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Dynamics of the non-ideal mechanical systems: A review

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Abstract

The paper is a review on the literature dealing with the main properties of non-ideal vibrating systems. The analytical and numerical methods applied for analysing such systems are shown. The practical examples of non-ideal systems are considered. The most common phenomenon for the systems are discussed. The specific properties for various models are also discussed. The direction of the future investigation are given.

Key words: non-ideal mechanical system, non-ideal energy sources, linear and nonlinear vibrations, stability conditions, analytical and numerical methods

1. Introduction

In 1889, Laval built a single-stage turbine and demonstrated that in the case of rapid passage through resonance with enough power, the maximum vibration amplitude may be reduced significantly compared with that obtained in the steady state resonant vibration. He was the first one who recognized the problem of interaction between the energy source and working machine during the passage through resonance (Balthazar et al. 2003). Namely, the excitation amplitude is not constant but influenced by the motion of the energy source. Such limited power energy source is named 'non-ideal source' and the system 'non-ideal system'. The phenomena explored by Laval was not known in the classical research on nonstationary vibrations during the passage through resonce rests of so called 'ideal mechanical systems' where it was assumed that the excitation frequency or excitation amplitude are constant parameters or prescribed functions of time independent on the motion of the electromechanical system. So, probably, Laval was the first to study the problem of non-ideal system via an experiment. Since that time a significant number of real mechanical systems are observed where the passage through resonance required more input power than the excitation source had available. The properties and the phenomena in such non-ideal mechanical systems are intensively investigated since 1904, when Sommerfeld signified that the vibrating system cannot pass the resonant frequency or requires an intensive interaction between the dynamical system and the motor to do it. A strong interaction results with fluctuating motor speed and fairly large vibration amplitudes. The consequence is the so-called Sommerfeld effect (Sommerfeld, 1904). To exceed this problem, the driving torque of the electromotor has to be large enough not to be affected by dynamic load variations during vibrations. However, Kononenko, 1980, presented the first detailed study on the non-ideal problem of passage through resonance. If we consider the region before resonance on a typical frequency-response curve, we note that as the power supplied to

the source increases, the speed of rotation of the motor increases accordingly. However, this behavior does not continue indefinitely. The closer the motor speed moves toward the resonant frequency, the more power the source requires to increase the motor speed, as part of the energy is consumed moving the supporting structure. A large change in the power supplied to the motor results in a small change in its frequency and a large incerase in the amplitude of the resulting oscillations. Thus, near resonance, it appears that additional power supplied to the motor only increases the amplitude of the response of the structure, while having little effect on the RPM of the motor. Jump phenomena and the increase in power required by a source operating near resonance are manifestations of a non-ideal energy source and are often referred as the 'Sommerfeld effect', in honor of the first man who observed it (Sommerfeld, 1904). Further contributions to non-ideal problems were presented in papers by Dimentberg and coworkers 1997. Dimentberg et al. 1997, included a limited power source in a simple one-degree of freedom dynamical system (Fig.1). Since that time many investigation in the matter are done. A complete review on different theories on non-ideal vibrating systems were discussed and presented by Balthazar et al. 2002 and 2003.

The aim of the paper is to show the methods which are applied for analysing of the mathematical models of the non-ideal systems, to give a review on various examples of non-ideal systems, to discuss the results and to give the direction for future investigations.

2. Methods for Dynamical Analysis

In spite of the fact that the physical models of the real ideal and non-ideal systems seem to be equal, the mathematical models are quite different. Namely, the fact that the energy supply in the non-ideal source is limited, in contrast with an ideal system, causes that the time variations of the driving torqe and the rotational speed are not known in advance, becouse they are influenced by time variations of the dynamic load. This fact requires that for non-ideal systems, governing equations of the corresponding ideal system must be completed by an additional equation that describes the behaviour (characteristics) of the energy source. From this reason, non-ideal systems have one more degree of freedom than corresponding ideal systems. The dynamic properties of various energy sources are described as the torque-velocity functions (Zukovic, 2008).

Nonstationary vibrations, which appear in a non-ideal system during the passage through critical rotational speed can be analyzed by the different methods. Various approximate analytical and numerical methods are developed. Mitropolsky, 1965, devised the method of averaging and used it to analyze the vibrating system with elastic coupling. Nayfeh and Mook, 1979, applied the method of multiple scales in various types of electromechanical systems which may encompass flexible structures. Pusenjak et al. 2009, present the Extended Lindstedt-Poincare (EL-P) method with multiple scales to treat nonstationary vibrations of the electromechanical system, which are forced by a non-ideal energy source. By using the extended Hamilton principle, governing nonlinear differential equations of the system are derived. By using multiple time scales, which correspond to the nonlinear frequencies of the system in addition to the slow time scale, which corresponds to the slowly varying parameter (see Pusenjak 2008), the system of partial differential equations is obtained, which is successively solved by using the proposed EL-P method.

By means of numerical simulations a practical problem of synchronization of a non-ideal (i.e. when the excitation is influenced by the response of the system) and non-linear vibrating system was posed and investigated by Balthazar et al. 2003. By using the variation of torque constants the control of the self-synchronizationa and synchronization in the system are observed at certain levels of excitations.

Tsuchida et al. 2003, analyzed the non-ideal problem with two degrees of freedom operating near a resonance by using numerical simulation. To prove the statement that the motion is regular or irregular the frequency spectrum are plotted and the maximum Lyapunov's exponents are calculated. It is evident that these two last methods are applicable for determination of the regular and irregular vibration in any non-ideal systems.

In the paper Dantas et.al. 2003, gave the local analysis of a special kind of non-ideal vibrations defined by Kononenko, 1980. The authors used the Bezout Theorem, for study of the stability of the equilibrium point of a non-ideal system with two degrees of freedom consisting of a dumped nonlinear oscillator coupled to a rotatory part. In the critical case the sufficient conditions in order to obtain an appropriate Normal Form are obtained. They got the conditions for the appearance of Hopf Bifurcation when the difference between the driving torque and the resisting torque is small are got. Dantas et al. 2004, used similar techniques to deal with the Hopf bifurcation in another nonlinear and non-ideal mechanical problem.

In the paper of Felix et al. 2005, the analytical averaging method is applied for analysing of a system of three coupled differential equations which describe the motion of a shear-building portal plane frame foundation that supports an unbalanced direct current motor with limited power supply.

Through a number of numerical simulation of self-synchronization and synchronization in pre-resonance and resonance region between four unbalanced DC motors, which are non-ideal exciters, interacting with their flexible structural frame foundation response has been analyzed by Balthazar et al. 2005. Using the obtained results the criteria for stability of synchronous solutions are obtained.

Kang et al. 2002, used an integration method for detecting the chaotic motions of a nonlinear system subjected to double excitation has been considered. Zukovic and Cveticanin 2007, used the averaging asymptotic solving method for obtaining of the steady state motion of a non-linear oscillator of Duffing type with cubic nonlinearity excited with a non-ideal source. The numerical simulation is applied for obtaining the conditions for period doubling bifurcation which gives the chaos. The Lyapunov's exponents are applied to prove the existence of the irregular motion.

Zukovic and Cveticanin 2009, used the Krylov-Bogolubov averaging approximate asymptotic method for obtaining of the transient and steady state motion of a non-ideal mechanical system with clearance described with two coupled second order linear but discontinual differential equations. The parameters for chaotic motion are obtained by calculating of the maximal Lyapunov's exponent. The criteria for period doubling of the system are stated.

The first control of motion of the system for resonant case was investigated by Iwatsubo et al. 1972, who consider the variations of the acceleration rate in order to minimize the motion during passage through resonance. The same was discussed in the earlier works of Suzuki, 1978_1 and 1978_2 , in the non-ideal dynamical systems field.

Recently, control techniques for chaos in non-ideal mechanical systems are developed. The most of them are the modified versions of those used in ideal systems. The aim is to transform the chaotic motion into the periodical one, or to eliminate chaos. Some of the methods will be presented.

Tereshko et al. 2004, suggest the control of chaotic motion in oscillators by altering their energy. Souza et al., 2005₁, use the impact dampers for controlling chaos in systems with limited power supply. Felix et al. 2005, adopted a control technique based on internal resonance and saturation phenomenon which was developed for ideal frame systems, to non-ideal portal

frames. The control of saturation of the system is done by introducing a controller with quadratic control law which gives the effects of the 2:1 internal resonance (Tsuchida et al. 2005). It is beleived that this technique will be widely applied in future.

A simple feedback control for a chaotic oscillator with limited power supply is introduced by Souza and his coworkers in 2007.

Chaos in non-ideal systems may be controlled by applying the Pyragos method (Zukovic and Cveticanin 2007). An external force control procedure is introduced, where the added force does not change the form of the desired unstable periodic solution, but can stabilize it under certain conditions.

Zukovic and Cveticanin 2009, introduced a new chaos control method based on a function that depends on the velocity of the oscillator vibrations. The control is directed at the oscillator and not at the motor as is usally done.

3. Discussion on Various Models of Non-Ideal Systems

The most often applied non-ideal system is an one-degree of freedom oscillator driven with an electromotor whose energy supply is limited. The model of the system is shown in Fig.1. The model is widely discussed in the long time period since 1904 when was for the first time experimentally investigated by Sommerfeld. The special attention is paid to the resonance case and the Sommerfeld's effect (Nayfeh and Mook 1979). The system is described with two coupled second order differential equations of motion which include the torque-velocity properties of the electromotor during run up and run down.



Fig. 1. Simplified model of a non-ideal system (Nayfeh & Mook 1979)



Fig. 2. Model of non-ideal system with two spur gears. (Balthazar et al 2003)

In recent years, the extention of the simple model of non-ideal system is done. The models have to describe the problems that are closed to real situations encountered in practice.

Souza et al. 2005, presented a system with two spur gears with different diameters and gaps between the teeth and suppose that the motion of one gear is given while the motion of the other is governed by its dynamics (Fig.2). In the ideal case, the driving wheel is supposed to undergo a sinusoidal motion with given constant amplitude and frequency. In non-ideal approach, one considers the motion of the driving wheel to be a function of the system response and a limit energy source is adopted. Analyzing this model the investigation confirm that besides the regular motion the chaotic behaviour is presented in the adopted model.

Zukovic and Cveticanin 2009, introduced the clearance in a non-ideal mechanical system (Fig.3). The system contains an oscillator connected with an unbalanced motor. The connection

of the oscillator to the fixed element is with clearance. Due to the existence of clearance the connecting force between motor and the fixed part of the system is discontinuous but linear. The transient and steady-state motion and also the stability of the system are analyzed.

Dimentberg et al. 1997, considered a one-degree of freedom dynamical system composed of a rigid base flexibly attached to the ground and excited by an unbalanced motor (Fig.4). The support stiffness was switched from a high value to a low value as the rotational speed of the motor increased. Dantas and Balthazar, 2003, extended the investigation by considering of a mechanical system excited by a DC motor with limited supply power which base is leaned on a spring with non-linear properties. Besides the DC motor, a small mass rotates. This electromechanism has the main properties of a machine known as centrifugal vibrator. It is assumed that the resistence of the oscillatory motion is a linear viscous force and the difference between the driving torque of the sorurce of energy (motor) and the resistence torque applied to the rotor is obtained experimentally. The oscillator is with cubic nonlinearity. Dantas et al. 2004, gave the local analysis of the oscillations of a non-ideal and non-linear mechanical model and obtained the conditions for Hopf bifurcation.





Fig. 3. Model of non-ideal system with clearance. (Zukovic & Cveticanin 2009)

Fig. 4. A centrifugal vibrator. (Dantas & Balthazar 2003)

Zukovic and Cveticanin 2007, studied the dynamics of the non-linear system (Fig.5) with nonideal excitation. An unbalanced motor with a strong non-linear structure of cubic type is





Fig. 5. Model of the motor-structure (Zukovic &Cveticanin 2007)

Fig. 6. Non-ideal system with system. shock absorber. (Pusenjak et.al. 2009)

considered. The steady state motions and their stability are studied. The Sommerfeld's effect is proved. For certain parameter values the chaotic motion occurs. The chaos is realized through

period doubling bifurcation. The control of chaotic motion is considered. The parameter values for transforming chaos into periodical motion are given.

In the paper of Pusenjak et al. 2009, a system with a rotating eccentric mass coupled by a nonlinear shock absorber, which is driven by a DC motor as a non-ideal energy source, is considered. The system consists from the driving electromotor, which is coupled by the rotating eccentric mass (Fig.6). The casing of the electromotor is isolated from the base by means of the shock absorber with nonlinear characteristics. The electromotor is considered as non-ideal source of energy with limited power. In this case the non-ideal electromechanical system is nonlinear and the corresponding nonlinear vibration model was used to analyze the no stationary response in the passage through fundamental resonance. The passage through fundamental resonance is conducted for motoring and braking mode of electromotor drive, respectively. The electromotor operates in the motoring mode, when the driving torque produces acceleration.



Fig. 7. Two-degree of freedom oscillatormotor system. (Tsuchida et al.2003)

Fig. 8. Model of the rotor excited with a non-ideal source. (Pusenjak et al. 2009)

Tsuchida et al. 2003, studied the possibilities of existence of regular and irreguloar motions in a non-ideal vibrating problem shown in Fig.7. This vibrating problem consists of a heavy block, a linear elastic spring and a linear damper with viscous damping. On the body of mass a non-ideal motor is placed with a driving rotor with eccentric mass. By means of a linear spring and a damper another heavy block has been attached to the previous one. The vibrating problem is analyzed taking into account the linear but also the nonlinear torque. The passage through resonance is obtained by varying the angular velocity of the DC motor with limited power supply. The maximum amplitudes of vibration before, during and after the passage through resonance, for the linear and nonlinear torque are investigated. The conditions for irregular motion and bifurcation are also considered.

In the paper of Pusenjak et al. 2009, an electromechanical system consisting from rotor system with rotating disc mounted on an elastic shaft which is forced by a non-ideal energy source is presented. The system consists form the driving electromotor coupled by an elastic shaft where a rotating disc is mounted in the midpoint of the weightless shaft (Fig.8). The model is assumed to be linear. The nonstationary motion of the rotor system at the passage through critical speed is analyzed.

In the paper of Krasnopolskaya and Shevts 1990, a crank mechanism coupled to the support point of a simple pendulum is analyzed, which is horizontally excited (Fig.9). The

crank mechanism is connected to a DC motor considered as limited power, providing the hypotheses of construction for a non-ideal system. The steady state behavior of the system was studied.



Fig. 9. Crank mechanism with electro-motorFig. 10. Self-excited non-idealsystem. pendulum non-ideal system (Balthazar et al. 2003)(Pontes et al. 2000)

Wauer and Burle 1997, considered also the dynamics of a flexible slider crank mechanism driven by a non-ideal energy source and analyzed the steady state behavior and the transient startup and rundown, using numerical simulations. Belato et al. 2001, gave the main solution during the fundamental resonance where the increasing of the frequency, lost of stability and the saddle-node bifurcation occur. The solution jumps and their escape from the potential well is shown. For the frequency diminishing the los of solution stability in a flip bifurcation and the followed boundary crisis are discussed. The destruction of the resonance periodic attractor and the appearance of chaotic attractor bounded in a single potential well is described.

Alifov and Frolov 1977, considered a limit power supply in the system with dry friction damper and found that it has a substantial influence on the nature of resonance behavior. The accuracy of result is proved analysing a slip-stick problem by Pontes et al 2000. The model contains a mass block on a non-ideal motor driven belt (Fig.10) and the sliding (stick) between the belt and the mass block. When the belt velocity surpasses a certain value, the dominant sliding (slip) mode and a limit cycle are plotted. The dynamical influence of the motor on the vibrating system is evident by the angular velocity time response and, due to the investigation carried out, it is possible to observe power supply influence on the vibrating system along with non-periodic motions with chaotic characteristics. The same model is studied by Dantas and Balthazar 2004. The interaction between source energy and motion is accomplished through special kind of friction. The stability and instability of the equilibrium point of the system is analysed. The most attention is paid to bifurcation investigation in the system. The Hopf bifurcation is specified for the set of parameters of the oscillating system.





Fig. 11. Non-ideal motor-portal frame model. (Balthazar et al. 2003)

Fig. 12. Non-ideal motor-portal model. (Felix et al. 2005)

Finally, the attention is focused on DC motor-frame interaction, too. The model in Balthazar et al. 2003, is with elastic portal frame and in Felix et al. 2005, the horizontal part is rigid. A simple portal frame of nonlinear behavior excited by a non-ideal motor (Fig.11) is considered by Felix et al. 2005. The simple flexible shear-building portal plane frame foundation excited by an unbalanced rotating machine (non-ideal system) is investigated. An unbalanced direct-current (DC) motor with limited power supply is placed at the midspan of the horizontal beam (part of a flexible portal plane frame). The performance of the saturation phenomenon is discussed and the saturation control of non-ideal vibrating system is considered. The interaction of the non-ideal structure with the saturation controller with quadratic control low and the special occurrences during forward passage through the several resonance states of the system are expressed. Special attention is focused on passage through resonance when the non-ideal system frequency is approximately twice the controller frequency (2:1 internal resonance).

Two rotating unbalanced motors connected with limited power supply mounted on the horizontal beam of a simple portal frame (Fig.12) are analyzed in the paper of Balthazar et al 2003. The phenomena associated with passage through resonance with limited power and also the self-synchronization and synchronization in the system are observed at certain levels of excitation. The self-synchronization problem of a vibrating system composed by two rotating unbalanced motors with limited power supply that is mounted on the horizontal beam of a simple portal frame is also considered.



Fig. 13. Two unbalanced motors on the portal a frame. (Balthazar et al. 2004)



Fig. 14. Four unbalanced DC exciters on flexible structural frame. (Balthazar et al. 2005)

The unbalanced DC motors are placed on the top of the columns (see Balthazar et al. 2004). The energy sources turn each rotor with the significant value of moment of inertia. Each rotor has an unbalanced mass at a distance r from the center of rotation which produces forces of inertia (Fig.13). A particular non-linear phenomenon of self-synchronization has been analyzed through numerical simulation. A flexible portal frame, excited by two non-ideal sources (DC motors of limited power supply) has been considered. The dynamic development of the system through the first and second resonance regions were considered and the Sommerfeld's effect is determined. The presence of the internal resonance between the first vibration modes of portal frame, causing the saturation phenomenon, was considered in this study.

Balthazar et al 2005, considered the system of four unbalanced direct current motors with limited power supply mounted on a flexible structural frame support. In Fig.14. the non-ideal system under investigations consists of a structural portal frame model with lumped masses. A horizontal beam rests on two identical columns, all assumed as linear elastic with negligible mass. Small masses are placed on the eccentric shaft of the four rotors to simulate possible unbalance. When the exciters turn counterclockwise from a horizontal direction the structural frame is capable only of horizontal motion. The influence of the response of the flexible structural frame on the DC motors, the appearance of jump phenomenon and the consequencies of Sommerfeld's effect are analyzed. The stability conditions for synchronous solutions are considered.

5. Conclusions and Remarks

It is of major importance to consider non-ideal energy sources in engineering problems. They act an oscillating system and at the same time experience a reciprocal action from the system.

The following is concluded:

1. Although non-ideal systems may be linear or nonlinear, they exhibit the same phenomena (Kononenko 1980). Jump phenomenon and the increase in power required by a source operating near resonance are manifestations of a non-ideal source called Sommerfeld's effect.

2. The analysis of the linear two mass system driven with a non-ideal source and linear torque, reveals that the model under 1:1 resonance has a regular vibration motion and the maximum amplitude just increases during the passage through the resonance (Tsuchida et al. 2003).

3. For the linear two degree of freedom mass system driven with a non-ideal source and nonlinear torque the vibration is regular before resonance and irregular during and after passage through the resonance 1:1 (Tsuchida et al. 2003).

4. In the problem of a non-ideal system with two degrees of freedom consisting of a dumped nonlinear oscillator coupled to a rotatory part (the example given by Kononeko, 1980) the existence of Hopf bifurcation is proved (Dantas et al. 2003).

5. The acceleration of gravity plays an important role in the search of Hopf bifurcation in non-ideal systems (Dantas et al. 2003). The authors believe that the occurence of Hopf bifurcation is a common property of non-ideal problems related to jump phenomenon. They suggest this way as the best to analyze this behavior of non-ideal system.

6. In the non-ideal one-degree of freedom oscillatory systems with linear viscous friction the asymptotically stable periodic orbits dependent on the gradient of the energy source characteristic exist (Kononenko 1980). Furthermore, in these self-excited systems Hopf bifurcation occurs (Dantas and Balthazar 2004).

7. Self-synchronization of shafts (a well-known nonlinear phenomenon, whereby two or more unbalanced shafts mounted on a common movable structure may rotate synchronously due to interaction via structural vibrations only, even in the absence of any direct kinematics coupling (Balthazar 2005) occurs in non-ideal systems like gas turbine engines with multiple shafts (Dimentberg et al. 2001).

8. In the non-ideal mechanical system with clearance the Sommerfeld's effect and also, for certain parameter values, the chaotic motion dependent on clearance exist (Zukovic and Cveticanin 2009).

9. In the DC motor-frame system with slow increase of power levels the Sommerfeld's effect appears in resonance, as there is not enough power to reach higher speed regimes with lower energy consumption (Felix et al. 2005). Saturation of high frequency low amplitude mode and transference of energy to low frequency high amplitude mode is possible if the non-ideal system frequency is twice of natural frequency of frame i.e. 2:1 internal resonance (Tsuchida et al. 2005).

10. In the non-ideal and nonlinear mechanical systems the irregular motion is evident. The chaos control is possible to be done. The control may be directed at the oscillator (Zukovic and Cveticanin 2009) and not at the motor by controling the voltage of the DC motor as is usually done.

Извод

Динамика неидеалних механичких система: један преглед

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Резиме

Рад представља један преглед литературе која се односи на основне особине неидеалних вибрационих система. Показане су аналитичке и нумеричке методе за анализу ових система. Посматрани су практични примери неидеалних система. Дискутовани су најчешћи феномени. Такође су дискутоване специфичне особине разних модела. Дат је правац даљих истраживања.

Кључне речи: неидеалан механички систем, неидеални извори енергије, линеарне и нелинеарне вибрације, услови стабилности, аналитичке и нумеричке методе

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