Journal of the Serbian Society for Computational Mechanics / Vol. 16 / No. 2, 2022 / pp 1-12
(10.24874/jsscm.2022.16.02.01)

FREE TRANSVERSE VIBRATIONS OF CANTILEVER BEAM FOR TAPERED THICKNESS PREPARED FROM VARIANT FIBERS REINFORCED POLYESTER

Nawal H. Al – Raheimy[1]* and Lekaa Hammed[2]

[1]College of Engineering, Mechanical Department, University of Babylon, Babylon, Iraq
e-mail: nawalalraheimy@yahoo.com
[2]College of Engineering, Mechanical Department, University of Babylon, Babylon, Iraq
e-mail: lekaahameed@gmail.com

**corresponding author*

Abstract

The natural frequency of the cantilever beam for the tapered thickness in the current paper is estimated using a Raleigh-Ritz approach. The study explores the effect of different parameters on the behavior of the beam such as the length of beam “L”, thickness at clamped end “h_c”, width “b”, the ratio of thickness at free end to thickness at clamped end “h_f/h_c”, types of fibers and the concentration of fibers “f” in the resin of unsaturated polyester representing the matrix. The resin can be reinforced by aligned long fibers such as E-fibers glass, Kevlar-49 and carbon fibers. When a tapered beam is formed from a composite material, the natural frequency decreases when the length of the beam and the ratio of the thickness “h_f/h_c” increase; however, the frequency increases as the width of the beam increases. The thickness at the clamped end likewise increases as the volume fraction of fibers in the resin increases. The carbon fiber beam has a higher natural frequency than the other types of fibers. Finally, the results were compared to other available results and were determined to be consistent.

Keywords: Composite materials, tapered thickness, different fibers, free vibrations, thickness ratio.

1. Introduction

The motions of the mechanical structure as fluctuations backwards and forwards are called vibration which occurs about the pivot point. The frequencies are called free vibrations when a structure is wavering by its own mass. Tapered beams have proved to be more efficient compared to uniform beams since the tapered beams have a better distribution strength and mass and can be used for special practical necessities in many engineering applications. A composite material is made of two or more materials in order to create a new material with better characteristics than when each component is used separately. The two components are a fortification and a matrix. The key benefits of composite materials are low density in contrast to high stiffness and strength. The reinforcing part supplies the stiffness and strength. In most cases, the reinforcement is stronger, stiffer and harder in comparison with the matrix. The final properties of composite materials depend on the kind and amount of the reinforcement; therefore, the continuous fibers provide us with highest strength and modulus. In practical applications, about 70 percent of volume reinforcement can be added to form composite materials. The composite materials can be used in numerous applications such as in satellites, robot arms, wind turbines, craft wings, tubes, boats and vessels, tanks, springs, brake pedals etc.

Byoung Koo Lee (2002) used numerical methods such as the Rung Kutta method to estimate the natural frequency for tapered beam for different types of boundary conditions. Abadi (2007) investigated the transverse free vibration of the Euler-Bernoulli beam with variable cross section under effect of axial force by using the Wentzel-Kramers-Brillouin (WKB) approximate method to attain differential equation in term of natural frequency.

Ali (2009) investigated the effect of reinforced fibers in different percentages on the mechanical properties such as impact strength, hardness and tensile for epoxy reinforced by Kevlar fibers. Hibba (2010) studied the behavior of damping of composite materials under effect of the reinforced fibers of the composite materials with risen from polyester and epoxy with different percentage of fibers from copper, carbon and glass to illustrate mechanical properties such as deflection stiffness, vibration damping and natural frequency**.** Mohamed Hussein (2008) studied vibration of non-uniform flexural beams, and concluded that the mass, stiffness and damping vary as a function with length of the beam during calculation of the characteristics of frequencies for a wide range of mass and damping intensities. Using the differential transformation method, Mahmoud A. (2013) investigated the free vibration of uniform and non-uniform beams. The natural frequency and mode shape were calculated for various cases of boundary conditions and cross section, and the differential equation of the system was solved using MATLAB code.

Aleksandra (2014) used the frame of Euler-Bernoulli beam theory to compute the natural frequency of beam with tapered width and constant thickness and compared the results with the other papers that proposed the same method. Rajesh (2016) calculated the natural frequency by using coupled displacement field method for tapered beam with rectangular cross section at hinged-hinged boundary conditions. Peng (2016) investigated the free transverse vibrations of tapered Bernoulli beams by using the finite point method under the effect of disparity of material properties and cross section through the length of beam. Zhou D. (2016) presented a theoretical study on tapered thickness of beam and tapered width of beam by using the Bernoulli-Euler theory to illustrate the motion of beam. Osama (2017) studied the free vibration of a laminated composite beam for different composite beam and different boundary conditions by using a finite element model. It was found that when the aspect ratio increased the natural frequency decreased. Khan (2018) showed that the composite material reinforced with jute or hemp is weaker than the composite reinforced with carbon, glass or Kevlar.

Shaik (2018) used the energy field method to evaluate the natural frequency for cantilever of tapered Timoshenko beam with rectangular cross section and compared the results with those obtained using Ansys software. Rajesh (2018) derived the parameter values of linear and nonlinear natural frequency of tapered beams in terms of taper ratio, large amplitude and slenderness ratio by using the coupled displacement field method for clamped-clamped and hinged-hinged beam boundary conditions. Nawras (2019) investigated the mechanical properties and vibration characteristics for composite material from glass-polyester and jute-polyester for weight of fiber equal to 18% for all composites.

In this study, we used the approximate method of Rayleigh-Ritz to estimate the natural frequency of cantilever beam with tapered thickness made from composite materials for different types of fibers in different fraction where the beam has a varying width and length. The natural frequency of structure at every arrangement decreases with increasing the ratio of (hf/hc) and the length of the beam. On the other side, the natural frequency increases with increasing the percentage of fiber, the width and the thickness of beam at clamped end. The frequency of composite beams with fiberglass is lower than that of the two others.

2. Theoretical Analysis

The cantilever beam of tapered thickness which is used in the recent study can be described in Fig. 1.

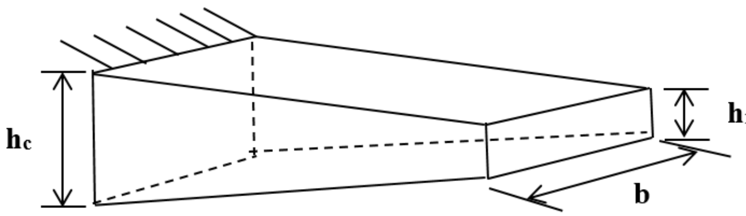


Fig. 1. Cantilever beam of tapered thickness**.**

In order to derive the formula of the tapered thickness of beam at length x of the beam, it is possible to use Fig. 2.

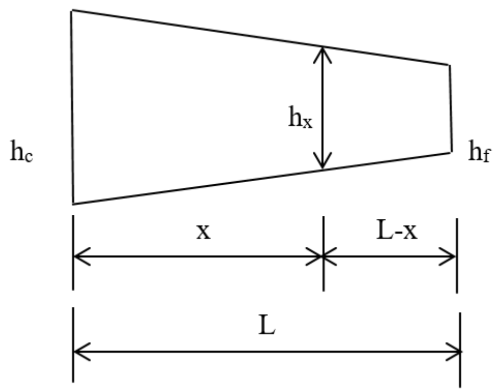


Fig. 2. Tapered thickness of a beam**.**

$$\frac{1}{2}\left(hx - hf\right) / \left(L - x\right) = \frac{1}{2}\left(hc - hf\right) / L \tag{1}$$

After simplification equation (1), we obtain the thickness of the beam at apart length of the beam (x)

$$hx = hc + \left(x / L\right) * \left(hf - hc\right) \tag{2}$$

The procedure of Rayleigh-Ritz is applied to derive the natural frequency of the cantilever beam with tapered thickness where two terms approximation is used, Benoraya (1998):

$$Y_r = c_1 y_1\left(x\right) + c_2 y_2\left(x\right) \tag{3}$$

where $y_1 = c_1 \left(\frac{x}{L}\right)^2$ and $y_2 = c_2 \left(\frac{x}{L}\right)^3$, Benoraya (1998).

The values of stiffness (kij) and mass (mij) can be expected from the following equations, Benoraya (1998)

$$k_{ij} = \int_0^L EI(x)\, y_i''\, y_j'' \tag{4}$$

$$m_{ij} = \int_0^L m(x)\, y_i\, y_j\, dx \tag{5}$$

where i =1,2 & j=1,2.

In the present study, the modulus of elasticity of composite materials of the beam for long (continuous) fibers is E_c which depends on the volume of fraction of fibers (f) shown below (Ashraf 2001)

$$E_c = E_f * f + E_m * (1 - f) \tag{6}$$

The cross section area Ax for the tapered beam at length x is equal to Ax = b *hx. In addition, the mass of the composite beam at apart length of the beam is identical to mc(x) =ρc * Ax, where the ρc is the mass density of composite materials comparable to (Ashraf 2001)

$$\rho_c = \rho_f * f + \rho_m * (1 - f) \tag{7}$$

Therefore, the mass of the beam per unit length can be calculated as follows

$$m_c(x) = \rho_c * b * \left\{ h_c + \left(\frac{x}{L}\right) * \left(h_f - h_c\right) \right\} \tag{8}$$

Now the second moment area of the beam at the clamped end and free end can be shown, respectively,

$$I_c = (b_c h_c^3)/12, I_f = (b_f h_f^3)/12$$

Now at the part length (x) of the beam the second moment of area Ix by means of equation (2) is corresponding to the formula

$$I_x = \frac{b}{12}\left[h_f^3\left(\frac{x^3}{L^3}\right) + 3h_f^2.h_c\left(\frac{x^2}{L^2} - \frac{x^3}{L^3}\right) + 3h_f.h_c^2\left(\frac{x}{L} - 2\frac{x^2}{L^2} + \frac{x^3}{L^3}\right) + h_c^3\left(1 - 3\frac{x}{L} + 3\frac{x^2}{L^2} - \frac{x^3}{L^3}\right)\right] \tag{9}$$

Using the last equation compensation in equation (4), the relations of stiffness of beam can be obtained

$$k_{11} = \frac{E_c * b_c}{12L^3}\left(h_f^3 + h_f^2.h_c + h_f.h_c^2 + h_c^3\right) \tag{10}$$

$$k_{12} = \frac{E_c * b_c}{L^3}\left(\frac{1}{5} * h_f^3 + \frac{3}{20} h_f^2 * h_c + \frac{1}{10} h_f * h_c^2 + \frac{1}{20} h_c^3\right) \tag{11}$$

$$k_{21} = k_{12} \tag{12}$$

$$k_{22} = \frac{3 * E_c * b_c}{L^3}\left(\frac{1}{6} * h_f^3 + \frac{1}{10} * h_f^2 * h_c + \frac{1}{20} * h_f * h_c^2 + \frac{1}{60} * h_c^3\right) \tag{13}$$

Now integration equation (5), according to the mass of beam for equation (8), can be obtained

$$m_{11} = \rho_c * b * L * \left\{\frac{1}{6} h_f + \frac{1}{30} h_c\right\} \tag{14}$$

$$m_{12} = \rho_c * b * L * \left\{\frac{1}{7} h_f + \frac{1}{42} h_c\right\} \tag{15}$$

$$m_{21} = m_{12} \tag{16}$$

$$m_{22} = \rho_c * b * L * \left\{\frac{1}{8} h_f + \frac{1}{56} h_c\right\} \tag{17}$$

The following matrix form represents the other form of mass and stiffness equations

$$\begin{bmatrix} k_{11} - \omega_n^2 m_{11} & k_{12} - \omega_n^2 m_{12} \\ k_{21} - \omega_n^2 m_{21} & k_{22} - \omega_n^2 m_{22} \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{18}$$

The general matrix notation can be rewritten as

$$\left[\{K\} - \omega_n^2 \{M\}\right]\{c\} = \{0\} \tag{19}$$

For a nontrivial solution of c_1 and c_2, the determinant of the coefficients of c_1 and c_2 must be zero

$$\begin{bmatrix} k_{11} - \omega_n{}^2 m_{11} & k_{12} - \omega_n{}^2 m_{12} \\ k_{21} - \omega_n{}^2 m_{21} & k_{22} - \omega_n{}^2 m_{22} \end{bmatrix} = 0 \tag{20}$$

However, $k_{12} = k_{21}$ and $m_{12} = m_{21}$ so

$$\left[(k_{11} - \omega_n{}^2 m_{11}) * (k_{22} - \omega_n{}^2 m_{22})\right] - \left[(k_{12} - \omega_n{}^2 m_{12}) * (k_{12} - \omega_n{}^2 m_{12})\right] = 0 \tag{21}$$

After the arrangement, the following equation can be obtained

$$\omega_n{}^4\left((m_{11} * m_{22}) - m^2{}_{12}\right) - \omega_n{}^2\left((k_{11} * m_{22}) + (k_{22} * m_{11}) + (2 * k_{12} * m_{12})\right) + \left(k_{11} * k_{22}\right) - k^2{}_{12} = 0 \tag{22}$$

The above equation (22) is called the characteristics equation or frequency because the results of its solution are the frequency of the system.

3. Results and Discussion

A composite material is a mixture of at least two materials. The first material represents the matrix, which is called the resin, while the other material is the reinforcement and it includes particles, fibers or layers. A composite material has brilliant mechanical properties that are not found in other materials. The use of composite materials in industry instead of metal materials helps to reduce weight in return, which increases the strength and toughness. Table 1 shows the mechanical properties of diverse materials that are used in the present paper and Table 2 shows the comparison of the frequencies in (cycle/sec) between the finite element method (FEM) in

Rishi Kumar (2013) and the Rayleigh-Ritz method in the current study for cantilever tapered beam in different values of thickness ratio (h_f/h_c) where b=0.0254m, h_c= 0.057m and L=1m. The results showed that the properties of material for the modules of elasticity are E=210 Gpa and for mass density, ρ=7995 kg/m^3.

Material	Modulus of Elasticity, Ec (GN/m2)	Mass Density (kg/m3)
Polyester	2.5	1380
E-glass	72	2500
Kevlar-49	154	1470
Carbon	224	1750

Table 1. Mechanical properties of materials, Hibba (2010).

(h_f/h_c)	Present study R- Ritz method	F.E.M. Rishi [2013]	Error δ (%)
0.5	51.566	54.46	5.6%
0.56	50.93	53.62	5.3%
0.625	49.974	53.03	6%
0.714	49.178	53.03	6%
0.83	48.22	52.27	8.4%
1	47.428	52.05	9.7%

Table 2. Frequencies of tapered cantilever beam for various (h_f/h_c), (cycle/sec).

Figures 3 and 4 show the natural frequency of the structure as a function of the thickness ratio “h_f/h_c” obtained with variant types of continuous fibers. It is apparent that the natural frequency increases with the increase in the width of beam “b” and clamped thickness of beam “h_c”. i=nO the other hand, the frequency decreases with increase in the ratio of thickness. Such behavior is clarified by the change in strain energy of the system causing an increase in stiffness of the structure due to the increase in the moment of inertia for the area, which leads to an increase in the natural frequency. The decrease in the natural frequency occurs with increasing the thickness ratio. In the other words, the increase in the cantilever thickness at the free end is due to the increase in the kinetic energy of the system and the reason for is is the increase in the mass of the system. Furthermore, it can be observed that the natural frequency of a composite tapered beam made of carbon fibers is greater than that the beam provided from the other fibers resulting that the carbon fibers have higher strength and modulus of elasticity compared with the Kevlar fibers and E-glass fibers. This increases the stiffness of the system and thus causes an increase in the natural frequency of the structure. Also, the natural frequency of the system increases due to an increase in the concentration of fibers “f” in the resin as a result of increasing the stiffness of the system which leads to an increase in the natural frequency.

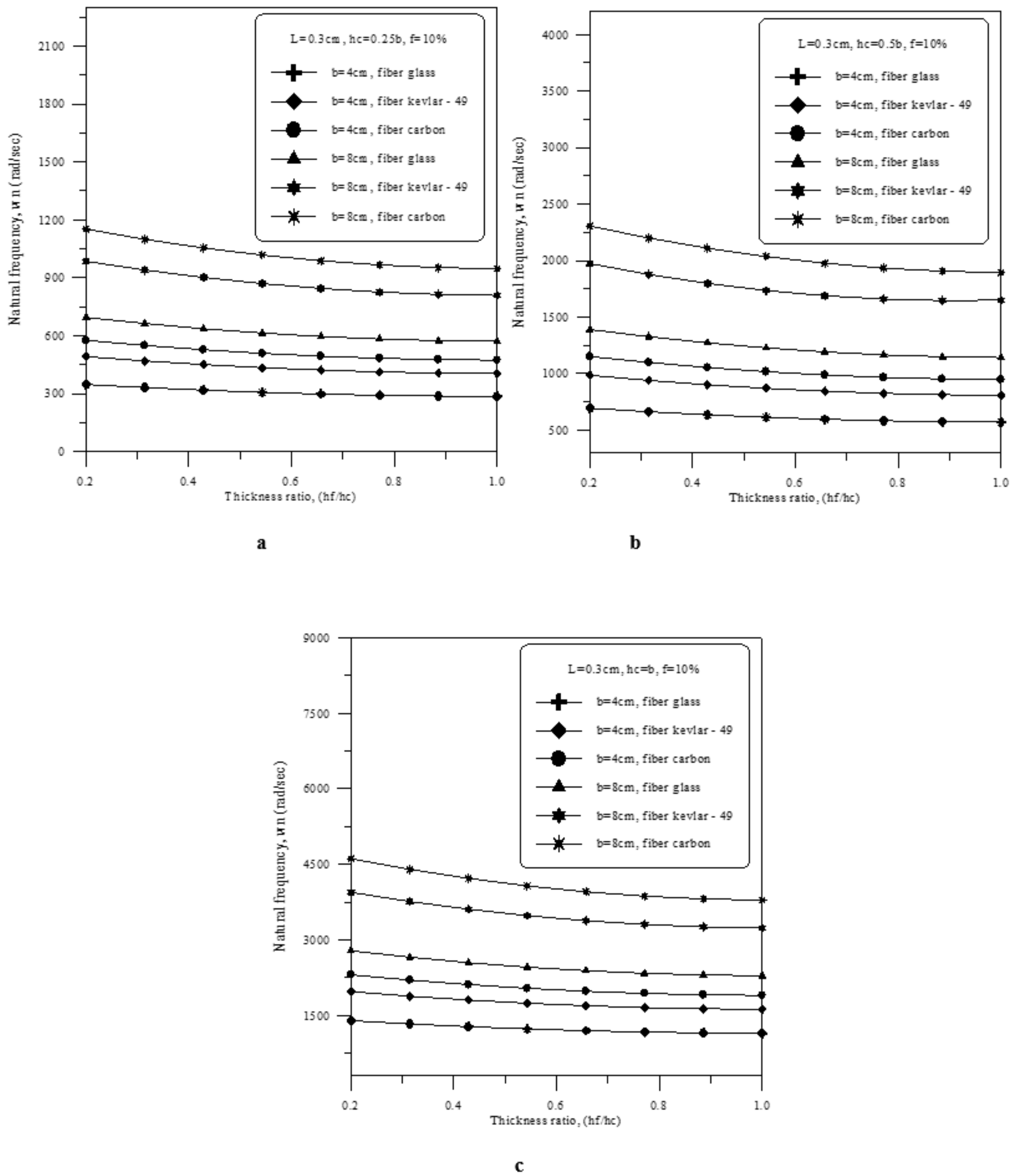
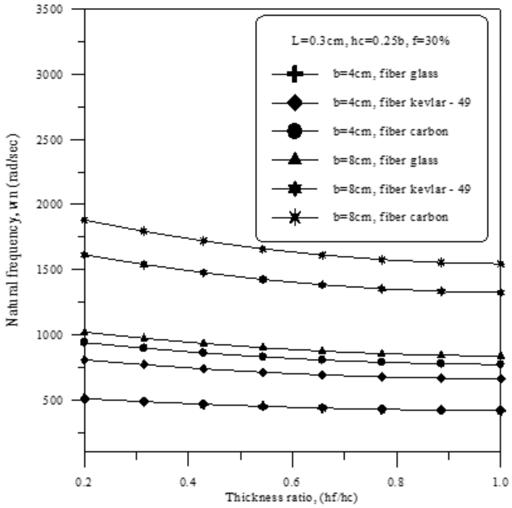
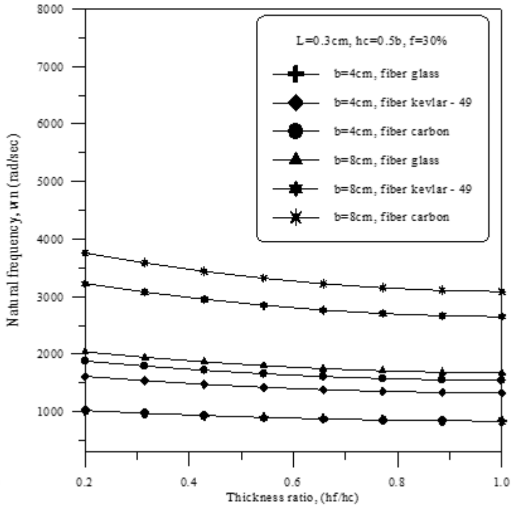


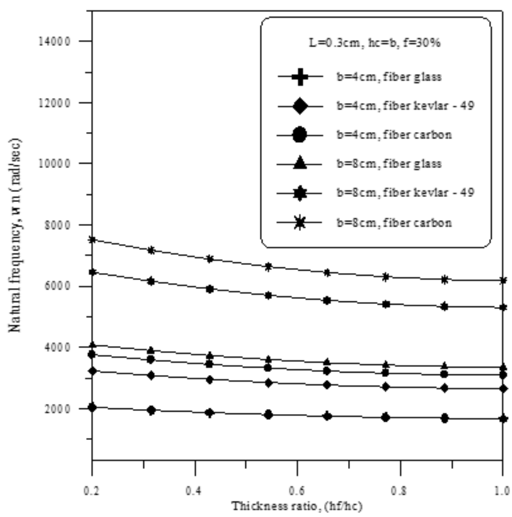
Fig. 3(a-c). Natural frequency as a function of thickness ratio (h_f/h_c) in different values of width (b) and different values of clamped thickness (h_c) at 10 % volume fraction (f) with 0.3 m of length.



a



b



c

Fig. 4(a-c). Natural frequency as a function of thickness ratio (h_f/h_c) in different values of width (b) and different values of clamped thickness (h_c) at 30 % volume fraction (f) with 0.3 m of length.

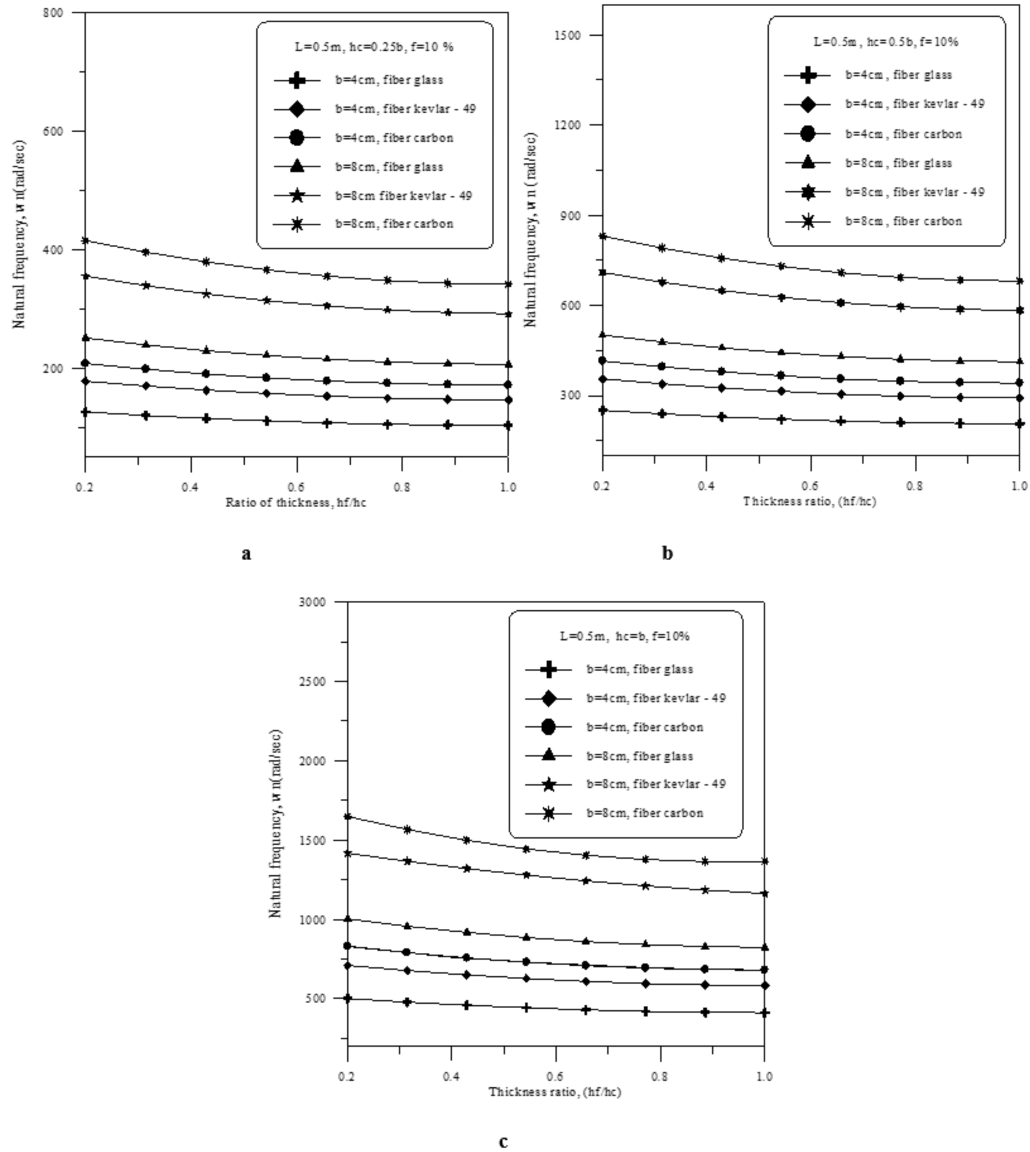
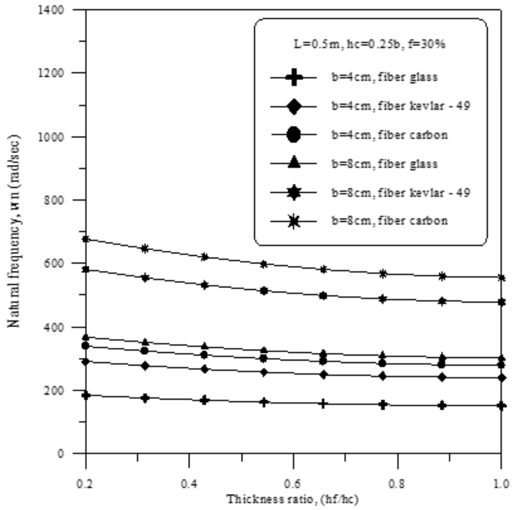
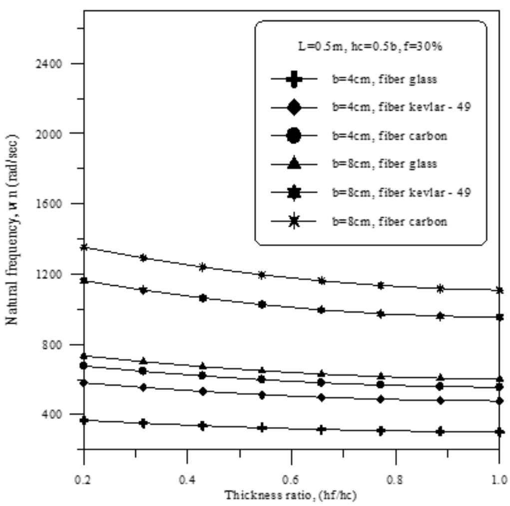
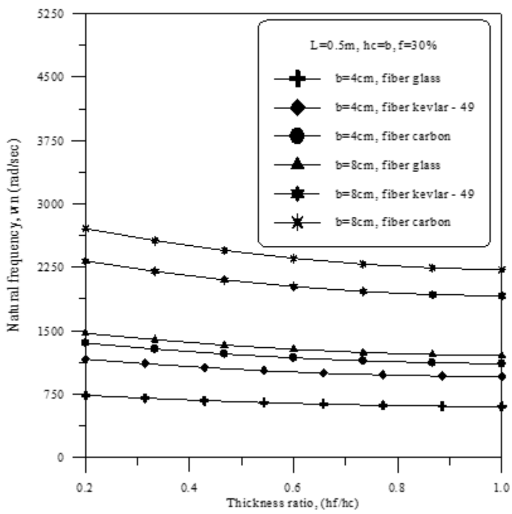


Fig. 5(a-c). Natural frequency as a function of thickness ratio (h_f/h_c) in different values of width (b) and different values of clamped thickness (h_c) at 10 % volume fraction (f) with half meter in length.





a b



c

Fig. 6(a-c). Natural frequency as a function of thickness ratio (h_f/h_c) in different values of width (b) and different values of clamped thickness (hc) at 30% volume fraction (f) with half meter in length.

Figures 5 and 6 illustrate the effect of the same variables on the natural frequency of the system that are shown in the previous figures at longer cantilever beam, where the same behavior of changing the natural frequency of the structure under the effect of any variable can be observed. It is important to be noted that there is a decrease in the natural frequency with an increase in the length of the beam. This change is due to the increase in the mass of the system, which leads to a decrease in the natural frequency of the system.

4. Conclusion

In the present study, the transverse free vibrations of a tapered cantilever beam made of different fibers in various concentrations, reinforcement polyester resin in different thickness ratio, width, thickness and length of beam. The approximate Rayleigh-Ritz method was apposite to predict the natural frequency of the system. The frequency increased with increasing the width and thickness of the beam at clamped end and the percentage of fibers; however, the frequency decreased with increasing the thickness ratio of the beam, thickness of the beam at free end and the length of beam. The results were compared with theoretical data, available in the other study for corroboration.

List of Symbols:

A_b	Area of cross section beam, (m^2).
b	Width of beam, (m^2).
c_1 & c_1	Constants.
E	Modulus of elasticity of materials, (N/m^2).
E_c	Modulus of elasticity of composite materials of beam, (N/m^2).
E_f	Modulus of elasticity of fiber, (N/m^2).
E_m	Modulus of elasticity of matrix, (N/m^2).
f	Concentration of fibers.
h_f	Thickness of beam for free end, (mm).
h_c	Thickness of beam for clamped end, (mm).
I	Second moment of area, (m^4).
I_b	Second moment of area of composite beam, (m^4).
k_{ij}	Stiffness matrix.
L	Length of the beam, (m).
m_{ij}	Mass matrix.
Y_1	Displacement amplitude of beam, (m).
ρ_c	Mass density of composite materials, (kg/m^3).
ρ_f	Mass density of fiber, (kg/m^3).
ρ_m	Mass density of matrix, (kg/m^3).
ω	Natural frequency of beam, (rad/sec).

References

Aleksandar V Nikolic (2014). Natural frequencies of a tapered cantilever beam of constant thickness and linearly tapered width, VIII International Conference “Heavy Machinery-HM 2014”, Zlatibor, 25-28, E.61-69.

Ali E. AL-Mousawy (2009). Study of Some Mechanical Properties for Polymeric Composite Material Reinforced by Fibers, Journal of AL- Qadisiya for Engineering Science, Vol.(3), No.(1), pp.(14-24).

Ashraf F. (2001). Mechanical Behavior of Composite Materials,

Benoraya, (1998) Mechanical Vibration, Prentice – Hill, Inc., U.S.A.

Byoung Koo Lee, et al. (2002). Free vibrations of tapered Beams with general boundary condition. Journal ofCivil Engineering, Vol. 6, Issue 3, p. 283-288.

Firouz-Abadi R.D., et al. (2007). An asymptotic solution to transverse free vibrations of variable-section beams. Journal of Sound and Vibration, vol.304, pp.530–540.

Hibba j. Jaffer, (2010) Effect of fibers on damping behaviors of composites materials thesis MSc., university of technology.

Khan T. et al. (2018). The challenges of natural fiber in manufacturing, material selection, and technology application, A review. J.ReinfPlast Compos., 37:770–9.

K. Rajesh and K.M. Saheb (2018). Large Amplitude Free Vibration Analysis of Tapered Timoshenko Beams Using Coupled Displacement Field Method, International Journal of Applied Mechanics and Engineering 23(3):673-688.

Mahmoud A. A., et al. (2013). Free vibrations of uniform and non-uniform Euler beam using differential transformation method. Asian Journal of Mathematics and Applications, Vol. 2013, p. 1-16.

Mohamed HussienTaha, Samir Abohadim (2008). Mathematical model for vibrations of non-uniform flexural beams, Journal of Engineering Mechanics, Vol. 15, Issue 1, p. 3-11.

Nawras H. Mostafa1 and Mustafa B. Hunain (2019). Mechanical Performance and Vibration Characteristics of Glass/Jute Fiber -Reinforced Polyester Hybrid Composites, 190404-3535-IJMME-IJENS © IJENSIJ E NS.

Osama Mohammed, et al. (2017). Free Vibration Analysis of Composite Laminated Beams, International Journal of Engineering Research and Advanced Technology (IJERAT), E-ISSN: 2454-6135, DOI: http://dx.doi.org/10.7324/IJERAT.2017.3138 Vol.3 (10).

Peng Liu, et al. (2016). Free Transverse Vibration Analysis of Axially Functionally Graded Tapered Euler-Bernoulli Beams through Spline Finite Point Method", Volume 2016 |Article ID 5891030 https://doi.org/10.1155/2016/5891030

Rajesh Korabathina, MeeraSaheb Koppanati (2016). Linear free vibration analysis of tapered Timoshenko beams using coupled displacement field method, Mathematical Models in Engineering., Volume 2, ISSUE 1.

Rishi Kumar Shukla (2013). Vibration Analysis of Tapered Beam, A Thesis in Machine Design and Analysis, Department of Mechanical Engineering National Institute of Technology, Rourkela Rourkela-769008, Odisha, INDIA.

Shik Mahmad Raffi and Tirumala Rao (2018). Linear free vibration analysis of tapered rectangular cantilever Timoshenko beams using energy field method, International Journal of Engineering &Technology, 7 (4.24) 119-123.

Zhou D., Cheung Y. K. (2016). The free vibration of a type of tapered beams. Journal of Computer Methods in Applied Mechanics and Engineering, Vol. 188, p. 203-216.