strong dependence of the effective behavior on the micro structural geometry, on the applied load, as well as on the size of the RVE, it was not possible to describe this complex behavior by an effective material model until now. For the post-buckling behavior a detailed analysis of the investigated material sample seems to be more reasonable.

5. Numerical model of a unit cell

First one open unit cell under uniaxial compression is computed. The initially cube shaped unit cell is modeled with eight node enhanced assumed strain elements suitable for finite deformations and Neo-Hookean material, Mueller-Hoeppel et al. 2010. In contrast to many other finite element formulations, this element has very good properties in bending, compression and also for incompressible materials. In Figure 5 a series of deformed configurations of the aluminum unit cell without filler material under uniaxial compression in vertical direction is displayed. One can clearly see the buckling behavior. The load in vertical direction is increased up to the self contact point.

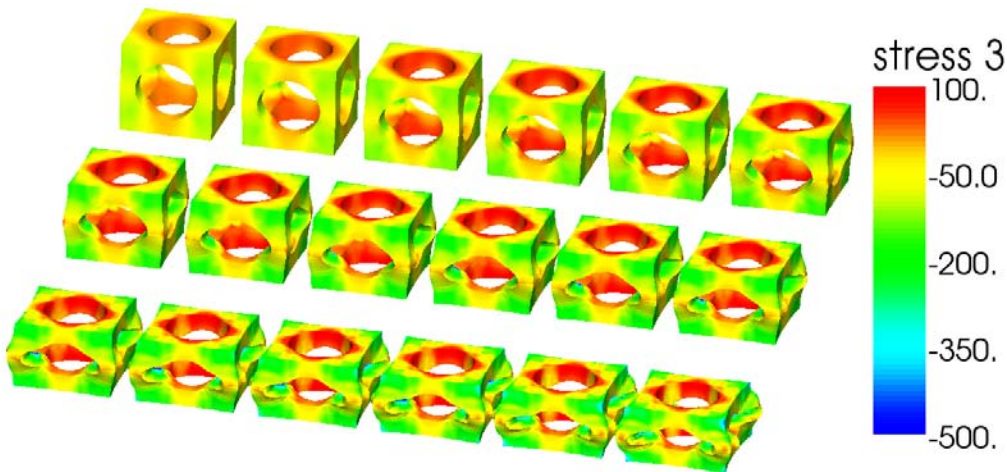


Fig. 5. Computational simulation of a meso-scale unit cell without the silicone filler material

Figure 6 shows the aluminum unit cell including the silicone filler material under uniaxial compression. The delamination between the silicone and the aluminum, as well as the damage

within the silicone, is not modeled, although it was observed in the experiments. In this case, as it was the case for the unit cell without the filler included (Fig. 5), the buckling of the aluminum matrix structure is the predominant failure mechanism.

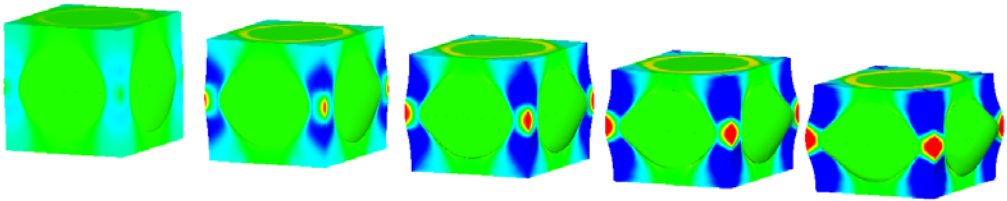


Fig. 6. Computational simulation of a meso-scale unit cell with the silicone filler material

6. Compression of a representative volume element

The unit cell model is not capable of reflecting the realistic behavior of a representative volume element consisting of multiple unit cells in each direction. This is due to the fact that the structure of the unit cell allows for a much softer buckling behavior than the real cellular structure including the filler consisting of many regularly placed unit cells exhibits.

A more realistic representative volume element consisting of $6 \times 6 \times 6$ unit cells is modeled (Fig 7). This model has the same shape as the specimen used for experiments (Fig. 2). Just like in the experiments, the modeled specimen is compressed in vertical direction. Debonding within the microstructure is not considered. The computation is performed up to the point where debonding within the microstructure would occur. At that point buckling within the matrix material cannot yet be observed. The material parameters for the used Neo-Hooke material for the matrix material, as well as for the filler material, are listed in Table 1.

	μ	Λ
Al matrix	$19663.46 \frac{N}{mm^2}$	$29495.19 \frac{N}{mm^2}$
silic. filler	$0.1483 \frac{N}{mm^2}$	$7.2677 \frac{N}{mm^2}$

Table 1. The Neo-Hooke material parameters

It could be observed that for this specific case a Neo-Hooke material model is the optimal choice for the effective material as well. The computed effective material parameters are $\mu_{\text{eff}} = 3901 \frac{N}{mm^2}$, $\Lambda_{\text{eff}} = 5450 \frac{N}{mm^2}$.

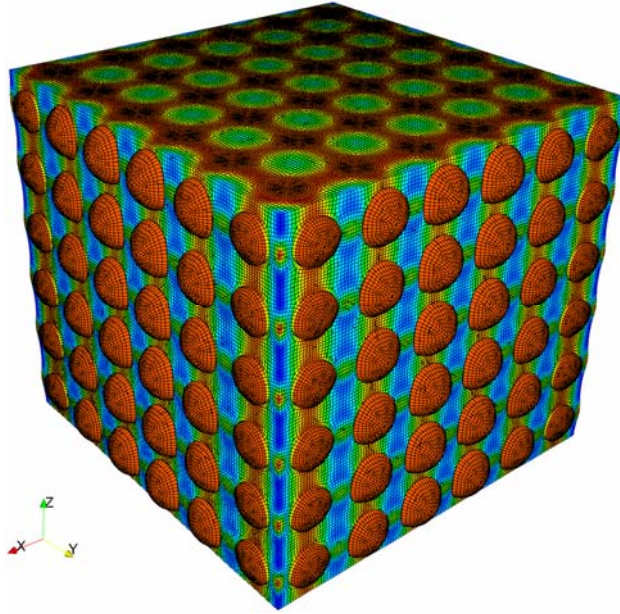


Fig. 7. Computational simulation of a specimen on macro-scale with the silicone filler material

7. Compression of an irregular open-cell structure

In addition to the calculation of the representative volume element of the regular open-cell cellular structure including filler material, an irregular open-cell structure without filler material is computed. The assumed material model is a finite deformation Neo-Hooke model including almost ideal von Mises plasticity based on Hencky strains. To regularize the equation system isotropic hardening with a small hardening parameter was used. The chosen material parameters are the same as for the regular open-cell cellular structure described in the last section. Additionally a yield stress of $\sigma_y = 91 \frac{N}{mm^2}$ and a hardening modulus of $H = 500 \frac{N}{mm^2}$ are used. Figure 8 shows the mesh and deformed configuration of the irregular open-cell cellular structure. In the deformed configuration the von Mises stresses are displayed. The computation was performed using quadratic tetrahedral elements and a mesh resolution of about 210000 nodes.

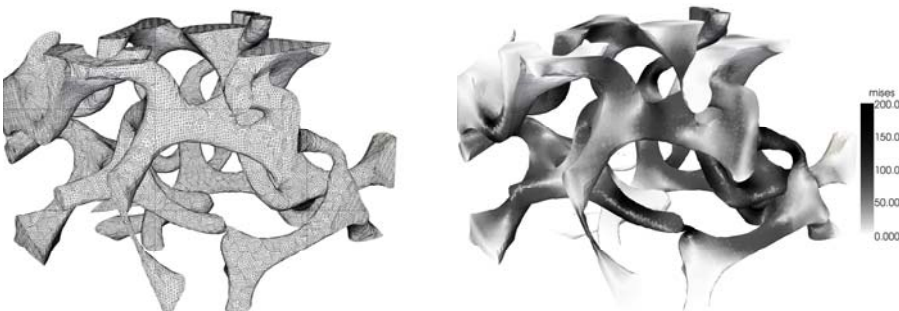


Fig. 8. The mesh and the deformed configuration of the irregular open-cell structure under compression

During the computation buckling could not be observed. The force deflection curve for the investigated sample shows the typical elasto-plastic behavior. Due to the fact that the deformation process is strongly path dependent, even though only isotropic hardening was considered, the effective response of the material sample shows strongly anisotropic behavior and until now, we were not able to find an effective material model to fit the resulting effective stress – strain curves appropriately.

8. Conclusions

A short overview of the influence of filler material to open-cell cellular structures shows the energy absorption benefits of the second phase material.

The regular open-cell cellular structure ensures the repeatability of results. It is used in the presented work as a base for a detailed investigation of filler material influence and numerical model development. Irregular open-cell cellular structures without filler material have been investigated as well. In future work, the influence of filler material on the mechanical behavior of irregular open-cell structures will be investigated.

Homogenization of representative volume elements (RVEs) is the reasonable choice when modeling such complex structures on the meso-scale and the whole structure on the macro-scale. The overview of basic homogenization principles needed for determination of effective material properties, including averaging theorems with their benefits and drawbacks, together with comments on setting boundary conditions, is presented as well.

Buckling of the structure leads to localization, which makes homogenization impossible. Therefore for the yield plateau a detailed analysis taking into account the real microstructure of the domain where localization occurs is necessary. The densification zone needs to be analyzed including self contact algorithms.

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Извод

Принцип хомогенизације заснован на моделирању на више скала ћелијске структуре

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Резиме

Ћелијски материјали су посебно значајни као енергијски апсорбери у бројним инжењерским применама. Због њихове унапређене структуре нумеричко моделирање ћелијских материјала представља посебан изазов. У овом раду је уочен утицај материјала попутне на апсорпционе карактеристике узорака са отвореним ћелијама. Да би се дубље изучио и упоредио његов ефекат, коришћени су узорци са отвореним регуларним ћелијама. Одговарајући нумерички приступ моделирања за ове материјале је хомогенизација репрезентативних запреминских елемената. Веома нелинеарно понашање материјала са отвореним ћелијама при притиску укључује и неколико консеквентних сценарија колапса. Ово захтева посебну пажњу при избору ефективног материјалног модела као и у хомогенизационом поступку.

Кључне речи: више скала, ћелијске структуре, хомогенизација

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