







LOAD-CARRYING CAPACITY OF PIVOTED CURVED SLIDER BEARINGS SUBJECT TO SURFACE ROUGHNESS PATTERN AND NON-NEWTONIAN FLUID FLOW BEHAVIOR

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Abstract

The present paper analyzes the Rabinowitsch fluid (RF) behavior on a pivoted curved slider with surface roughness (SR) with the aid of the perturbation technique. In order to account for the random roughness of the bearing surfaces, a stochastic random variable with a nonzero mean, variance, and skewness was taken into consideration. The averaged Reynolds equation (RE) was developed using a stochastic random variable. The load carrying capacity (LCC) and centre of pressure (COP) were obtained by solving the relevant stochastically averaged RE with appropriate boundary conditions. Bearing performance was observed to suffer as a result of transverse SR; nevertheless, the bearing system's efficiency can be increased in the case of negatively-skewed SR. It was demonstrated that, unlike a traditional lubricant, the bearing can withstand a LCC even in the absence of flow. The outcomes of the current paper can be helpful in various modern mathematical physics engineering applications.

Keywords: Pivoted curved slider bearing, surfaces roughness, non-Newtonian fluid, perturbation technique, Rabinowitsch fluid model.

1. Introduction

Commercial lubricants are optimized to accommodate different machine systems implemented with various additives. These characteristics of complex fluids exhibit non-Newtonian (n-N) fluids behavior after incorporating a few other additives in fluids. We can improve the performance of bearings and increase their life cycle using n-N fluids in industrial and scientific applications (Cameron and Mc Ettles 1981; Pinkus and Sternlicht 1961). In the last few decades, researchers have completed a decent amount of work to build the productivity of settling properties of n-N fluids (Hsu 1967). The rheological impacts of n-N fluids in light of various

fluid models, for example, power law, couple stress, micropolar, Bingham, etc. have been investigated in the theoretical part of the research. Using different fluid models of n-N fluid, several researchers have investigated the effect of n-N fluids. For slider bearing, Kapur (1969) and Hashimoto (1994) have investigated the effect of n-N fluids and inertial flow regimes considering high-speed bearing. They calculated that bearing LCC and friction forces decrease due to the n-N effect. Wada and Hayashi (1971) have explored the performance journal bearing with the n-N fluid and they deduced that the film pressure, LCC, and friction forces are decreased for pseudoplastic fluids (PPF), which was proven by the experimental analysis. RF is one of the n-N fluid models which was used by several researchers made to an assumption for the characteristics and performance of hydrodynamic film bearings. Moreover, Singh (2013) has investigated the impact of RF between pivoted slider bearings while Lin et al. (2016) have derived the modified RE for the slider bearing lubricated with n-N fluid.

In most theoretical studies, the feature of bearings lubrication is centered on the perfectly smooth surfaces of bearings, whereas, in practice, all bearings' surfaces are rough. The impact of SR assumes a huge part in the improvement of science and innovation of tribology. Christensen (1969) was the first to employ stochastic theory to study the effect of SR on two moving surfaces. He has considered journal bearing and calculated that bearing suffers from azimuthal SR case. In the context of SR, several researchers have used Christensen's stochastic theory. For example, Naduvanamani et al. (2016) have studied the performance of circular stepped rough plates. They observed that bearing performance improved for dilatant fluid (DF) but in the case of PPF the bearing performance suffered. The performance of pivoted curved slider bearing based on a SR and a couple of stress fluids have been investigated by Naduvanamani et al. They observed that couple of stress fluids improves the bearing performance with SR. Recently, for conical bearing geometry, Rao and Rahul (2019) have investigated the impact of SR in the context of RFM. They have studied the effect of SR with RF with viscosity variation for circular stepped plates. They have observed that bearings performance decreases with radial roughness patterns and PPF.

In the present paper, the problem of a pivoted curved slider bearing using an RF model as a lubricant with SR effects has been analyzed. The modified RE is derived and its general solution for the pivoted curved slider bearing is shown. The current study is an extension of the work of Christensen's stochastic rough model with the variations of the different formations of RE to analyze the different types of SR. Analytically, the analysis of LCC and COP has been presented with a random variable with a non-zero mean, variance, and skewness.

2. Problem formulation

RF model has the following nonlinear cubic power relationship between shear stress-strain and is taken to be (Wada and Hayashi, 1971):

$$\tilde{\tau}_{xy} = \tilde{\mu} \frac{\partial \tilde{u}}{\partial y} - \tilde{k} \tau_{xy}^3 \quad (1)$$

where $\tilde{\mu}$ and \tilde{k} denotes zero shear rate viscosity and non-linear factor discretizes the n-N fluid. According to the RF model, $\tilde{k} > 0$ the lubricant will be PPF and if $\tilde{k} < 0$ the lubricant will be DF, for Newtonian fluid (NF) the value of the nonlinear parameter is $\tilde{k} = 0$. A schematic diagram of pivoted curved slider bearing with SR is demonstrated in Fig. 1. The bottom surface moves at an even speed \tilde{U} , while the curved surface is at rest. The RF is used as a lubricant that lubricates the lower and upper surfaces of bearings.

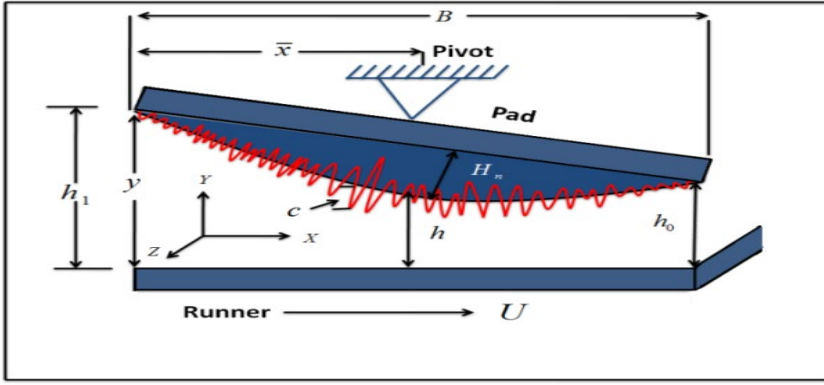


Fig. 1. Physical diagram of rough pivoted curved slider bearing

We have considered incompressible fluid, absence of body forces and body couples, and thin film (Cameron and Mc Ettles, 1981; Pinkus and Sternlich, 1961) between moving surface and fixed curved surface, which is applicable for hydrodynamic lubrication. The governing equation and boundary condition for pivoted curved slider bearing are given as follows:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \quad (2)$$

$$\frac{\partial \tilde{p}}{\partial \tilde{x}} = \frac{\partial \tilde{\tau}_{xy}}{\partial \tilde{y}} \quad (3)$$

$$\frac{\partial \tilde{p}}{\partial \tilde{y}} = 0 \quad (4)$$

The corresponding boundary conditions become:

$$\tilde{u} = \tilde{U}, \tilde{v} = 0 \text{ at } \tilde{y} = 0 \quad (5)$$

$$\tilde{u} = \tilde{U} \text{ at } \tilde{y} = \tilde{H} \quad (6)$$

$$\tilde{v} = 0 \text{ at } \tilde{y} = \tilde{H} \quad (7)$$

$$\tilde{p} = 0 \text{ at } \tilde{x} = 0, \tilde{B} \quad (8)$$

The film thickness needs to be calculated in two parts, in the presence of SR on bearing surfaces:

$$\tilde{H}(\tilde{x}) = \tilde{h}(\tilde{x}) + \tilde{h}_s \quad (9)$$

$$\tilde{h}(\tilde{x}) = \tilde{H}_n \left[4 \left(\frac{\tilde{x}}{\tilde{B}} - \frac{1}{2} \right)^2 - 1 \right] + \tilde{h}_0 \left[1 + (d-1) \left(1 - \frac{\tilde{x}}{\tilde{B}} \right) \right] \quad (10)$$

3. Solution of the problem

After integrating the eq. (3) w.r.t. \tilde{y} , with the use of expression (5-6), we obtain

$$\tilde{u} = \frac{1}{\tilde{\mu}} \left[\frac{1}{2} \tilde{y}(\tilde{y} - \tilde{h}) \frac{\partial \tilde{p}}{\partial \tilde{x}} + \tilde{k} \left(\frac{\partial \tilde{p}}{\partial \tilde{x}} \right)^3 \left\{ \frac{1}{4} \tilde{y}^4 - \frac{1}{2} \tilde{h} \tilde{y}^3 + \frac{3}{8} \tilde{h}^2 \tilde{y}^2 - \frac{1}{8} \tilde{h}^3 \tilde{y} \right\} \right] + \tilde{U} \left[1 - \frac{\tilde{y} + \tilde{k} \left(\frac{\partial \tilde{p}}{\partial \tilde{x}} \right)^3 \left(\tilde{y}^3 - \frac{3}{2} \tilde{h} \tilde{y}^2 + \frac{3}{4} \tilde{h}^2 \tilde{y} \right)}{\tilde{h} \left(1 + \frac{1}{4} \tilde{k} \left(\frac{\partial \tilde{p}}{\partial \tilde{x}} \right)^2 \tilde{h}^2 \right)} \right] \quad (11)$$

Plug in the expression (11) in eq. (2) and integrating w.r.t. y , we obtain the RE for the pivoted curved slider bearing [eq. (12)]:

$$\frac{\partial}{\partial \tilde{x}} \left[\tilde{H}^3 \frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{3}{20} \tilde{k} \tilde{H}^5 \left(\frac{\partial \tilde{p}}{\partial \tilde{x}} \right)^3 \right] = 6 \tilde{\mu} \tilde{U} \frac{\partial \tilde{H}}{\partial \tilde{x}} \quad (12)$$

Multiplying $\tilde{f}(\tilde{h}_s)$ on both sides of Eq. (12) and integrating from $-\tilde{c}$ to \tilde{c} , using roughness parameter from Eqs. 14-17, the modified RE (eq. 12) takes the form:

$$\frac{d}{d\tilde{x}} \left[\tilde{f}_1(\tilde{h}, \tilde{\alpha}, \tilde{\sigma}, \tilde{\varepsilon}) \frac{d\tilde{p}}{d\tilde{x}} + \frac{3}{20} \tilde{k} \tilde{f}_2(\tilde{h}, \tilde{\alpha}, \tilde{\sigma}, \tilde{\varepsilon}) \left(\frac{d\tilde{p}}{d\tilde{x}} \right)^3 \right] = 6 \tilde{\mu} \tilde{U} \frac{d\tilde{H}}{d\tilde{x}} \quad (13)$$

where

$$\tilde{f}_1(\tilde{h}, \tilde{\alpha}, \tilde{\sigma}, \tilde{\varepsilon}) = \tilde{h}^3 + 3\tilde{h}^2\tilde{\alpha} + 3\tilde{h}(\tilde{\sigma}^2 + \tilde{\alpha}^2) + \tilde{\varepsilon} + \tilde{\alpha}(3\tilde{\sigma}^2 + \tilde{\alpha}^2)$$

$$\begin{aligned} \tilde{f}_2(\tilde{h}, \tilde{\alpha}, \tilde{\sigma}, \tilde{\varepsilon}) = & \tilde{h}^5 + 5\tilde{h}^4\tilde{\alpha} + 5(\tilde{\sigma}^2 + \tilde{\alpha}^2)(5\tilde{h}^3 - \tilde{\alpha}^3) + \{\tilde{\varepsilon} + \tilde{\alpha}(3\tilde{\sigma}^2 + \tilde{\alpha}^2)\}(10\tilde{h}^2 + \tilde{\alpha}^2) \\ & + (\tilde{\sigma}^2 + \tilde{\alpha}^2)^2(5\tilde{h} + 3\tilde{\alpha}) + 2\tilde{\alpha}^5 + \tilde{\varepsilon}(\tilde{\alpha}^2 + \tilde{\sigma}^2) \end{aligned}$$

The following SR parameters are given:

$$\tilde{\alpha} = E(\tilde{h}_s) \quad (14)$$

$$\tilde{\sigma} = E(\tilde{h}_s - \tilde{\alpha})^2 \quad (15)$$

$$E(*) = \int_{-\infty}^{\infty} (*) \tilde{f}(\tilde{h}_s) \quad (16)$$

$$\tilde{\varepsilon} = E(\tilde{h}_s - \tilde{\alpha})^3 \quad (17)$$

The roughness parameters α and $\tilde{\varepsilon}$ vary with negative and positive values, whereas $\tilde{\varepsilon}$ varies with positive values. Moreover, the parameters $\tilde{\alpha}$, $\tilde{\sigma}$, $\tilde{\varepsilon}$ are all independent from \tilde{x} .

Using the following non-dimensional parameters:

$$\bar{x} = \frac{\tilde{x}}{\tilde{B}}, \bar{h} = \frac{\tilde{H}}{\tilde{h}_0}, \bar{h}_s = \frac{\tilde{h}_s}{\tilde{h}_0}, \bar{p} = \frac{\tilde{h}_0^2 \tilde{p}}{\tilde{\mu} \tilde{U} \tilde{B}}, \gamma = \frac{\tilde{k} \tilde{\mu}^2 \tilde{U}^2}{\tilde{h}_0^2}, \alpha = \frac{\tilde{\alpha}}{\tilde{h}_0}, \sigma = \frac{\tilde{\sigma}}{\tilde{h}_0}, \varepsilon = \frac{\tilde{\varepsilon}}{\tilde{h}_0^3}, z = \frac{\tilde{H}_n}{\tilde{h}_0}$$

The non-dimensional fluid film thickness as well as dimensionless stochastic modified RE can be obtained as:

$$\bar{h} = z \left[4 \left(\bar{x} - \frac{1}{2} \right)^2 - 1 \right] + [1 + (d-1)(1-\bar{x})] \quad (18)$$

$$\frac{d}{d\bar{x}} \left[\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon) \frac{d\bar{p}}{d\bar{x}} + \frac{3}{20} \gamma \bar{f}_2 \left(\frac{d\bar{p}}{d\bar{x}} \right)^3 \right] = 6 \frac{d\bar{h}}{d\bar{x}} \quad (19)$$

where:

$$\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon) = \bar{h}^3 + 3\bar{h}^2\alpha + 3\bar{h}(\sigma^2 + \alpha^2) + \varepsilon + \alpha(3\sigma^2 + \alpha^2)$$

$$\begin{aligned} \bar{f}_2(\bar{h}, \alpha, \sigma, \varepsilon) = & \bar{h}^5 + 5\bar{h}^4\alpha + 5(\sigma^2 + \alpha^2)(5\bar{h}^3 - \alpha^3) + \{\varepsilon + \alpha(3\sigma^2 + \alpha^2)\}(10\bar{h}^2 + \alpha^2) \\ & + (\sigma^2 + \alpha^2)^2(5\bar{h} + 3\alpha) + 2\alpha^5 + \varepsilon(\alpha^2 + \sigma^2) \end{aligned}$$

The dimensionless stochastic RE (19) is observed to be non-linear, so it is difficult to find the film pressure analytically. Therefore, the small perturbation is applied to calculate the film pressure. The film pressure of the bearing can be demonstrated as:

$$\bar{p} = \bar{p}_0(\bar{x}) + \gamma \bar{p}_1(\bar{x}) \quad (19)$$

Substituting Eq. (20) into Eq. (19) and omitting the higher order terms of γ , the two separated equations governing the film pressure p_0 and p_1 are obtained respectively:

$$\frac{d}{d\bar{x}} \left[\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon) \frac{d\bar{p}_0}{d\bar{x}} \right] = 6 \frac{d\bar{h}}{d\bar{x}} \quad (21)$$

$$\frac{d}{d\bar{x}} \left[\left\{ \frac{3}{20} \bar{f}_2(\bar{h}, \alpha, \sigma, \varepsilon) \left(\frac{d\bar{p}_0}{d\bar{x}} \right)^3 + \bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon) \left(\frac{d\bar{p}_1}{d\bar{x}} \right) \right\} \right] = 0 \quad (22)$$

The boundary conditions are as below:

$$\bar{p} = 0 \text{ at } \bar{x} = 0, 1 \quad (23)$$

Solving Eqs. (21) and (22) by (23), we obtain

$$\bar{p} = \int_0^{\bar{x}} \frac{6\bar{h} + C_{00}}{\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon)} d\bar{x} + \gamma \left[C_{01} \int_0^{\bar{x}} \frac{1}{\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon)} d\bar{x} - \frac{3}{20} \int_0^{\bar{x}} \frac{\bar{f}_2(\bar{h}, \alpha, \sigma, \varepsilon) (6\bar{h} + C_{00})^3}{\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon)^4} d\bar{x} \right] \quad (24)$$

where:

$$C_{01} = \frac{\frac{3}{20} \int_0^1 \frac{\bar{f}_2(\bar{h}, \alpha, \sigma, \varepsilon) (6\bar{h} + C_{00})^3}{\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon)^4} d\bar{x}}{\int_0^1 \frac{1}{\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon)} d\bar{x}} \quad (25)$$

$$C_{00} = - \frac{6 \int_0^1 \frac{\bar{h}}{\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon)} d\bar{x}}{\int_0^1 \frac{1}{\bar{f}_1(\bar{h}, \alpha, \sigma, \varepsilon)} d\bar{x}} \quad (26)$$

The non-dimensional LCC (\bar{W}) is derived by the expression (24) and integrated into the domain $[0,1]$.

$$\bar{W} = \int_0^1 \bar{p} dx \quad (27)$$

We have the non-dimensional COP of bearing derived by expression (24) and (27) in the domain $[0,1]$:

$$\bar{X} = \frac{1}{\bar{W}} \int_0^1 \bar{x} \bar{p} d\bar{x} \quad (28)$$

The Gaussian quadrature method is applied to carry out the results of \bar{W} and the COP with the use of MATHEMATICA Software.

4. Results and discussion

This investigation mainly concerns the impact of SR and RF behavior on the pivoted curved slider bearing. The LCC and COP are analyzed to show the characteristics of the bearing. With the variations of five non-dimensional parameters, the results have been presented as follows.

- SR parameters
- $\alpha = -0.1, -0.05, 0.0, 0.05, 0.1$; $\varepsilon = -0.1, -0.05, 0.0, 0.05, 0.1$; $\sigma = 0.0, 0.1, 0.2$.
- Non-linear parameter ($\gamma = -0.005, 0.0, 0.005$)
- Film thickness ratio ($d = 2.0, 2.5, 3.0, 3.5$)
- Curvature parameter ($z = 0.5, 0.0, 0.25$)

Figure 2 demonstrates the results of LCC (\bar{W}) with curvature parameter (z) for distinct values of the non-linear parameter (γ). The execution of rough pivoted curved slider bearing displays that \bar{W} increases with an increase in curvature parameter z . Moreover, the RF behavior was perceived that \bar{W} higher with DF and lower for PPF w.r.t NF. Figure 3 displays the feature of \bar{W} against film thickness ratio (d) under the impact of z and γ . It is noticed that an increment in the curvature parameter (z) is responsible for a decrement in the LCC of bearing. In addition, the LCC gradually increases up to the step ratio $d \approx 2.4$ thereafter it decreases.

Figure 4 has been plotted to display the influence of \bar{W} with z under the impact of α and γ . It is revealed that the mean parameter α acts as an increasing function of \bar{W} and curvature parameter (z). Moreover, the DF and roughness parameters (α) provide higher LCC. Figures 5

and 6 analyze the variation of \bar{W} w.r.t curvature parameter (z) for the distinct value of γ , ε and σ respectively. One can observe from these outcomes the \bar{W} gradually increases with increasing curvature parameter (z). Because of the rough bearing surfaces, it is reported that the roughness of the negatively skewed surface improves the LCC for DF ($\gamma = -0.005$), whereas the LCC is reduced for a positively skewed roughness pattern. Figure 7 shows the profile of \bar{X} vs. z for distinct values of γ and d . It can be demonstrated that each estimation of d , the profile \bar{X} fluctuated towards the inlet of bearing with the increase of the curvature parameter z and nonlinear parameter γ respectively. The COP moves towards the inlet for PPF, and it is towards the exit for DF. SR is accounted to decrease the COP for DF and increase for PPF w.r.t. NF. The COP (\bar{X}) concerning d distinct values of the curvature parameter (z) and the non-linear parameter (γ) is shown in Fig. 8. It is obtained that with an increase in d the COP shifts towards the outlet of the bearing. Due to SR, the COP is above (up to $d \approx 1.5$) for NF, thereafter, it goes below for the rest of all values d . It is also revealed from the figure that the effect of the central pressure is very significant from $z = 1.6$.

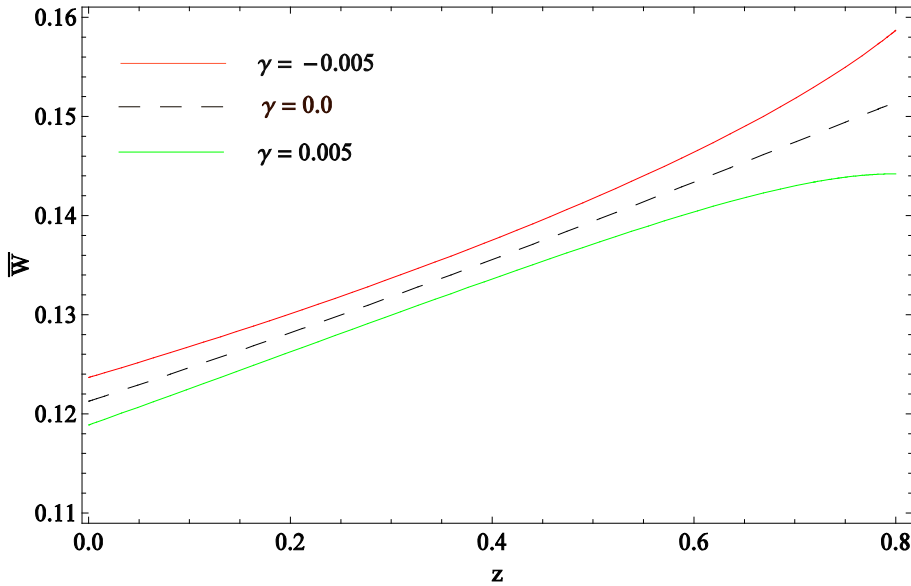


Fig. 2. Plots of \bar{W} vs. z for γ and $d = 2.0$ with $\alpha = 0.1$, $\sigma = 0.2$ and $\varepsilon = 0.05$

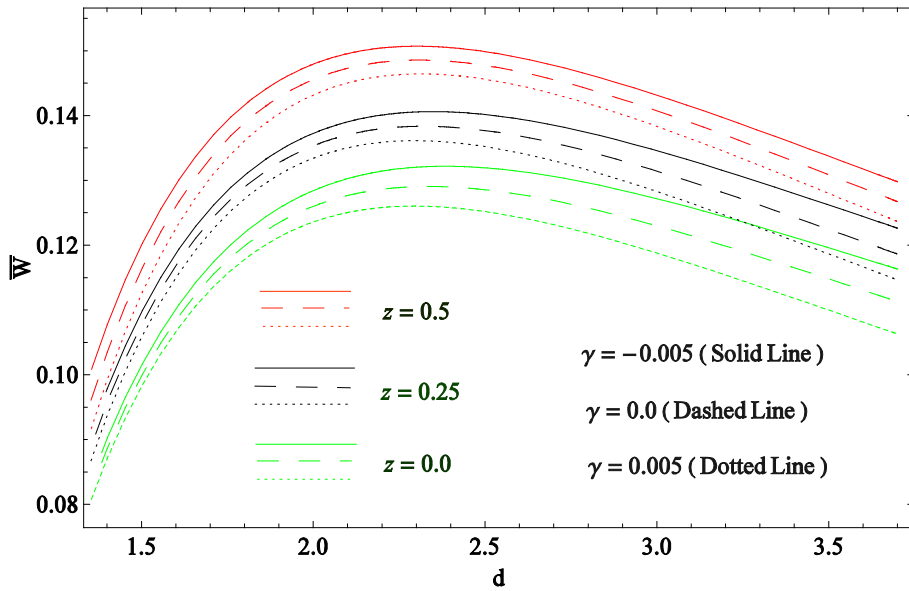


Fig. 3. Plots of \bar{W} vs. d for z and γ with $\alpha = 0.1$, $\sigma = 0.1$ and $\varepsilon = 0.05$

The variations of \bar{X} against z for various values of α , σ and ε as shown in Figures (9-11). It is indicated with the increment of curvature parameter (z), that the COP shifts towards the inlet of the bearing. The SR accounted that the COP is larger for PPF with negatively skewed SR whereas the COP lowers for DF with a positively skewed SR pattern.

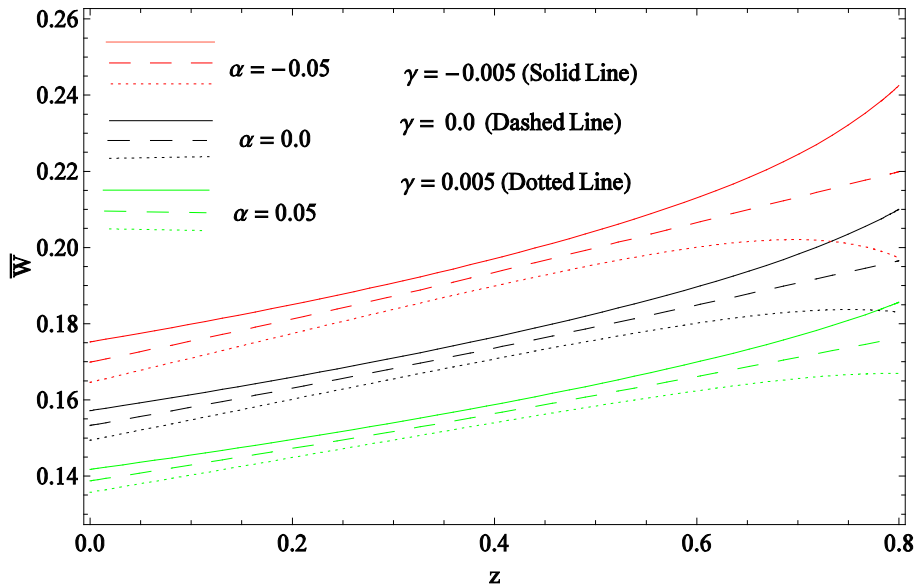


Fig. 4. Plots of \bar{W} vs. z for α and γ with $\sigma = 0.1$, $\varepsilon = 0.05$ and $d = 2.0$

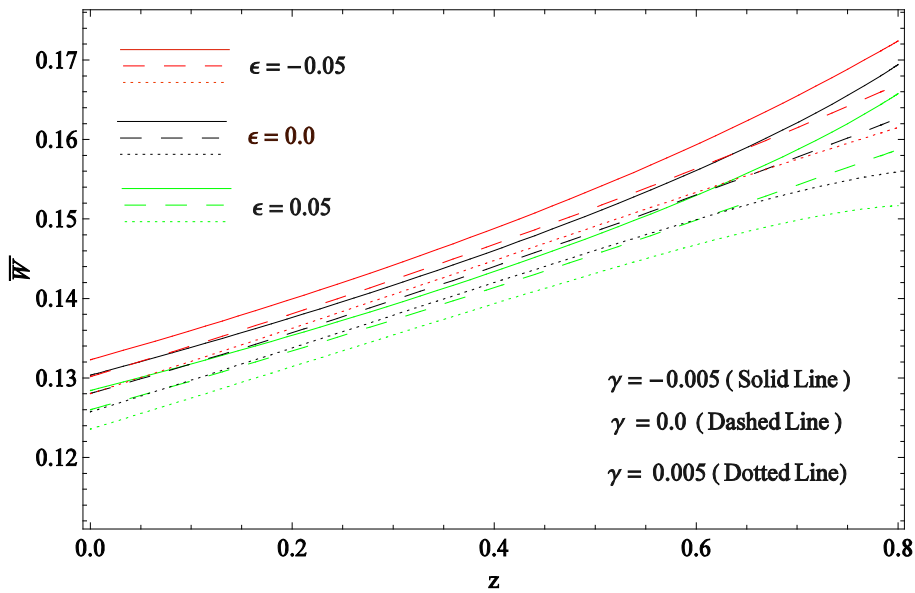


Fig. 5. Plots of \bar{W} vs. z for ϵ and γ with $\sigma = 0.1$, $\alpha = 0.1$ and $d = 2.0$

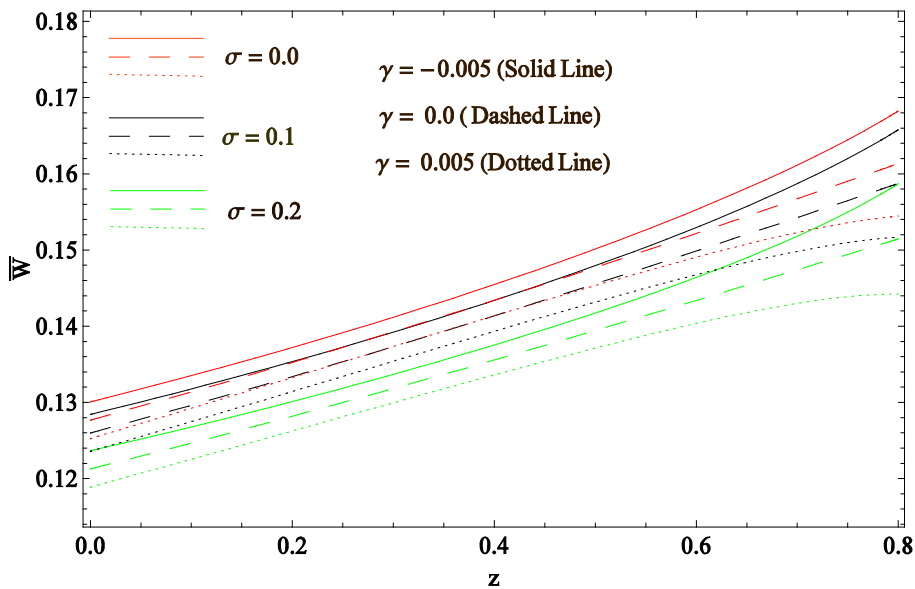


Fig. 6. Plots of \bar{W} vs. z for σ and γ with $\sigma = 0.1$, $\epsilon = 0.05$ and $d = 2.0$

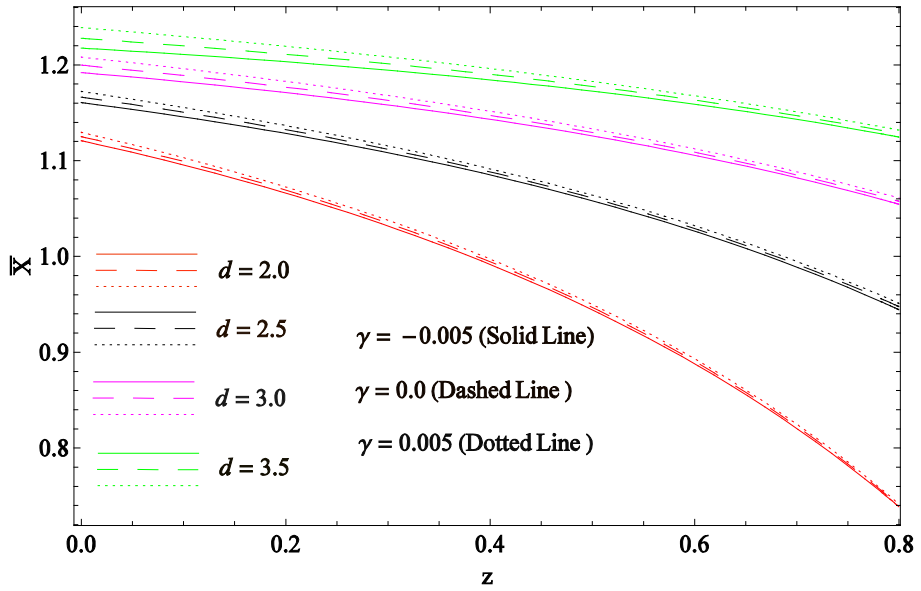


Fig. 7. Plots of \bar{X} vs. z for d and γ with $\sigma = 0.1$, $\varepsilon = 0.05$ and $\alpha = 0.1$

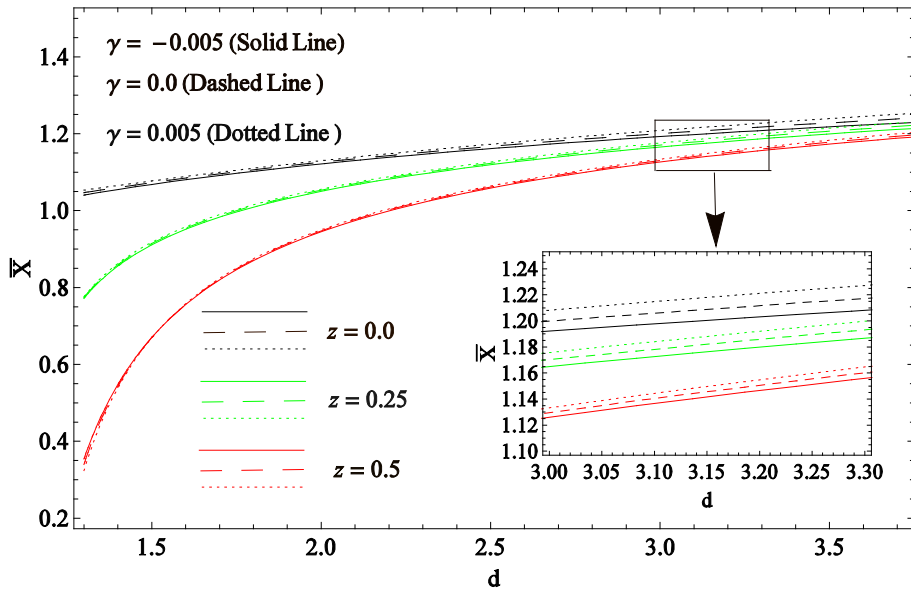


Fig. 8. Plots of \bar{X} vs. d for z and γ with $\sigma = 0.1$, $\varepsilon = 0.05$ and $\alpha = 0.1$.

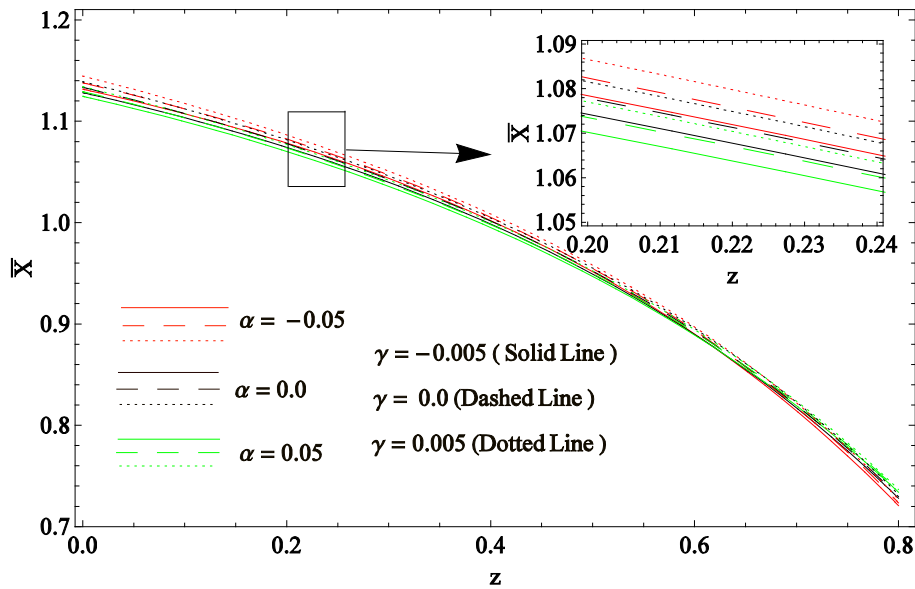


Fig. 9. Plots of \bar{X} vs. z for α and γ with $\sigma = 0.1$, $\varepsilon = 0.05$ and $d = 2.0$.

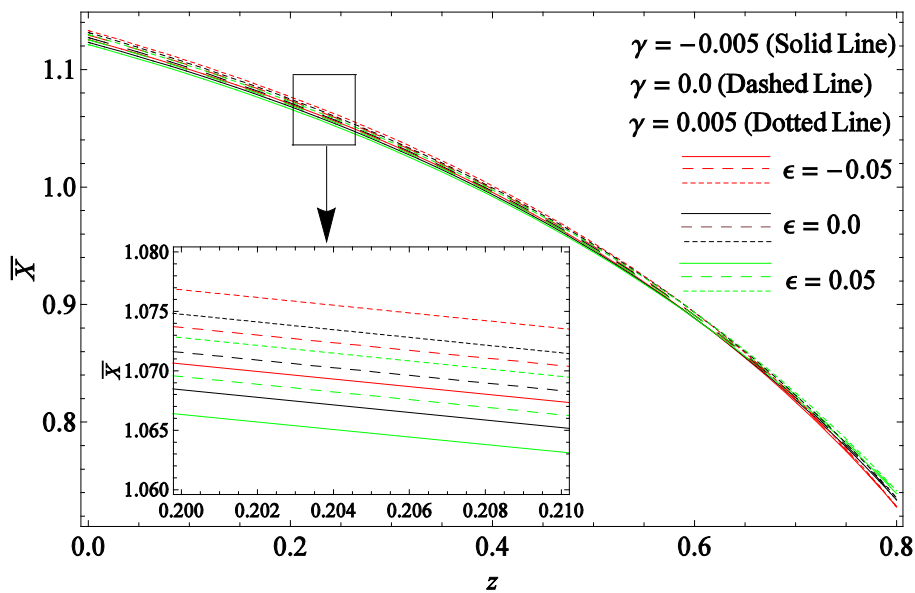


Fig. 10. Plots of \bar{X} vs. z for ε and γ with $\sigma = 0.1$, $\alpha = 0.1$ and $d = 2.0$

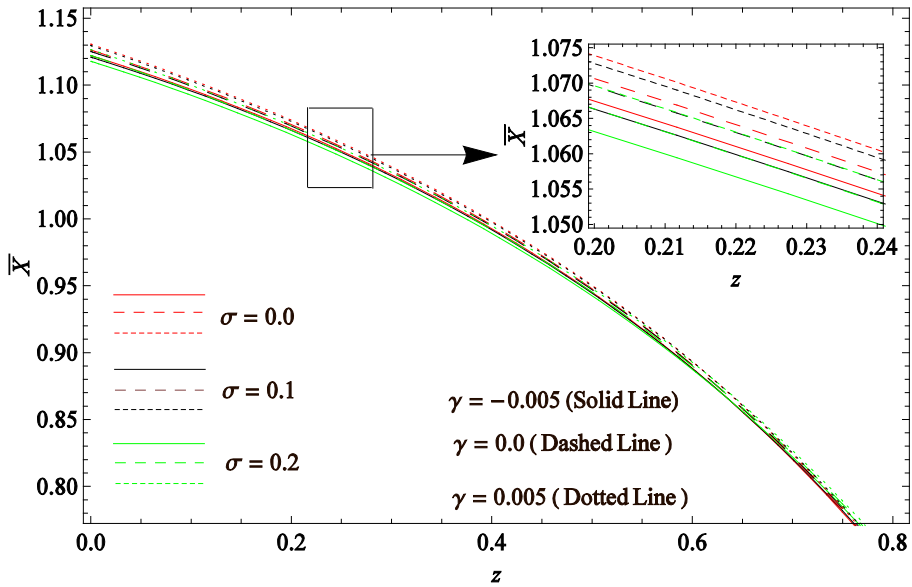


Fig.11. Plots of \bar{X} with z for σ and γ with $\varepsilon = 0.05$, $\alpha = 0.1$ and $d = 2.0$

5. Conclusions

Utilizing a stochastic approach for deriving the modified Reynolds equation, this study explores the influence of surface roughness (SR) and Rabinowitsch fluid (RF) within pivoted curved slider bearings. The results are summarized as follows:

1. An increase in the variations of SR leads to a slowdown in the fluid flow.
2. The impact of n-N fluids and SR of the bearing surface carried out that load carrying capacity is increased for dilatant fluid ($\gamma < 0$) and decreased for the pseudoplastic fluids ($\gamma > 0$) as compared to Newtonian fluid.
3. The load carrying capacity is enhanced for negatively skewed SR with DF as compared to positively skewed SR with PPF.
4. The centre of pressure shifts towards the outlet for negatively skewed SR with pseudoplastic fluids, whereas positively skewed SR with dilatant fluid shifts towards the inlet with the increasing curvature z .

Nomenclature

Variable	Meaning	Variable	Meaning
d	film thickness ratio (h_1/h_0)	l	width of bearing
B	length of bearing	u	velocity of X-direction
v	velocity of Y-direction	τ_{xy}	shear stress component
μ	zero shear rate viscosity	k	Non-linear factor
H_n	height of the crown segment of the slider bearing	E	expectancy operator
c	maximum deviation from the mean film thickness	h_0	outlet film thickness
σ^*	the standard deviation of the film thickness	$H(x)$	film thickness
\bar{h}	non-dimensional film thickness (H/h_0)	h_1	inlet film thickness
α^*	mean of the stochastic film thickness,	σ^{*2}	variance
ε^*	a measure of the symmetry of the stochastic random variable	p	lubricant film pressure
$\alpha, \sigma, \varepsilon$	non-dimensional form of $\alpha^*, \sigma^*, \varepsilon^*$, respectively	z	curvature parameter (H_n/h_0)
P	the expected value of the film pressure p	W	load carrying capacity
\bar{x}	distance of the center of pressure (x/B)	\bar{X}	x coordinate
\bar{P}	non-dimensional form fluid film pressure ($h_0^2/\mu UB$)	\bar{W}	non-dimensional load-carrying capacity ($Wh_0^2/\mu UB^2$)
γ	the dimensionless non-linear factor of lubricants ($k\mu^2U/h_0^2$)	H	film thickness measured between the nominal mean levels of the bearing surface
h_s	stochastic film thickness measured from nominal mean levels of the bearing surfaces		

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