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Optimization Design for Graded Porous Tubular Structures

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Abstract

Many natural and advanced biomedical materials are functionally graded porous architectures. Recently, studies on these graded porous structures have been attracted lots of research attentions due to their outstanding strength-to-weight performance. It has been a challenge to analyze this kind of structures mechanically due to the gradation in physical properties within the material. In this article, an optimization technique, which was incorporated with a finite element model, is proposed to design graded porous tubular structures for maximizing their bending strength. The gradation of the physical property of a tubular structure was described mathematically by parametric functions. A parametric finite element model was constructed to mimic the flexural response of a graded porous tubular structure using the finite element code ANSYS. With the concept of damage mechanics, an optimization procedure developed for maximizing the flexural strength and at the same time minimizing the weight was integrated with the finite element code using the ANSYS parametric design language. Using a case example, the effectiveness of the proposed technique was demonstrated. This study provides a method for designing graded porous structures and also contributes to better understanding of functionally graded porous materials.

Keywords: Graded porous; Optimization; Finite element

1. Introduction

The concept of functionally graded materials (FGMs) was first introduced in 1984 by a group of Japanese scientists (Yamanouchi et al. 1990, Koizumi 1993), and then the FGMs have been increasingly used in contemporary engineering applications because of their multi-functional properties. FGMs are non-homogeneous and usually made of two constituents whose volume fractions vary gradually in predefined directions to give a predetermined composition profile, resulting in corresponding changes in the material properties. Their mechanical behaviors have been studied extensively in recent years (Erdogan 1995, Tang et al. 2006, Tang et al. 2007, Wang and Mai 2005, Jin and Batra 1998).

Functionally graded tubular structural members are being increasingly used for space, biomechanical and aeronautical applications because hollow members have high strength and rigidity especially in bending or torsion, and mechanical analysis of functionally graded tubular structural members has been recently an important research topic (Iesan and Scalia 2007, Singh et al. 2006, Elmaimouni 2005, Hematiyan and Doostfatemeh 2007, Arghavan and Hematiyan 2008). In the material of a graded tubular structure, there are pores densely distributed in the core region and sparsely distributed in the near the outer surface region with the porosity change along the radial direction. The porosity reduces both the strength and the density of the material. Therefore, the material structure is constructed in such a way that strength is provided only where it is needed (Low and Che 2006). Intelligent design of functionally graded tubular structures can perform outstanding mechanical properties. Due to heterogeneous nature of the problem, conventional methods in structural design may not be able to provide an optimal solution.

Elmaimouni et al. (Elmaimouni et al. 2005) investigated the wave propagation in hollow infinite FGM cylinders made up of macroscopically inhomogeneous composite materials, wherein the mechanical properties vary continuously from one surface to the other. Andertova et al. (Andertova et al. 2007) investigated the preparation of bio-inert alumina ceramics with gradient of porosity by slip casting to the porous molds, and obtained the physical and mechanical properties of one-component with layers of variable controlled porosity. Valeria et al. (Valeria et al. 2009) presented the effect of porosity on the elastic properties of porcelainized stoneware tiles by a multi-layer model based on the finite element simulation. Zhang et al. (Zhang et al. 2000) carried out a series of static indentation and impact tests at different speeds to study the deformation behavior and damage resistance of reformed bamboo/aluminum laminate composites. Iesan and Scalia (Iesan and Scalia 2007) investigated the problem of extension and bending of functionally graded porous elastic cylinders using the linear theory of isotropic elastic materials. Arghavan and Hematiyan (Arghavan and Hematiyan 2008) performed a mechanical analysis on functionally graded tubular structures of arbitrary shape using finite element method. In their study, the effect of varying thickness and volume fraction of the constituent phases were also investigated.

Markworth et al. (Markworth and Saunders 1995) and Hedia et al. (Hedia and Malmound 2004) developed a simple model for optimizing the composition of a metal/ceramic functionally graded material for either maximizing or minimizing the heat flow through the material. Nadeau et al. (Nadeau and Ferrari 1999) reported the microstructural optimization of a layer which is free of tractions, transversely isotropic, infinite and subjected to a prescribed thermal gradient. In their reports, a reproduction-algorithms-based finite element method is presented for optimizing the porous distribution function with layers of variable controlled porosity, wherein the evolution of stresses of functionally graded porous tubular structure members is constrained in a certain range.

The result of a literature survey indicates that studies on functionally graded porous tubular structures have been mainly focused on the mechanical properties under variable porosity based on experimental or numerical measures. However, there is little attention paid on analyzing the effect of porosity distribution on the mechanical response of a functionally graded tubular structure. There are only few reports on optimal design of this kind of structures. It is essential to understand better the mechanical properties of this kind of structure for the design of high strength-to-weight ratio structures.

2. Computational modeling and optimization

Many natural materials are designed intelligently. Scientists and engineers have been trying to mimic the design of these materials to synthesize advanced aerospace structures and biomedical materials. Bamboo is a typical example of graded porous natural material and is in form of a tubular structure. It has an interesting microstructure and a macrostructure with hierarchical features which contribute to its structural integrity. Specifically, bamboo consists of long and parallel cellulose fibers embedded in a ligneous matrix. The density of the fibers in the cross-section of a bamboo shell varies along its thickness (Zhang et al. 2000, Iyer 2002, Li 2004). Bamboo, moreover, has functional gradient properties in which there is a distribution of Young's modulus across the stem cross-section.

To illustrate the optimization method proposed in this paper, bamboo was chosen as a case example of a functionally graded porous structural member. The method can also be used for designing other man-made graded porous structures such as implants for bone repair. Fig.1 (*a*) shows the cross-section of a bamboo stem which is adopted from Reference (Li 2004). It can be seen the size of the vascular bundle is large and denser in the outermost layer but smaller and sparser in the inner layer, thus the outermost layer has significantly higher mechanical strength than the inner layers. Fig.1 (*b*) is an idealized model for representing the porosity distribution. In this model, R_i and R_o denote the inner radius and the outer radius, respectively, and x denotes the distance from the centreline of the tubular structural member. These pores were assumed to vary in the direction of the thickness according to an unknown radial variation law.

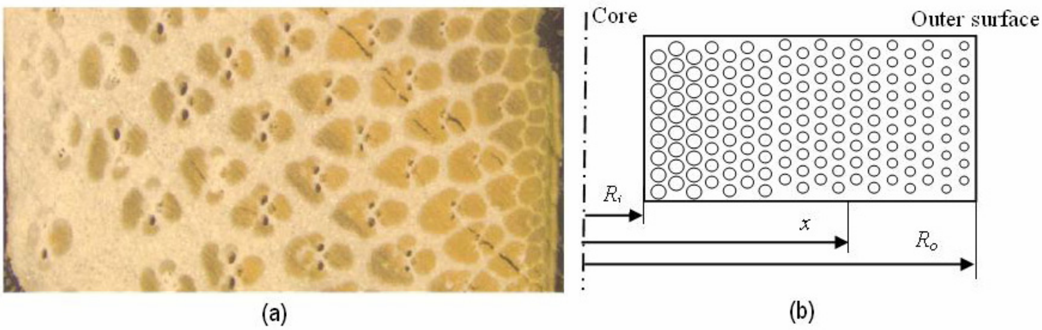

Fig. 1. (*a*) Cross-section of a bamboo stem (Magnification 10X) adopted with permission from Ref (Li 2004); and (*b*) a model of the graded porous material.

The physical and mechanical properties of a tubular structural member are strongly dependent on its porosity distribution (Low and Che 2006). Pores are generally densely distributed in the inner region and sparsely distributed in the outer region and the porosity ratio may be written as a function of x:

$$f(x) = \left(\frac{R_o - x}{R_o - R_i}\right)^{\alpha} \quad \text{for } R_i < x < R_o \tag{1}$$

where α is the power index of relative radius ratio, $f(x)$ is the function of porosity ratio of the tubular structural member.

Fig.2 shows the porosity ratio curves under different values of α. It can be seen that the variation of the porosity distribution depends strongly on the value of α.

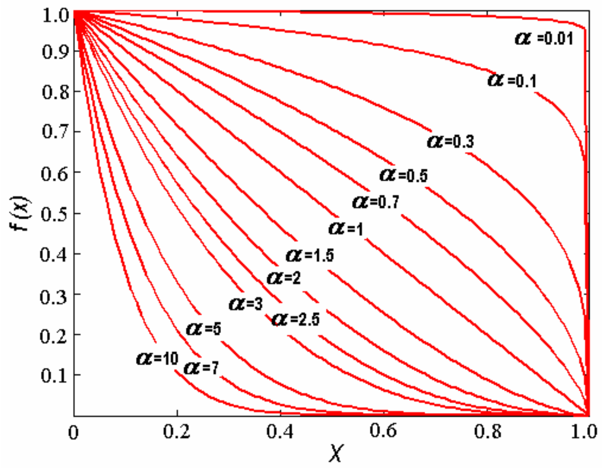

Fig. 2. Curves of porosity ratio of graded porous structures with different values of α.

The density of the structural material at x may be expressed by

$$\rho(x) = \rho_0(1 - f(x))^{\beta} \tag{2}$$

where ρ_o is the density of the outermost layer of the tubular structural member and β is the power index.

Similarly, Young's modulus of the graded porous tubular structural member at x may be expressed by

$$E(x) = E_0(1 - f(x))^{\gamma} \tag{3}$$

where E_0 is Young's modulus of the outermost layer of the tubular structural member and γ is the power index.

Based on the concept of damage mechanics, an intact material element is usually free of voids and cavities. Fig.3 shows a 'damaged' material element which consists of a pore at the central. The effective stress of the 'damaged' material element will be higher than that of the intact one due to the existence of the pore. Damaged materials are often badly stressed when a macro-crack initiates (Tsui et al. 2004). It is expected that the effective stress increases with the porosity of a material.

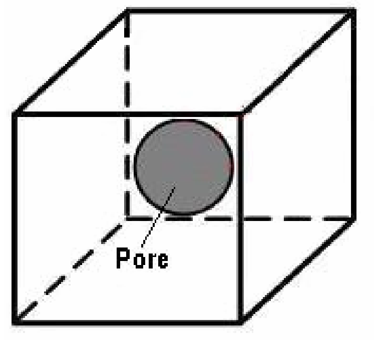

Fig. 3. The pore is considered as a kind of damage to the material.

According to the theory of damage mechanics, the damage variable D can be defined as a function of porosity ratio p as below.

$$D = D(f(x)) = D(p) \tag{4}$$

in which $p = f(x)$.

Moreover, the damage effective Young's modulus $\tilde{E}$ and the damage effective stress $\tilde{\sigma}$ may be written as follows.

$$\begin{aligned} \tilde{E} &= E(p) \\ \tilde{\sigma} &= \frac{\sigma}{1-\tilde{E}/E} \end{aligned} \tag{5}$$

where E and σ are the intact Young's modulus and the stress without damage, respectively.

Some natural materials, like bone and bamboo, possess the characteristics of a graded porous tubular structure. These structures are usually characterized by not only the presence of compositional or other gradients but also the sophisticated mechanical behavior of the FGM components in comparison with conventional materials (Low and Che 2006). They are shaped in such a special manner that strength is provided only where it is needed. The porosity of these functionally graded tubular structures provides evolutional mechanical properties (Berezovski et al. 2003, Pender et al. 2001, Suresh and Mortensen, 1998). The mechanical strength is determined by the porosity and the manner in which the porosity is structured.

By making use of the possibilities inherent in the FGMs coupling with the concept of damage mechanics, the materials can be improved with creation of new functions. On the other hand, some analytical and computational studies (Berezovski et al. 2003) on the effective stresses in FGMs subjected to an applied load show that the utilization of an optimized graded interface between two dissimilar layers can reduce stress significantly.

The evolution stress of a natural material may be taken as the damage effective stress. It may be written as

$$\tilde{\sigma}(x) = \sigma(x)/(\frac{x-R_i}{R_o-R_i})^{\alpha^{\lambda}} \tag{6}$$

where $\tilde{\sigma}(x)$ is the evolution stress, $\sigma(x)$ is the conventional bending stress, and the λ is the power index of the porosity ratio.

To maximize the load bearing ability, it is targeted to design a structure in which the material is loaded with uniform damage effective stress. Therefore, the optimization objective is to maintain the evolution stress being constant in the cross-section of a functionally graded porous tubular structural member, i.e.

$$\tilde{\sigma}(x) = \text{constant} < \sigma_{\mathrm{f}}$$

where σ_{f} is the fracture strength of the tubular structural member.

The objective function can be expressed as

$$\begin{aligned} &\min(\text{Error} = \sum_{i=1}^{n}(\sigma_s(n) - \sigma_s(i))) \\ &S.t. \quad \tilde{\sigma}(n) < \sigma_{\mathrm{f}} \\ &\qquad 0 < \alpha \le 10 \\ &\qquad 0 < \gamma \le 10 \\ &\qquad 0 < \lambda \le 10 \end{aligned} \tag{7}$$

where n represents the number of layers in the cross-section of the tubular structure member, $\sigma_s(n)$ is the von Mises stress in the n^{th} layer, and $\sigma_s(i)$ is the von Mises stress in the i^{th} layer, and $\tilde{\sigma}(n)$ is the evolution stress in the n^{th} layer. The computation is reiterated until the magnitude of error of the objective function is minimized.

In general, the porosity and the density as well as Young's modulus and the porosity ratio are related. It is further proposed that

$$\begin{aligned} \beta &= \alpha^{\xi} \\ \gamma &= \alpha^{\eta} \end{aligned} \tag{8}$$

where ξ and η are the coefficients of the β and γ, respectively. The values of ξ and η are taken as 1 and 0, respectively.

It is a double-variable single objective optimization problem. A special optimization procedure was developed by writing a sub-program using the ANSYS parametric design language. The solution has been achieved when the error of the objective function becomes the smallest. As there are only two unknown parameters, the zero order optimization method of ANSYS has been adopted. The optimization algorithm can only find a local minimum within a specified tolerance. Whether that local minimum is the global minimum is generally unknown. For this reason, a given optimization should be initiated from a number of different starting points.

A multilayer tubular model with functionally graded porosity has been constructed as shown in Fig.4. The geometrical parameters and material properties of the model used are based on References (Iyer 2002, Li 2004). An internodal length of the tubular structure member is 200 mm, the inner radius R_i is 30 mm, the outer radius R_o is 40mm, and the thickness is 10 mm. The finite element model consists of ten layers (n=10) and each layer possesses different material properties. The density and Young's modulus of the outermost layer are taken as 0.66 g/cm^3 and 30 MPa, respectively.

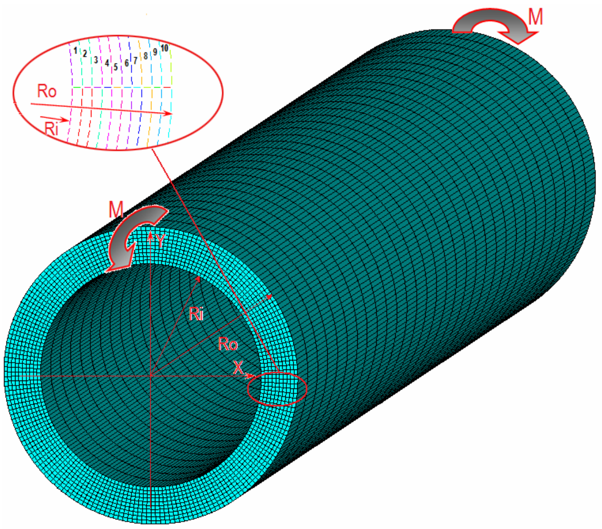

Fig. 4. Multilayered finite element model of the tubular structural member.

The finite element model for the tubular structural member was constructed by using 8-node hexahedral brick elements and a pair of bending moments M was applied to the two ends of the model. As an illustrative example for studying the flexural strength of a tubular structural member, an isotropic linear elastic material model is used. An anisotropic analysis may be required for a combined bending and torsion case.

3. Results and discussion

As shown in Fig.2, the distribution of porosity in the cross-section of the tubular structural member depends strongly on the value of α . The optimization variables are very sensitivity to the change in the porosity ratio. Therefore, the optimization range has been divided into two subranges: (*i*) for $0<\alpha<1$ and (*ii*) for $1<\alpha<10$.

Table 1. Iterative process for optimizing the porosity ratio.

					$0<\alpha<1$						
α	0.01	0.85	0.39	0.54	0.75	0.79	0.79	0.87	0.8169	***0.8169***	0.8179
λ	-0.01	-0.69	-1.04	-0.93	-0.70	-1.41	-1.20	-1.33	-1.218	***-1.229***	-1.218
$\tilde{\sigma}(n)$	243	66.18	1087	165.0	69.8	83.57	72.57	65.55	69.49	***69.86***	69.29
Error	1560	117.26	1925.9	230.1	112.82	73.07	66.9	71.02	63.27	***62.98***	63.46
					$1<\alpha<10$						
α	1.01	8.62	4.49	5.84	1.13	***1.48***	1.66	1.20	1.50	1.48	
λ	0.01	0.46	0.69	0.62	0.12	***0.089***	0.097	0.076	0.107	0.103	
$\tilde{\sigma}(n)$	61.78	6863	6740	11034	59.24	***59.89***	65.53	58.13	61.42	60.62	
Error	111.3	11616	11786	18111	73.38	***9.15***	43.55	59.37	15.21	10.85	

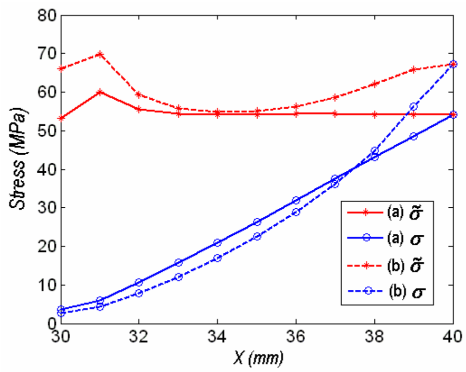

Fig. 5. Variation of $\tilde{\sigma}$ and σ along the radial direction of the tubular structural member.

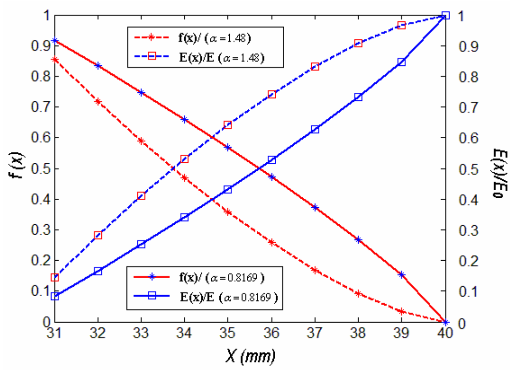

Fig. 6. Optimized porosity ratio and elastic modulus distributions.

The optimization procedure is found to be efficient since the optimal solution can be obtained in a few iterative steps. Table 1 lists the computation results. For part (i), $0<\alpha<1$, the optimal values of α and λ are found to be 0.8169 and -1.229 respectively through eleven iterative steps. For part (*ii*), $1<\alpha<10$, the optimal values of α and λ are found to be 1.48 and 0.089 respectively also through ten iterative steps. Fig.5 shows the distributions of the evolution stress and the conventional bending stress. It can be observed that the optimization results of the two parts are quite similar. However, the error and the stress of part (*i*) were greater than those of part (*ii*). The conventional bending stress of the outermost layer was much greater than that of the inner layer, but the evolution stress was almost constant in any layer. It may imply that the evolution performance of natural functionally graded porous materials, like bamboo or bone, the strength of which is provided only where it is needed. The damage effective stress may be a mechanical stimulus for evolution and micro-damage may stimulate growth of living cells. Fig.6 shows the distributions of the porosity ratio and Young's modulus based on the optimization results. The porosity ratio decreases and Young's modulus increases with the increase in the radius of the tubular structure.

Under pure bending, the bending radius of curvature R of a tubular structural member may be written as $R = L/\theta$, where L is the length of the structural member and θ is the bending angle. Fig.7 shows the change in θ due to different values of R_o/R_i. The smaller the bending angle, the larger is the flexural strength of the structural member. Therefore, the bending strength-to-weight S_b is often used as the performance index of a structural member (Ashby 2005), i.e.

$$S_b = \frac{R}{\bar{\rho}} \tag{9}$$

in which

$$\bar{\rho} = \frac{m_w}{V} = \sum_{i=1}^{n}(\rho_i V_i) \Big/ \sum_{i=1}^{n}(V_i) \tag{10}$$

$\bar{\rho}$ denotes the mean density of tubular structural member, m_w is the total mass of the tubular structural member, V is the total volume of the tubular structural member, ρ_i is the density of the i^{th} layer of the tubular structural member, and V_i is the volume of the i^{th} layer of the tubular structural member.

Fig.8 shows the strength-to-weight ratio S_b under different porosity distributions. When $\alpha = 0$, the tubular structural member is homogeneous having a constant Young's modulus being equal to the average value for the case α =1.48. It can be seen that the graded porous designs often exhibit better mechanical performance in terms of the strength-to-weight ratio. The strength-to-weight ratio also increases with the porosity ratio. However, when α =0.48, the strength-to-weight ratio of the graded porous tubular structural member will be worse than that of a uniform structure member. It may be the result of weakened mechanical performance due to excessive porosity.

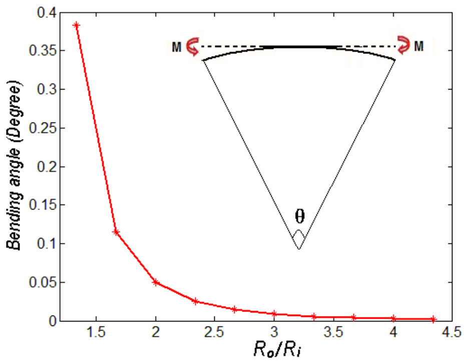

Fig. 7. Bending angle curve versus radius ratio.

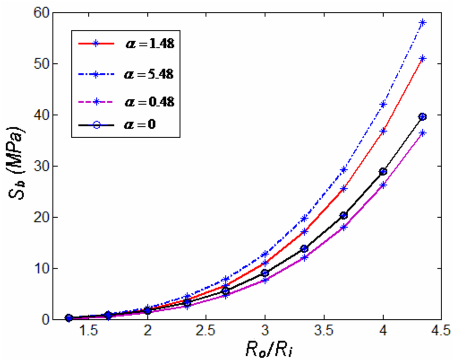

Fig. 8. Strength-to-weight ratios under different designs.

4. Conclusions

Based on the concept of damage mechanics and the parametric finite element modeling technique, a multi-variables single objective optimization method has been proposed for designing graded porous tubular structural members in this study. By running a case example, the method has been found to be efficient in arriving at the optimal solution in a few iterations. With appropriate modification, the proposed method may be used for designing other kinds of graded porous structures or materials. Moreover, this study also paves the way for future biomechanical analysis of graded porous natural materials.

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