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Finite Element Model for Steady-State Rolling Tire Analysis

N. Korunović[1], M. Trajanović[2], M. Stojković[3]

Mechanical Engineering Faculty, University of Niš,
Aleksandra Medvedeva 14, 18000 Niš, Serbia
[1] nikola.korunovic@masfak.ni.ac.yu
[2] traja@masfak.ni.ac.yu
[3] miloss@masfak.ni.ac.yu

Abstract:

The paper describes a finite element model for steady state rolling tire analysis. The model, which is improved in comparison to earlier models created by the authors, enables a larger number of analyses to be conducted in less time and with greater accuracy. The descriptions of model structure and model building procedure are given, followed by the algorithm according to which the improvement of future tire performance may be achieved. It includes the analyses of inflation, vertical loading and steady state rolling. Example analysis of each of mentioned types and typical results are given in the last part of the paper.

Keywords: finite element analysis (FEA), tire design, steady-state rolling analysis

Introduction

Tires are designed to satisfy required standards and to have as good driving performance as possible. Tire performance is evaluated trough a set of criteria, which are most frequently based on physical tire testing, conducted either in laboratory or on test vehicles.

During driving, the tire finds itself in the state of rolling. In general, this state is not stationary, but is constantly changing over time. Nevertheless, the duration of transient states is very short comparing to the time which a tire spends in completely or nearly steady rolling state. After a driver-induced change of an input parameter, like steering wheel angle or breaking moment, steady state is usually reached after a few rotations of the tire, while in laboratory it gets achieved after one or two tire rotations. In other words, tire related phenomena experienced by the driver are mostly related to its behavior under steady state rolling.

Steady state rolling of the tire, as defined by Kelsey (2000), is considered to be rolling where the set of all measurable tire values - axle loads, internal stresses and strains, deformations, footprint shears and footprint shapes - as viewed from a reference frame translating with the tire axes, remains constant over time. Steady state rolling state may, in addition to vertical load, include the action of side forces that arise during cornering, driving or breaking torque, or any combination of those.

Rolling tire behavior may successfully be analyzed using **finite element (FE) models**. For this purpose an optimal approach is usually found in **mixed Eulerian-Lagrangian**

formulation. It assumes that the kinematics of rolling problem is defined in terms of a coordinate frame that moves together with the ground motion of the tire, in which rigid body rotation is described in Eulerian (spatial) manner and the deformation in Lagrangian (material) manner. In this way time dependence is removed, and the problem may be treated as purely spatially dependent. From practical point of view, it is very convenient as it requires less computational resources and thus less time for a typical analysis. A short overview of published papers on steady state rolling tire analysis may be found in earlier paper by Korunović et al. (2007a).

In this paper a finite element model developed by the authors is shown, which is improved in comparison to earlier ones described in paper by Korunović et al. (2007a), considering its simplicity, creation time, accuracy and suitability for analysis of rolling tires. In the first part of the paper the finite element model is described. The second part describes the analyses conducted using this model, which aim at proving its practical value in tire design.

Finite element tire model for steady state analysis

FE tire models for steady state rolling analysis, described in this chapter, are created on basis of 2D parametric CAD model of tire profile, developed by the authors (shown in Fig. 1), which is going to be described in more details elsewhere. This model is based on 3D CAD model developed earlier, described by Stojković et al. (2003, 2005) and may easily be combined with it, in cases when tire tread is going to be modeled in detail. The advantage of the new model is that, besides basic parameterized geometry of tire profile, it contains a parameterized network of lines and points, which represent the basis for creation of FE mesh (Fig. 2).

FE model described here may be used for all tire types whose profile is constructed in the same or in the similar way. If an essentially different type of tire is to be analyzed, a new CAD model needs to be created, by remaking the current one. Very important acceleration in the analysis process is achieved by writing the code that translates the set of points from the CAD model to set of nodes of FE model. In that way the FE model, that represents one tire type, needs to be created only once. After each change in the geometry of CAD model, a new set of nodes is created from CAD model data using the translation code. Those nodes are automatically used in a new analysis, to form the new FE model in its very beginning. In such way, it becomes possible to create a large number of different FE models in very short time. This enables a much more efficient examination of the influence of changes in tire design parameters to its rolling behavior.

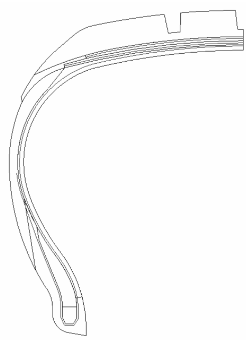


Fig. 1. 2D parametric CAD model of an existing type of tire, designated as 165/70 R13, which provides for easy change of tire profile and structural components.

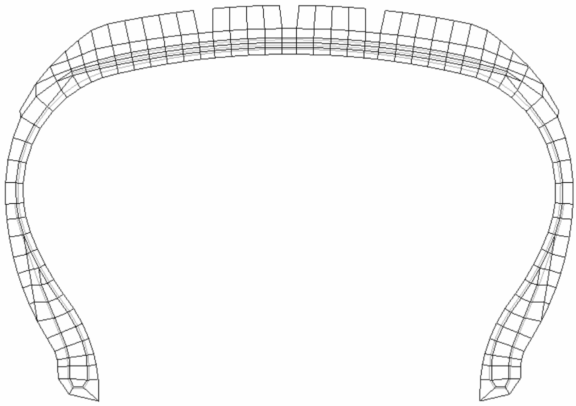


Fig. 2. Following dimensional change of tire profile and structural components, parameterized lines and points of the CAD model allocate accordingly, to form the basis for mapped FE mesh.

Fig. 3. represents the axisymmetric FE tire model, based on 2D CAD model described in this chapter. The model is created in FE preprocessor, using geometry entities exported from CAD model (Fig. 4).

The surface of tire profile is segmented using exported curves, in order to form a finite element mesh. The elements are then grouped to represent the purely rubber components of tire, like tread, sidewall or bead filler. Composite structural components of tire, carcass and belts, are created by embedding of surface elements in volume ones. Inside of surface elements rebar layers are defined, that represent steel or rayon cords. The definition of one rebar layer contains cord area, distance between cords, cord angle and cord material.

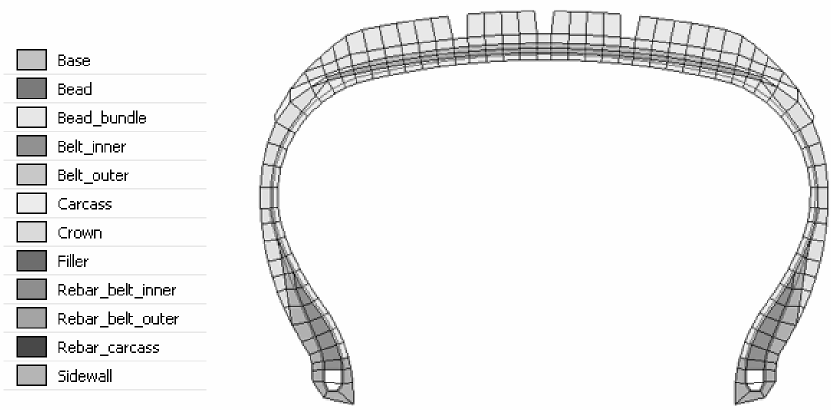


Fig. 3. Axisymmetric FE tire model with structural components shown.

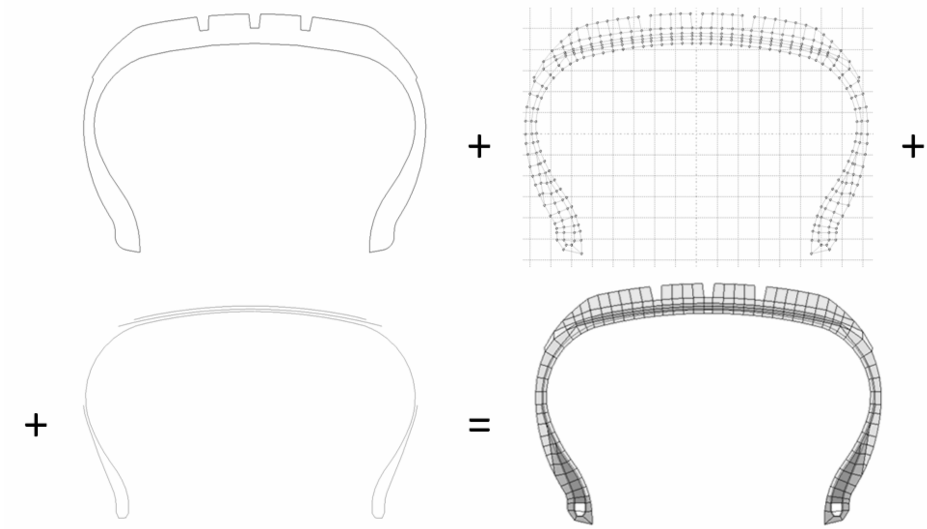


Fig. 4. The creation of FE model based on curves and surfaces exported from CAD model.

Bead wire is modeled as isotropic material, in contrast to earlier models where it was also modeled using surface elements with rebar layers, as described in papers by Korunović et al. (2007b, 2007a). The rim, which has earlier been modeled as rigid surface, is substituted by rigid supports. Those two changes, introduced after positive results of sensitivity studies had been obtained, simplify the model and shorten analysis time.

As the material model for rubber components, Mooney-Rivlin form has been selected, its coefficients being taken from earlier models (determined as described in paper on rubber modeling for FEA by Korunović, 2004). Steel and rayon have been modeled as linearly elastic materials.

In FE model following types of finite elements have been used:

- axisymmetric hybrid elements with twist (with four nodes and three nodes where needed), to model purely rubber structural components,
- axisymmetric surface elements with twist (two nodes) – for carcass and belts modeling,
- axisymmetric solid elements with four nodes – to model the bead wire.

Finite elements with twist enable capturing of deformation outside the symmetry plane, namely twisting around the symmetry axis which is constant in circumferential direction. This is a very useful option in tire analysis, as tire belts are not symmetric around wheel plane and thus, when the tire is inflated, some twisting around tire axis occurs.

After FE model is created, it is used for analysis of tire inflation. Then 3D FE tire model is created by rotation of axisymmetric model around tire axis. Dense mesh needs to be created only in the vicinity of tire footprint. Mesh density in meridional direction is determined by mesh density of axisymmetric model, while mesh density in circumferential direction is determined by user input and it is defined via selection of rotation angle increments in different model zones (Fig. 5).

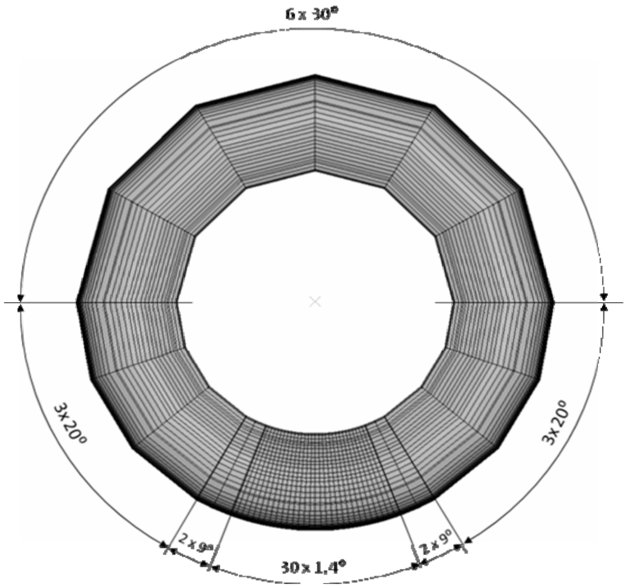


Fig. 5. One possible incrementation in meridional direction, during creation of 3D FE tire model by rotation of axisymmetric model. The mesh density is large only in the vicinity of tire footprint, which is possible when mixed Eulerian-Lagrangian formulation is used.

3D FE model, created using increments defined in Fig. 5, is shown in Fig. 6, while Fig. 7 shows its inner structure.

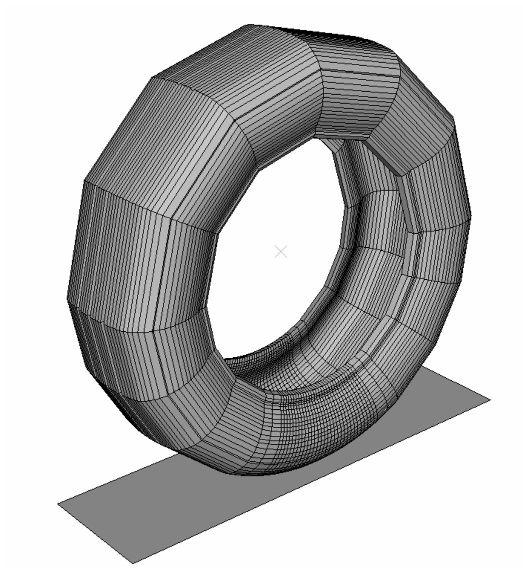


Fig. 6. 3D tire model which is used to analyze tire inflation, vertical loading and steady state rolling. The rim is substituted by rigid supports, while ground surface is modeled as rigid surface.

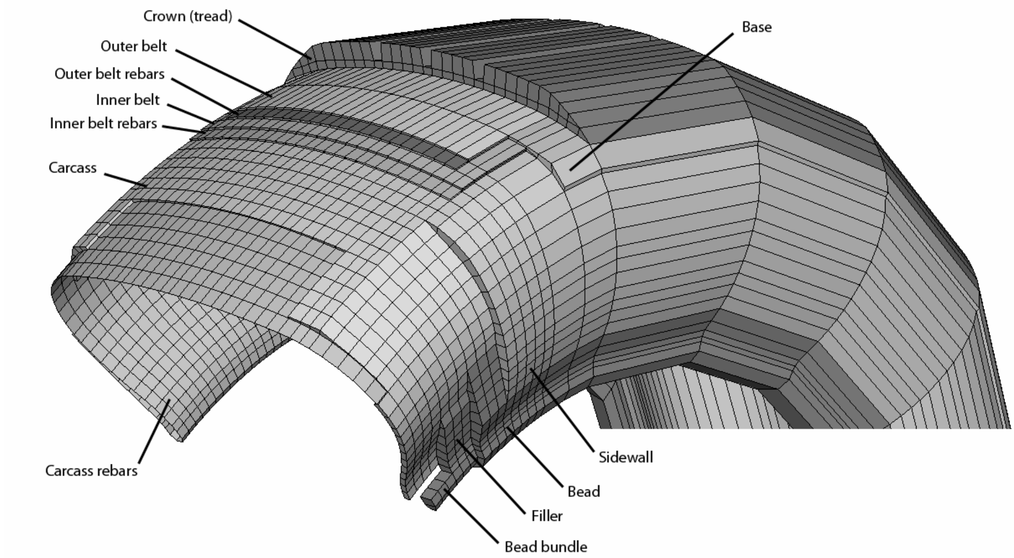


Fig. 7. Segments of 3D tire model which are created as a close approximation of its structural components.

During rotation of axisymmetric model 3D finite elements are created, which represent the equivalents to corresponding axisymmetric ones. Thus, 3D model is composed of:

- hybrid elements with eight or six nodes, which are used to model rubber,
- surface elements with four nodes – for modeling of carcass,
- solid elements with eight nodes – used to model the bead wire.

Using the 3D model created in such way, the following analyses have been conducted:

- inflation analysis,
- analysis of vertically loaded tire,
- straight line rolling under the action of driving or breaking torque,
- straight line rolling analysis in fine increments, to find the angular velocity of free rolling,
- free-rolling cornering analysis.

The flexibility of FE model, which is achieved by the introduction of the code that translates the CAD model to FE model, enables the studies to be preformed, which as a goal have the improvement of tire performance, especially the improvement of its maneuverability and acceleration-braking potential. One possible algorithm, which may be used for such studies, is shown in Fig. 8.

1. Analyses and results

This chapter describes the analyses conducted on FE tire models described in previous part of the paper. The sequence of the analyses corresponds to the algorithm shown in Fig. 8.

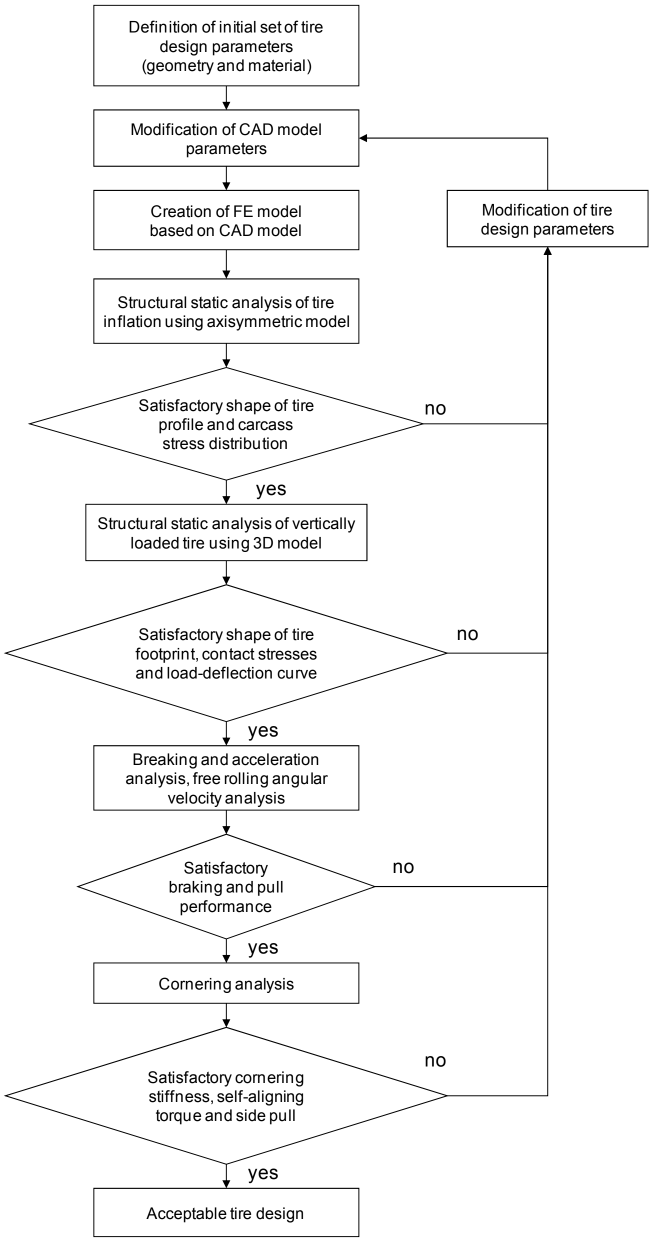


Fig. 8. An algorithm according to which, using FE model, tire performance may be improved.

1.1. Inflation analysis

Inflation analysis on axisymmetric model is primarily performed because of:

- model integrity checking,
- finding the deformed shape of the model (Fig. 9),
- finding the stresses in carcass, belts and rubber components (Fig. 10),
- setting up the basis for creation of 3D FE model.

Stress distribution in structural components of the tire may also be used as an early indicator of some aspects of rolling tire performance, like maneuverability or durability (Cho, 2002, Ridha, 1994).

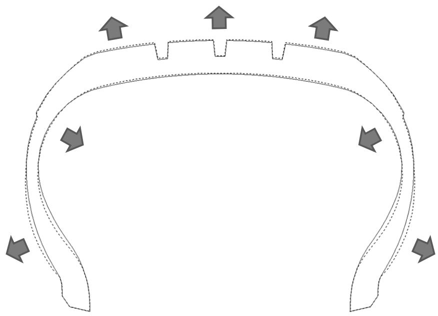


Fig. 9. Deformed shape of tire profile, after the inflation to the pressure of 2 bar (0,2 N/mm^2). The upper parts of the sidewall, in the vicinity of tire shoulders, have deflected towards the inside of the tire, while the lower parts, closer to beads, have moved towards the outside. Such tendency is in accordance with observed behavior of tires constructed according to RCOT (Rolling Contour Optimization Theory), as noted by Ridha (1994). According to this theory, the geometry of carcass profile is chosen in a way which enables its equal distribution in a rolling tire, which in turn leads to lessening of rolling resistance.

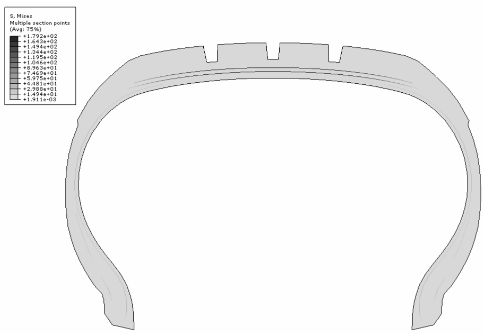


Fig. 10. Stresses in the tire inflated to 2 bars, obtained as a result of axisymmetric model analysis. Detailed results on tire inflation analysis performed by the authors may be found in paper by Korunović, 2007b.

1.2. Analysis of vertically loaded tire

Inflation analysis of axisymmetric model is followed by 3D static analysis of vertically loaded tire. The analysis consists of several steps. In step zero the 3D model is created, by rotation of axisymmetric model around tire axis. In step one tire is inflated, while in step two ground surface gets gradually moved towards tire axis, until it reaches the goal value (which in given example equals to 20mm). In step three equilibrium is reached, with constant force acting on surface. This step is needed as the basis for subsequent steady state rolling analysis. Basic results of vertically loaded tire analysis are shown in Fig. 11.

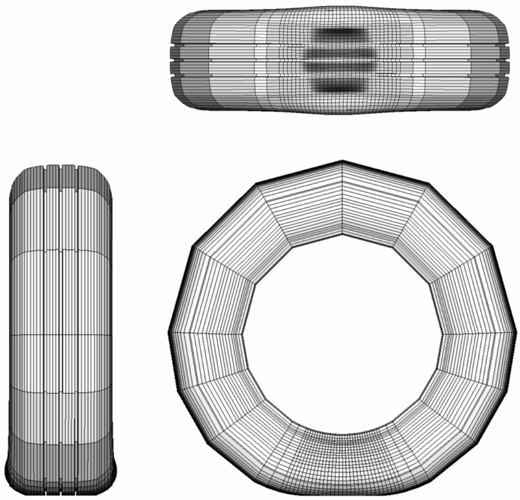


Fig. 11. Deformed shape of 3D tire model inflated to 2 bars, under action of vertical load of 2750N (which is equal to the load that a tire of given dimensions bears when mounted on a small passenger car with four passengers inside).

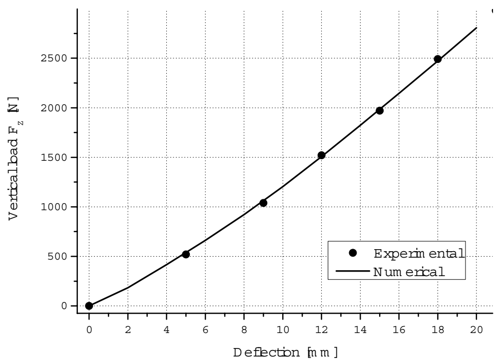


Fig. 12. Comparison of load-deflection curves obtained numerically and experimentally. The numerical results shown here, are the closest to experimental ones considering all previous models built by the authors (Korunović 2003, Korunović 2007b, Korunović 2007a).

1.3. Steady state rolling analysis

The analyses described in previous chapters consider statically loaded tire. Nevertheless, they need to be conducted before steady state rolling analysis is performed. Zero load step of vertically loaded tire analysis is also very important for state rolling analysis, because it is in this step when streamlines for material movement during rotation are formed, which are required by analysis formulation. Some basic concepts of steady state rolling analysis and short review of relevant papers are given in paper by Korunović, Trajanović and Stojković (2007a).

Rolling analysis results are usually given as functions of tire forces and moments. Resulting forces and moments are going to be represented in tire coordinate system according to SAE, which is, for example, shown in monograph by Gent et al. (2005).

In general, tire forces and moments represent complex non-linear functions of tire usage variables that are established by drivers' inputs and vehicle responses. Tire usage variables are: tire load (characterized in terms of normal force F_Z), loaded radius, R_l, slip angle, α, inclination angle, γ, wheel torque, T and slip ratio, SR. Definitions of those variables may be find in monograph by Gent et al., 2005. Wheel torque, when greater than zero, is called driving torque and when less than zero, braking torque. Slip ratio is calculated according to the formula:

$$SR = \frac{\Omega}{\Omega_0} \tag{1}$$

where Ω_0 stands for angular velocity at free rolling, i.e. when T equals zero.

The results of steady state rolling analyses are highly dependent on the definition of friction between tire and ground. Rubber friction is a very complex phenomenon and its description still represents an attractive scientific challenge. One of the commonly used approaches to description of tire rubber friction the use of phenomenological models, which are based on laboratory tests of rubber specimens. According to this approach, the coefficient of friction for contact pair rubber-ground surface is represented as a function of one or more variables, usually contact pressure, sliding velocity and/or temperature. In the analyses described in this paper the coefficient of friction is described in two ways: as a constant value or as a set of tabular values, as shown in Table 1.

μ		**p (kPa)**			
		220	**230**	**440**	**550**
v_s [m/s]	**0.010**	0.510	0.380	0.312	0.298
	0.018	0.515	0.392	0.337	0.325
	0.032	0.528	0.412	0.364	0.354
	0.056	0.550	0.438	0.391	0.382
	0.100	0.590	0.477	0.428	0.418
	0.178	0.644	0.518	0.461	0.444
	0.316	0.690	0.557	0.490	0.460
	0.562	0.726	0.586	0.513	0.473
	1.000	0.736	0.602	0.522	0.476
	1.778	0.713	0.597	0.502	0.469
	3.162	0.652	0.577	0.492	0.459
	5.623	0.619	0.541	0.478	0.441
	10.000	0.572	0.503	0.451	0.413
	17.783	0.543	0.464	0.419	0.387
	31.623	0.522	0.437	0.384	0.354

Table 1. Friction coefficient values of a tire tread compound. The values are extrapolated from friction coefficient curves, shown in paper by Guo et al. (2004), which are obtained by laboratory testing of tread rubber specimens on asphalt surface. In the picture *p* stands for contact pressure and V_S for sliding velocity.

1.3.1. Breaking and acceleration during straight line rolling

According to the algorithm shown in Fig. 8, as the first stage in steady state rolling analysis straight line rolling analysis is conducted, in which the angular velocity of the tire is changed in given increments while translational speed of the tire is held constant. In this way the value of slip ratio is changed from the negative one, which provides for full slipping in contact during braking (when all points in contact with the ground are slipping), to the positive one, which provides for full slipping during acceleration (Fig. 13).

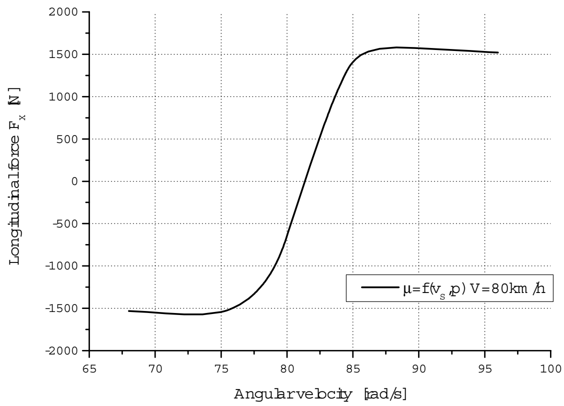


Fig. 13. The correlation between longitudinal force and angular speed, in the range of angular velocities that covers the cases from full slipping during braking to full slipping during acceleration. The coefficient of friction between the tire and ground surface is defined as a tabular function of contact pressure and slip velocity (see Table 1).

Fig. 14 shows the deformed shape and contact pressure distribution at tire footprint in the case of full braking, at the moment when slip ratio has the smallest value.

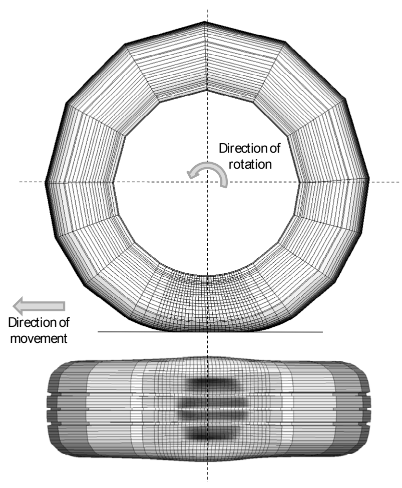


Fig. 14. Deformed shape and contact pressure distribution at full braking.

In order for tire behavior at various speeds and various definitions of friction to be compared, it is necessary to substitute the angular velocity on abscise of Fig. 13 with slip ratio. The definition of slip ratio (1) requires the angular velocity at free rolling to be known, which is found by fine incrementation around the value obtained from straight line rolling analysis, as shown in Fig. 15. One such comparison is shown in Fig. 16.

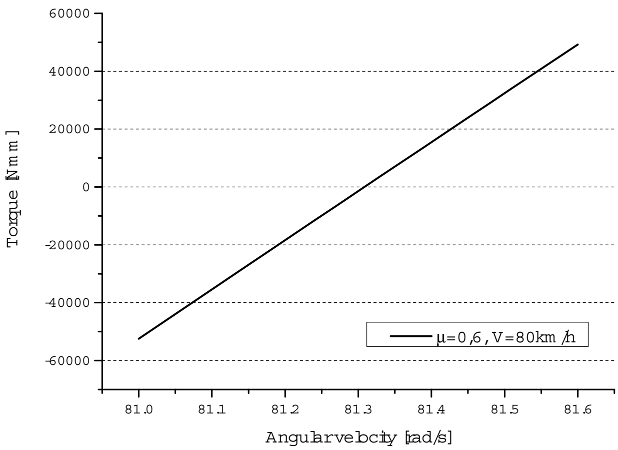


Fig. 15. Determination of angular velocity at free rolling by fine incrementation around initial value, obtained from straight line rolling analysis.

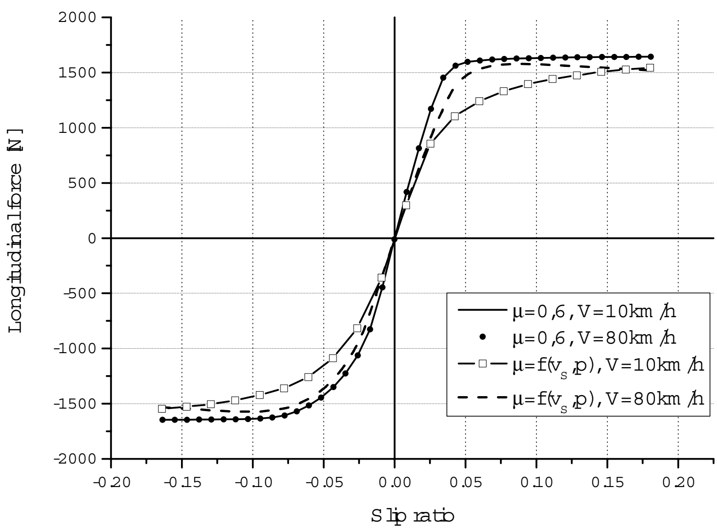


Fig. 16. Longitudinal force, which acts between the tire and ground surface, obtained from four different analyses, where different combination of friction definitions and tire velocities have been used. The left side of the graphic (where slip ratio < 0) corresponds to braking, while the right side (where slip ratio > 0) corresponds to acceleration.

In the first of the examined cases, tire velocity is equal to 10km/h, while friction coefficient is constant and equal to 0.6. For absolute values of slip ratio larger then approximately 0.1, the curve is horizontal, which means that full slipping has been achieved. The value of longitudinal force in this case equals 1650N, which is the value equal to the product of vertical force and friction coefficient. Very similar curve is obtained in the second case, when tire speed is enlarged to 80km/h. In the third case, speed is once again set to 10km/h, while the definition of friction is changed to tabular. In this case, for given slip ratio the corresponding curve never becomes horizontal, as the maximum friction potential of rubber material is not reached, i.e. the values of slip velocity are too small. In the fourth case, friction is still defined as tabular, while tire velocity is raised to 80km/h. Then, for the absolute value of slip ratio of about 0.09 the maximal value of longitudinal force is reached, while for larger values is gradually falls. Those results correspond to experimental ones, which may be found in literature, like, for example in Monograph by Gent et al. (2005).

1.3.2. Free-rolling cornering analysis

In order for tire maneuverability to be assessed, breaking-to-acceleration analysis is followed by cornering analysis. In this case, a free-rolling cornering analysis has been conducted. In such an analysis, after equilibrium state at straight line free rolling is established, the slip angle gets incrementally changed, until it reaches the predefined value. In this way a series of steady state rolling solutions at different slip angles is obtained.

The tire has been inflated to the pressure of 2 bars and vertically loaded by 2750N force. Then it was brought to the state of free rolling at the straight line. Slip angle has then been gradually changed from zero to 8º and then from zero to -8º, in increments of 0.5º. Tire response to slip angles from -8º to 8º has then been created, which combines the results of those two analyses. Slip angle sign convention according to SAE is shown in Fig. 17. When slip angle is positive, side force that acts on tire is negative.

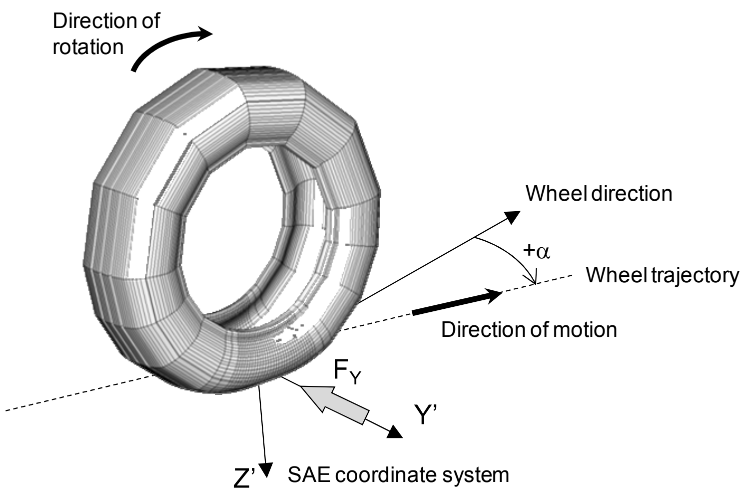


Fig. 17. FE tire model in SAE coordinate system, in the left turn (positive slip angle).

Fig. 18 shows numerically obtained footprint shape and contact stress distribution for a cornering tire in left turn, moving at 80 km/h and at slip angle of 8°. The trapezoidal shape of footprint is similar to experimentally obtained one, shown by Gent et al. (2005), although the two tires are not of the same design. It may also be seen that the contact pressure is higher on the right side of the footprint, which lies on the outside of the turn.

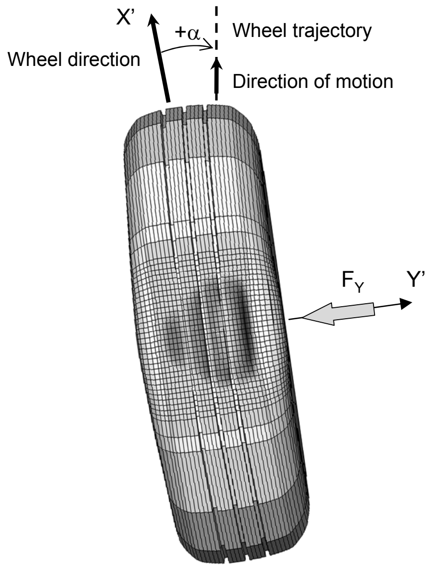


Fig. 18. Deformed shape of FE tire model and contact pressure distribution in left turn, The picture has been inverted like a mirror image, in order to simulate the view from above, i.e. "trough the tire".

In the rest of the paper results of cornering analyses are presented as slip angle vs. side force (Fig. 19), or slip angle vs. self-aligning torque (Fig. 20) curves. Two mentioned pictures relate to tire model with variable coefficient of friction moving at 80km/h. It can be seen from Fig. 19 that for slip angles between 7° and 8° the maximum friction potential of the tire has been reached.

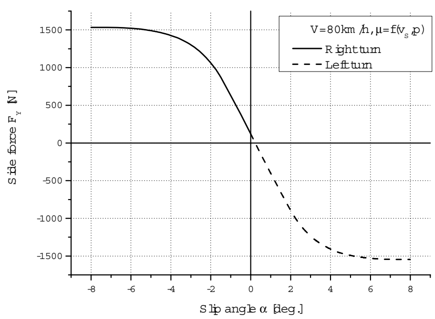


Fig. 19. Correlation between slip angle and side force obtained by free-rolling cornering analysis at tire speed of 80km/h, for FE model with variable coefficient of friction. The right side of the graphic, where values of slip angle according to SAE coordinate system are positive, corresponds to left turn, while the left side corresponds to right turn.

Self-aligning torque (Fig. 20), which is the source of the resistance that the driver feels on the steering wheel, for given tire moving at 80km/h, reaches its extreme values at slip angles of around +2.2 and -1,8°. At greater absolute values of slip angle, absolute value of self-aligning torque begins to fall down. In the case of right turn, for values of slip angle greater than 7° it even changes the sign, which gives the driver a feeling similar do driving on ice.

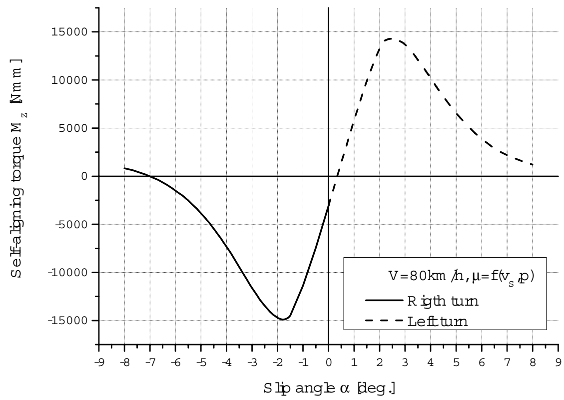


Fig. 20. Correlation between self-aligning torque and slip angle, for tire model with variable friction coefficient moving at 80km/h.

It may be seen that, for zero slip angle, the values of side force and self-aligning torque do not also equal zero. This is the consequence of anti-symmetry of tire belts in relation to wheel plane. When those non-zero force and moment values are not in balance on a pair of tires which lie on the same axle, they make the vehicle move on a large circle instead of the straight line, i.e. pull to one side, which is quite unpleasant for the driver. Details about this effect may be found in literature, for example in monograph by Gent et al (2005).

The importance of friction definition in FE model may be seen from Fig. 21. and Fig. 22.

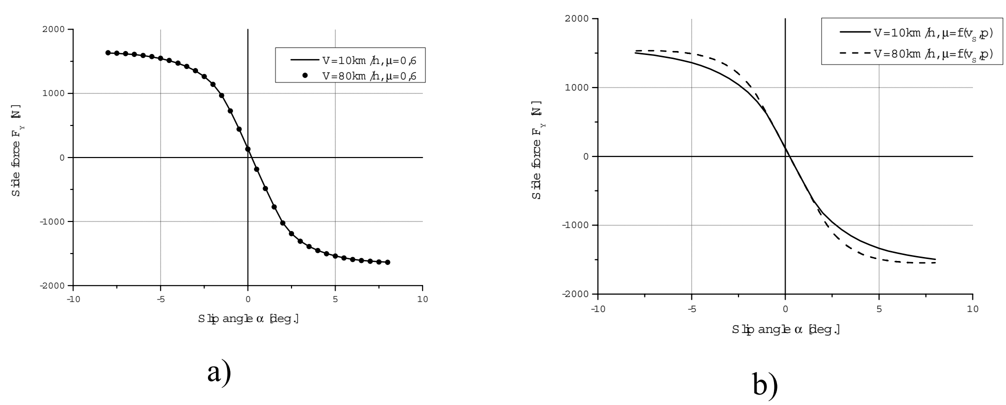
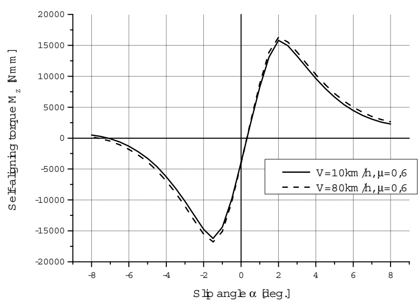
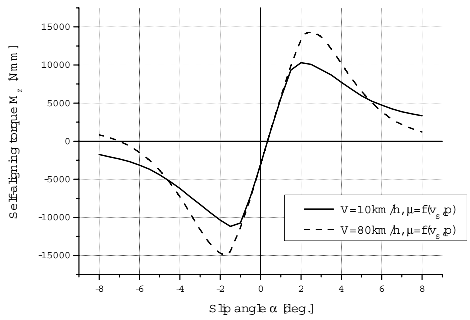


Fig. 21. The comparison between side force values for FE models with: a) variable coefficient of friction and b) constant coefficient of friction, at speeds of 10 and 80 km/h. While in the first case the difference between model responses is very small, in second it gets notable, as the coefficient of friction of tread rubber depends on sliding velocity.



a)



b)

Fig. 22. The comparison between the values of self-aligning torque during cornering for the models with: a) variable coefficient of friction and b) constant coefficient of friction, at speeds of 10 and 80 km/h. In both cases there exists a difference between tire responses, but in the second case it is much more notable.

2. Conclusions

In this paper, the description of a finite element model, which may be used for steady state rolling tire analysis, in order to speed up and improve tire design process, has been given. An algorithm which defines the sequence of necessary analysis steps, from geometry definition to cornering analysis, has also been shown.

The improvements which this model brings, in comparison to earlier models of the same authors, primarily relate to its accuracy, comprehensiveness and flexibility. Particular convenience of this model lies in automatic translation of CAD model to FE one, which enables a large number of design changes to be analyzed in time that is much shorter than before. This makes it possible to conduct the studies, which as the goal have the optimization of tire design parameters.

The verification of the accuracy of the results is expected to come after their comparison to experimental ones. It is probable that the model may further be improved by introduction of viscoelastic material model, which however requires that the new laboratory tests of rubber specimens are conducted.

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