Journal of the Serbian Society for Computational Mechanics / Vol. 16 / No. 1, 2022 / pp 29-42
(10.24874/jsscm.2022.16.01.03)

FLEXURAL ANALYSIS OF THERMALLY LOADED SYMMETRIC SANDWICH BEAM

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Abstract

In the present paper, an attempt has been made to study the effect of temperature gradient on simply supported symmetric sandwich beam. A Navier's solution technique is used. The temperature profile is assumed to be linear across the thickness of a sandwich beam. A higher order beam theory (HBT) is used to include the effect of shear deformation on thermal flexural response of the sandwich beam. The theory satisfies the shear stress free boundary condition at the top and bottom surfaces of the sandwich beam. No shear correction factor is required. The principle of virtual work is used to obtain the governing equations and boundary conditions. A program has been developed in FORTRAN-77 to obtain thermal stresses and displacements in the sandwich beam for various aspect ratios. The numerical results are presented for moderately thick and thin sandwich beams to assess the performance of the theory. The validity of the present theory is verified by comparing the results with the results available in the literature. The present results are in good agreement with the results of other theories.

Keywords: Thermal stresses, sandwich beam, shear deformation, higher order beam theory, equivalent single layer theory

1. Introduction

Sandwich beams are most widely used in building, vehicle and airplane construction. This special class of composite material has high bending stiffness and low density. Sandwich beams with soft core are used in various industrial applications. They consist of two composites which are separated by thick core and lightweight face sheet material.

In the literature, the sandwich beams subjected to mechanical load are analysed by various researchers. Recently, Kulkarni and Ghugal (2020) presented the analysis of simply supported sandwich beams subjected to linear thermal load across the thickness of the beam using an equivalent single layer theory. The review of current trends in theoretical development, new designs and modern applications of sandwich structures was presented by Birman and Kardomateas (2018). The field experimental analysis of sandwich beams under bending is presented by Giordano et al. (2021). The bending of sandwich beam under three-point loads is taken into consideration. An analytical study of transverse vibration of sandwich beam with thermally induced non-uniform sectional properties was presented by Chen et al. (2019). A thermal stress analysis of functionally graded material sandwich beams subjected to thermal shocks was presented by Pandey and Pradyumna (2017). A layer wise higher order theory was adopted by the authors. The problem of influence of restrictions on the displacements in sandwich beams subjected to thermal load was examined by Pozorska and Pozorski (2014). The classical theory of sandwich beam has been applied to study the influence of restrictions on the displacements. The behaviour of sandwich laminates with low density core material under thermal loading was investigated by Padhi and Pandit (2016) using higher order zig-zag theory. The theory satisfies continuity condition of transverse shear stress at the layer interfaces and zero transverse shear stress at the top and bottom surfaces of the laminated plate.

The bending analysis of laminated and sandwich beams subjected to mechanical load was presented by Sayyed et al. (2015) using trigonometric beam theory. The displacement field of the theory includes the bending and shear components of transverse displacements. A simple four variable theory was presented by Sayyad and Ghugal (2017) to study the free vibration of antisymmetric soft core sandwich plates. This theory consists of only four unknowns. The new trigonometric shear deformation plate theory was developed for the analysis of isotropic, composite laminated and sandwich plates subjected to mechanical load by Mantari et al. (2012). The new displacement field of this theory gives results closest to the 3D elasticity solutions. The novel shear deformation theory was presented by Pawar et al. (2015) for the analysis of laminated and sandwich beams subjected to mechanical load. The effect of shear and normal deformation is included in the theory. The work is specially devoted to thick laminated and sandwich beams under plane stress conditions. The bending of laminated and sandwich plates subjected to mechanical and thermal load was presented by Cetkovic (2015) using layer-wise displacement model. The thermal displacements were evaluated under linear thermal load. A unified theory was presented by Sayyad and Ghugal (2020) to study the buckling response of functionally graded sandwich beam. This theory was built upon a classical beam theory. Bhaskar et al. (1996) presented exact thermoelastic solutions for orthotropic and anisotropic composite laminates under cylindrical and bidirectional bending. However, exact thermoelastic solutions for sandwich beams are missing in the literature.

The aim of this paper is to study the effect of temperature gradient on a symmetric sandwich beam using higher order beam theory (HBT). The thermoelastic analysis of a sandwich beam with various aspect ratios is presented. The sandwich beam is subjected to linear sinusoidal and uniform thermal loads. The results of the present equivalent single layer higher order theory are compared with the existing results to verify the accuracy of the theory.

2. Theoretical Formulation

The sandwich beam has a rectangular uniform cross section of height or thickness h and width b. The beam is assumed to be straight and has a length a along x axis. The beam is made up of two face sheets and soft core. The soft core is in between two orthotropic face sheets. In this case the width b along the y axis is very small as compared to the length a along x axis. The beam occupies the domain in Cartesian coordinate (x, y, z) as given below.

$$0 \le x \le a,\ 0 \le y \le b, -\frac{h}{2} \le z \le \frac{h}{2} \tag{1}$$

The z direction is assumed to be positive in downward direction. The upper surface of the laminated beam $\left(z=-\frac{h}{2}\right)$ is subjected to temperature distribution $T\left(x,z\right)$. The geometry of a beam is as shown below.

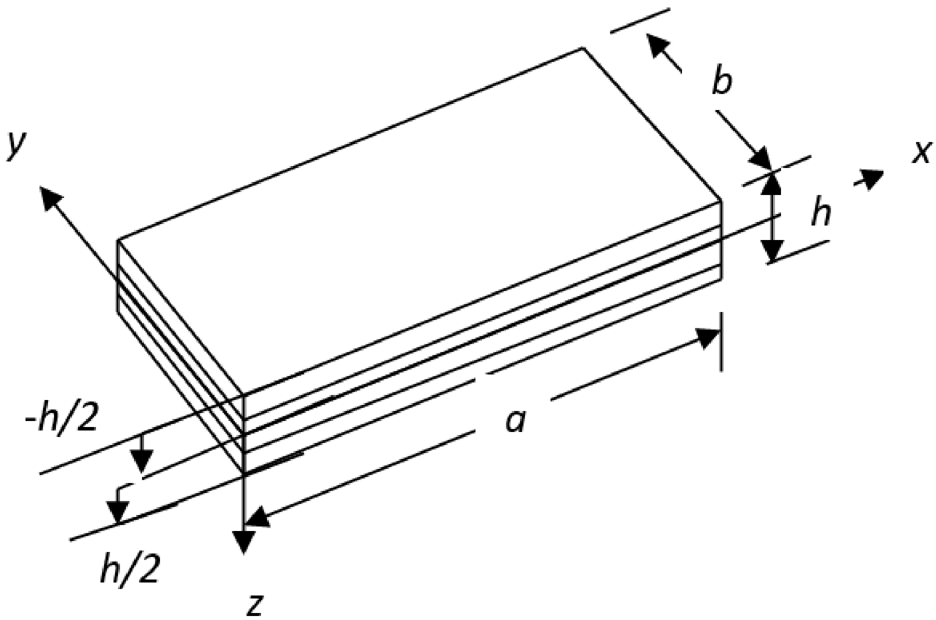

Fig. 1. Geometry and coordinate system of a beam.

2.1 Displacement field

The displacement field of higher order beam theory is obtained by expanding the displacements as cubic functions of the thickness coordinate. This represents better kinematic and avoids the need of shear correction factor. The displacement field of the higher order beam theory is of the form (Ghugal and Shimpi, 2001) and can be expressed as follows when applied to sandwich beam:

$$u\left(x,z\right)=u_0\left(x\right)+z\left[\psi\left(x\right)-\frac{4}{3}\frac{z^2}{h^2}\left(\psi\left(x\right)+\frac{\partial w}{\partial x}\right)\right] \tag{2}$$

$$w\left(x\right)=w_0\left(x\right) \tag{3}$$

Here, $\left(u,w\right)$ are the axial and transverse displacements along x and z directions respectively and $\left(\psi\right)$ is the shear rotation in the xz plane due to bending only.

2.2 Strain-displacement relationship

With reference to the definition of strains from theory of elasticity, the linear strain displacement relationships associated with the displacement field can be obtained as follows.

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + z\frac{\partial \psi}{\partial x}\left(1 - \frac{4}{3}\frac{z^2}{h^2}\right) - \frac{4}{3}\frac{z^3}{h^2}\frac{\partial^2 w}{\partial x^2} \tag{4}$$

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \left(1 - \frac{4z^2}{h^2}\right)\left[\psi + \frac{\partial w}{\partial x}\right] \tag{5}$$

2.3 Stress-strain relationship

The thermo-elastic stress-strain relationship for kth orthotropic lamina in a laminated beam can be written as:

$$\begin{Bmatrix} \sigma_x \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & 0 \\ 0 & \overline{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x T \\ \gamma_{zx} \end{Bmatrix} \tag{6}$$

where $\overline{Q}_{ij}^{(k)}$ are the reduced stiffness coefficients as given below:

$$\overline{Q}_{11}^{(k)} = \frac{E_1}{\left(1 - \mu_{12}\mu_{21}\right)},\ \overline{Q}_{55}^{(k)} = G_{13}^{(k)} \tag{7}$$

where E_1 is Young's modulus; G_{13} is shear modulus and α_x is the coefficient of thermal expansion in *x* direction.

2.4 The temperature distribution

The temperature distribution across the thickness of laminated beam is assumed to be in the form as given below:

$$T\left(x,z\right) = \frac{2z}{h} T_0\left(x\right) \tag{8}$$

In the above equation, T is the temperature change from a reference state which is a function of *x* and *z*. The thermal load T_0 is linearly varying across the thickness of laminated beam and is a function of *x*. The temperature distribution through the thickness of sandwich beam is as shown below in Fig. 2.

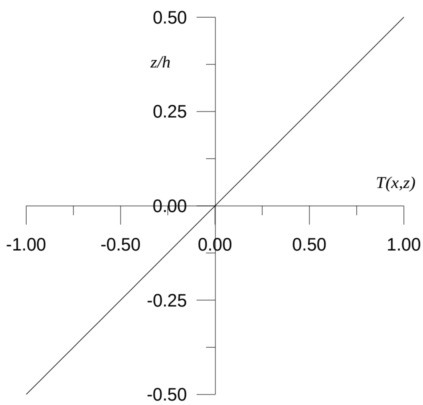

Fig. 2. Temperature distribution through the thickness of sandwich beam.

2.5. Governing equations and boundary conditions

The principle of virtual work is used to obtain the governing equations and boundary conditions. These governing equations are generalized and can be applied to laminated and sandwich beams. The principle of virtual work when applied to the laminated and sandwich beams leads to:

$$b\int_{-h/2}^{h/2}\int_{0}^{a}\left(\sigma_x \delta\varepsilon_x + \tau_{zx}\delta\gamma_{zx}\right)dx\,dz = 0 \tag{9}$$

The symbol δ denotes variational operator. The governing equations of equilibrium can be derived from the above Eq. (9) by integrating the displacements gradients in ε_i by parts and setting the coefficients of $\delta u_0, \delta w$ and $\delta\psi$ to zero separately. The governing equations obtained are as follows:

$$\delta u_0 : \left(-A_{11}\frac{\partial^2 u_0}{\partial x^2}\right) - B_{11}\frac{\partial^2 \psi}{\partial x^2} + E_{11}\frac{4}{3h^2}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right) + TB_{11}\frac{2}{h}\frac{\partial T_0}{\partial x} = 0 \tag{10}$$

$$\begin{aligned}\delta w : &\left(-E_{11}\frac{4}{3h^2}\frac{\partial^3 u_0}{\partial x^3}\right) - F_{11}\frac{4}{3h^2}\frac{\partial^3 \psi}{\partial x^3} + H_{11}\frac{16}{9h^4}\left(\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^4 w}{\partial x^4}\right) + \frac{8}{h^2}D_{55}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right)\\ &\left(-A_{55}\right)\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) - \frac{16}{h^4}F_{55}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + \frac{8}{3h^3}TF_{11}\frac{\partial^2 T_0}{\partial x^2} = 0\end{aligned} \tag{11}$$

$$\begin{aligned}\delta\psi : &\left(-B_{11}\frac{\partial^2 u_0}{\partial x^2}\right) - D_{11}\frac{\partial^2 \psi}{\partial x^2} + F_{11}\frac{4}{3h^2}\left(2\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right) + E_{11}\frac{4}{3h^2}\frac{\partial^2 u_0}{\partial x^2}\\ &\left(-H_{11}\frac{16}{9h^4}\right)\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right) + A_{55}\left(\psi + \frac{\partial w}{\partial x}\right) - \frac{8}{h^2}D_{55}\left(\psi + \frac{\partial w}{\partial x}\right)\\ &+\frac{16}{h^4}F_{55}\left(\psi + \frac{\partial w}{\partial x}\right) + TD_{11}\frac{2}{h}\frac{\partial T_0}{\partial x} - \frac{8}{3h^3}TF_{11}\frac{\partial T_0}{\partial x} = 0\end{aligned} \tag{12}$$

The variationally consistent boundary conditions associated with present beam theory at $x = 0$ and $x = a$ are as given below.

$$A_{11}\frac{\partial u_0}{\partial x} + B_{11}\frac{\partial \psi}{\partial x} - E_{11}\frac{4}{3h^2}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) - TB_{11}\frac{2}{h}T_0 = 0 \text{ or } u_0 \text{ is prescribed.} \tag{13}$$

$$\begin{aligned}&E_{11}\frac{4}{3h^2}\frac{\partial^2 u_0}{\partial x^2} + F_{11}\frac{4}{3h^2}\frac{\partial^2 \psi}{\partial x^2} - H_{11}\frac{16}{9h^4}\left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^3 w}{\partial x^3}\right) + A_{55}\left(\psi + \frac{\partial w}{\partial x}\right) + \frac{16}{h^4}F_{55}\left(\psi + \frac{\partial w}{\partial x}\right)\\ &-\frac{4}{h^2}D_{55}\left(2\psi + 2\frac{\partial w}{\partial x}\right) - \frac{8}{3h^3}TF_{11}\frac{\partial T_0}{\partial x} = 0 \text{ or } w \text{ is prescribed.}\end{aligned} \tag{14}$$

$$-E_{11}\frac{4}{3h^2}\frac{\partial u_0}{\partial x} - F_{11}\frac{4}{3h^2}\frac{\partial \psi}{\partial x} + H_{11}\frac{16}{9h^4}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + \frac{8}{3h^3}TF_{11}T_0 = 0 \text{ or } \frac{dw}{dx} \text{ is prescribed.} \tag{15}$$

$$B_{11}\frac{\partial u_0}{\partial x}+D_{11}\frac{\partial \psi}{\partial x}-F_{11}\frac{4}{3h^2}\left(2\frac{\partial \psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)-E_{11}\frac{4}{3h^2}\frac{\partial u_0}{\partial x}+H_{11}\frac{16}{9h^4}\left(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)$$
$$-TD_{11}\frac{2}{h}T_0+\frac{8}{3h^3}TF_{11}T_0=0 \text{ or } \psi \text{ is prescribed.} \tag{16}$$

The beam stiffness coefficients appeared in the governing equations of laminated beam are as given below.

$$\left(A_{11},B_{11},D_{11},E_{11},F_{11},H_{11}\right)=\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}\bar{Q}_{11}^{(k)}\left(1,z,z^2,z^3,z^4,z^6\right)dz \tag{17}$$

$$\left(TB_{11},TD_{11},TF_{11}\right)=\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}\bar{Q}_{11}^{(k)}\alpha_x^{(k)}\left(z,z^2,z^4\right)dz \tag{18}$$

$$\left(A_{55},D_{55},F_{55}\right)=\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}\bar{Q}_{11}^{(k)}\left(1,z^2,z^4\right)dz \tag{19}$$

3. Application of the theory

To assess the performance of the theory under sinusoidal and uniform temperature distribution, a simply supported symmetric sandwich beam (0/core/0) is considered as shown in Fig. 3.

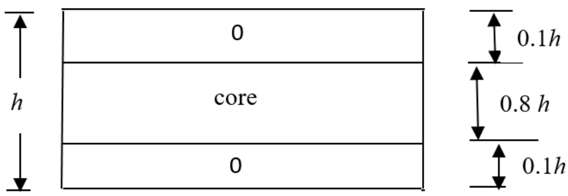

Fig. 3. Thicknesses of face sheets and soft core of sandwich beam.

The thicknesses of top and bottom face sheet, central soft core and total thickness of three-layer sandwich beam are as shown in Fig. 3.

The material properties of high modulus Graphite-Epoxy orthotropic layer are taken from Bhaskar et al. (1996) as:

$$\frac{E_L}{E_T}=25;\ \frac{G_{LT}}{E_T}=0.5;\ \frac{G_{TT}}{E_T}=0.2;\ \mu_{LT}=\mu_{TT}=0.25$$

where L and T refer to directions parallel and perpendicular to the fibres respectively. The coefficients of thermal expansions in the fibre and transverse directions are denoted by α_L and α_T respectively.

Material properties:

The material properties for sandwich beam are taken from Sayyad et al. (2015) as:

For 0^0 layers: $Q_{11}=25;\ Q_{55}=0.5$

For 90^0 layers: $Q_{11} = 1$; $Q_{55} = 0.2$

For core: $Q_{11} = 4$; $Q_{55} = 0.06$

Coefficient of thermal expansions:

$\frac{\alpha_L}{\alpha_0} = 0.3333$, $\frac{\alpha_T}{\alpha_0} = 1$

For core: $\alpha^{core} = \alpha_L = \alpha_T = 1.36\alpha_0$.

where α_0 is a normalization factor for the thermal expansion coefficients.

A simply supported symmetric sandwich beam (0/core/0) subjected to sinusoidal and uniform temperature distribution with the material properties as mentioned above is considered for the numerical analysis.

3.1. The solution scheme

Following are the boundary conditions used for simply supported sandwich beam at the edges $x = 0$ and $x = a$.

$$w = 0,\ M_x = 0,\ N_x = 0 \tag{20}$$

The closed-formed analytical solution of simply supported symmetric sandwich beams subjected to temperature distribution has been presented in this paper. The following is the solution form for $u_0(x), w(x)$ and $\psi(x)$, which satisfies the boundary conditions exactly.

$$\begin{Bmatrix} u_0 \\ w \\ \psi \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} u_{0m} \cos\frac{m\pi x}{a} \\ w_m \sin\frac{m\pi x}{a} \\ \psi_m \cos\frac{m\pi x}{a} \end{Bmatrix} \tag{21}$$

where u_{0m}, w_m, ψ_m are arbitrary constants to be determined. Thermal loads are expanded in Fourier sine series as follows:

$$T_0(x) = \sum_{m=1}^{\infty} T_{0m} \sin\frac{m\pi x}{a} \tag{22}$$

where the series constants T_{0m} are expressed as follows:

$T_{0m} = T_0$ for sinusoidal temperature distribution, $m = 1$

$T_{0m} = 4T_0/m\pi$ for uniform temperature distribution, $m = 1, 3, 5, ...$

In which T_0 is the intensity of thermal load. Substitution of solution form given by equations 21 and 22 into governing equations (10-12) results in a system of algebraic equations which can be written into a matrix form as follows:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} u_{0m} \\ w_m \\ \psi_m \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} \tag{23}$$

$$[K]\{\delta\} = \{f\} \tag{24}$$

where $[K]$ is symmetric stiffness matrix, $\{\delta\} = \{u_{0m}, w_m, \psi_m\}$ and $\{f\}$ is the generalized force vector. Solving the set of algebraic equations, unknown coefficients $\{\delta\}$ can be obtained, further substituting these coefficients into equations (21) displacements and stresses can be obtained.

The coefficients of stiffness matrix $[K]$ are as given below.

$$K_{11} = A_{11} \frac{m^2\pi^2}{a^2} \tag{25}$$

$$K_{12} = K_{21} = -E_{11} \frac{4}{3h^2} \frac{m^3\pi^3}{a^3} \tag{26}$$

$$K_{13} = K_{31} = B_{11} \frac{m^2\pi^2}{a^2} - E_{11} \frac{4}{3h^2} \frac{m^2\pi^2}{a^2} \tag{27}$$

$$K_{22} = H_{11} \frac{16}{9h^4} \frac{m^4\pi^4}{a^4} + A_{55} \frac{m^2\pi^2}{a^2} - \frac{8}{h^2} D_{55} \frac{m^2\pi^2}{a^2} + \frac{16}{h^4} F_{55} \frac{m^2\pi^2}{a^2} \tag{28}$$

$$K_{23} = K_{32} = -F_{11} \frac{4}{3h^2} \frac{m^3\pi^3}{a^3} + H_{11} \frac{16}{9h^4} \frac{m^3\pi^3}{a^3} + A_{55} \frac{m\pi}{a} - \frac{8}{h^2} D_{55} \frac{m\pi}{a} + \frac{16}{h^4} F_{55} \frac{m\pi}{a} \tag{29}$$

$$K_{33} = D_{11} \frac{m^2\pi^2}{a^2} - \frac{8}{3h^2} F_{11} \frac{m^2\pi^2}{a^2} + H_{11} \frac{16}{9h^4} \frac{m^2\pi^2}{a^2} + A_{55} - \frac{8}{h^2} D_{55} + \frac{16}{h^4} F_{55} \tag{30}$$

The elements of thermal load vector $\{f\}$ are as given below.

$$\{f\} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} \tag{31}$$

$$f_1 = -TB_{11} \frac{2}{h} \frac{m\pi}{a} T_{0m} \tag{32}$$

$$f_2 = \frac{8}{3h^3} TF_{11} \frac{m^2\pi^2}{a^2} T_{0m} \tag{33}$$

$$f_3 = -TD_{11} \frac{2}{h} \frac{m\pi}{a} T_{0m} + \frac{8}{3h^3} TF_{11} \frac{m\pi}{a} T_{0m} \tag{34}$$

4. Results and discussion

In this paper, thermal displacements and stresses are determined for a symmetric simply supported sandwich beam subjected to sinusoidal and uniform temperature distribution linearly varying across the thickness of a sandwich beam. The results are presented for side to thickness ratios 4, 10 and 100. The side to thickness ratio is also termed as aspect ratio and denoted by (S). The results are presented in the following normalized forms for the purpose of discussion.

$$\bar{u}\left(0,-\frac{h}{2}\right)=\frac{u}{\alpha_1 T_0 a},\ \bar{w}\left(\frac{a}{2},0\right)=\frac{wh}{\alpha_1 T_0 a^2},\ \bar{\sigma}_x\left(\frac{a}{2},-\frac{h}{2}\right)=\frac{\sigma_x}{E_2\alpha_1 T_0},\ \bar{\tau}_{zx}\left(0,0.4h\right)=\frac{\tau_{zx}}{E_2\alpha_1 T_0} \tag{35}$$

Sinusoidal Thermal load (0/core/0)						
Source	Model	S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{EE}$
Present	HBT	4	0.4330	0.2882	-10.9900	0.7485
Kulkarni and Ghugal (2020)	PSDT	4	0.4330	0.2917	-11.0686	0.7485
Kulkarni and Ghugal (2020)	SSDT	4	0.4575	0.2839	-10.9570	0.7805
Kulkarni and Ghugal (2020)	CBT	4	0.4592	0.2924	-11.0687	0.7824
Present	HBT	10	0.4548	0.2923	-10.7186	0.3107
Kulkarni and Ghugal (2020)	PSDT	10	0.4548	0.2653	-11.0687	0.3107
Kulkarni and Ghugal (2020)	SSDT	10	0.4601	0.2986	-11.1628	0.3138
Kulkarni and Ghugal (2020)	CBT	10	0.4592	0.2924	-11.0687	0.3130
Present	HBT	100	0.4592	0.2924	-11.0654	0.0313
Kulkarni and Ghugal (2020)	PSDT	100	0.4592	0.2921	-11.0651	0.0313
Kulkarni and Ghugal (2020)	SSDT	100	0.4589	0.2922	-11.0731	0.0313
Kulkarni and Ghugal (2020)	CBT	100	0.4592	0.2924	-11.0687	0.0313
Uniformly Distributed Thermal load (0/core/0)						
Present	HBT	4	0.6153	0.3566	-11.4442	9.9560
Kulkarni and Ghugal (2020)	PSDT	4	0.6153	0.3600	-10.7163	9.9643
Kulkarni and Ghugal (2020)	SSDT	4	0.7042	0.3501	-10.5995	9.9451
Kulkarni and Ghugal (2020)	CBT	4	0.7068	0.3607	-10.7172	9.9618
Present	HBT	10	0.6767	0.3606	-11.4996	3.8600
Kulkarni and Ghugal (2020)	PSDT	10	0.6767	0.3335	-10.7179	3.9844
Kulkarni and Ghugal (2020)	SSDT	10	0.7074	0.3690	-10.8683	3.9797
Kulkarni and Ghugal (2020)	CBT	10	0.7068	0.3607	-10.7172	3.9847
Present	HBT	100	0.7062	0.3607	-10.7606	0.3946
Kulkarni and Ghugal (2020)	PSDT	100	0.7042	0.3604	-10.7603	0.3946
Kulkarni and Ghugal (2020)	SSDT	100	0.7064	0.3605	-10.8124	0.3986
Kulkarni and Ghugal (2020)	CBT	100	0.7068	0.3607	-10.7172	0.3985

Table 1. Thermal displacements and stresses in three-layer symmetric sandwich beam (o/core/0).

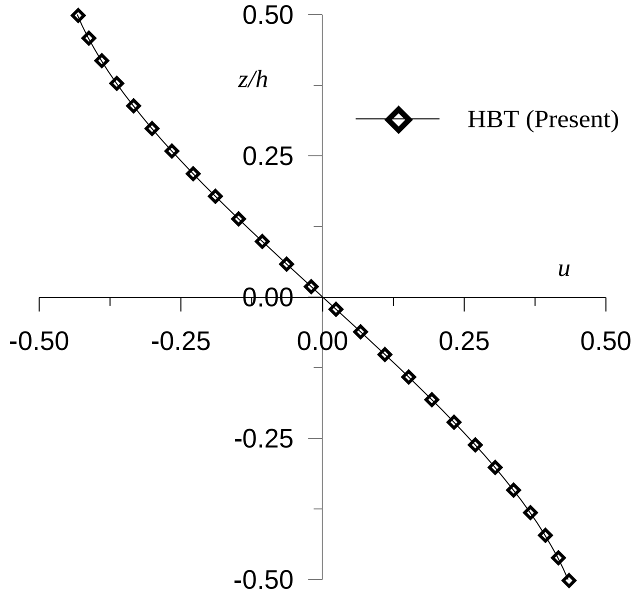

Fig. 4. Through thickness variation of normalized in-plane displacement $(\bar{u})$ in three-layer sandwich beam (0/core/0) for aspect ratio 4 under sinusoidal thermal load.

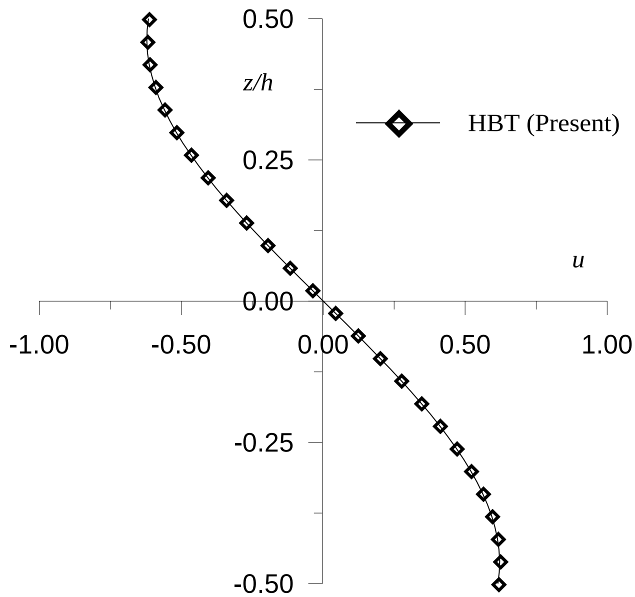

Fig. 5. Through thickness variation of normalized in-plane displacement $(\bar{u})$ in three-layer sandwich beam (0/core/0) for aspect ratio 4 under uniformly distributed thermal load.

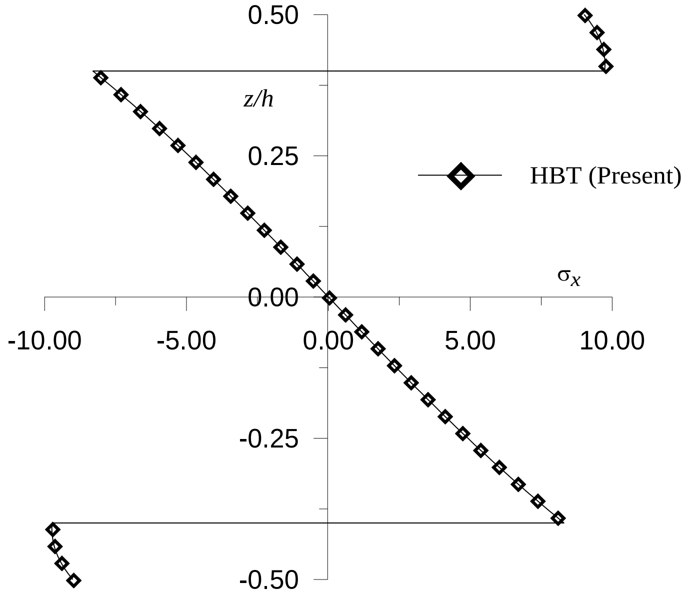

Fig. 6. Through thickness variation of normalized inplane normal stress $\left(\bar{\sigma}_x\right)$ in three-layer sandwich beam (0/core/0) for aspect ratio 4 under single sinusoidal thermal load.

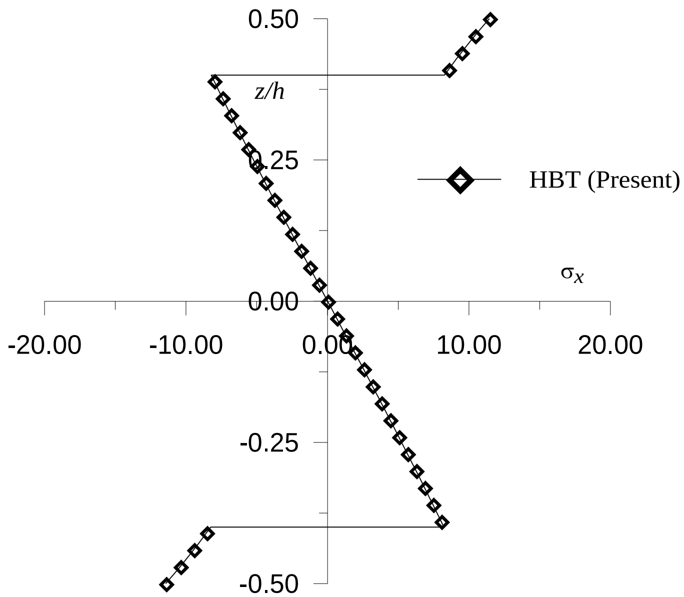

Fig.7. Through thickness variation of normalized inplane normal stress $\left(\bar{\sigma}_x\right)$ in three-layer sandwich beam (0/core/0) for aspect ratio 4 under uniformly distributed thermal load.

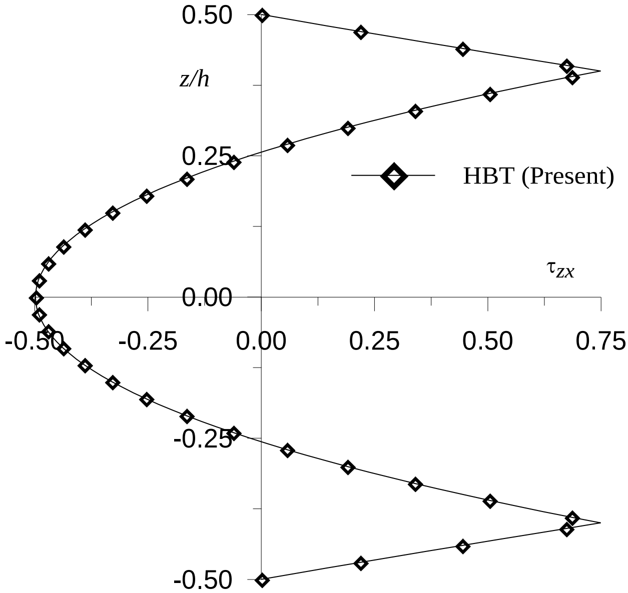

Fig. 8. Through thickness variation of normalized transverse shear stress $\left(\bar{\tau}_{zx}\right)$ in three-layer sandwich beam (0/core/0) for aspect ratio 4 under sinusoidal thermal load.

In-plane Displacement $\left(\bar{u}\right)$: The results of in-plane displacement obtained by the present theory are shown in Table 1 and are in good agreement with parabolic shear deformation theory (PSDT) and sinusoidal shear deformation theory (SSDT) presented by Kulkarni and Ghugal (2020). The present theory shows a significant change in in-plane displacement when aspect ratio changes from 4 to 100 under sinusoidal and uniform thermal loads. The through thickness variation of in-plane displacement across the thickness of sandwich beam under sinusoidal and uniform thermal load is shown in Figs. 4 and 5, respectively. The realistic variation of this displacement across the thickness of the beam is observed under both thermal loads. The variation of this displacement is more nonlinear under uniform temperature compared to that under the sinusoidal linear thermal load.

Transverse Displacement $\left(\bar{w}\right)$: The results of transverse displacements under sinusoidal and uniform thermal load obtained by the present theory are shown in Table 1. The results of transverse displacement obtained by the present theory (HBT) are in good agreement with the results presented by Kulkarni and Ghugal (2020). This shows that the present theory is suitable for thick and thin sandwich beams. The results of this displacement converge to classical beam theory at higher aspect ratio due to attenuation of shear deformation effect in thin beam limit.

Normal Stress $\left(\bar{\sigma}_x\right)$: The results of this stress are shown in Table 1. This stress does not show significant variation under sinusoidal and uniform thermal loads for various aspect ratios. The results of normal stress obtained by the present theory (HBT) are almost identical with those of parabolic shear deformation theory (PSDT) presented by Kulkarni and Ghugal (2020) under sinusoidal and uniform thermal loads. The through thickness variations of these stresses under sinusoidal and uniform thermal loads are shown in Figs. 6 and 7, respectively. A realistic variation is observed across the thickness of a sandwich beam under sinusoidal and uniform thermal loads. The variation of this stress is more nonlinear in each layer under sinusoidal temperature compared to that under the uniform linear thermal load.

Transverse Shear Stresses($\bar{\tau}_{zx}^{EE}$): Transverse shear stresses in a sandwich beam are obtained by using 2D elasticity equations under sinusoidal and uniform thermal loads. These stresses are shown in Table 1. Transverse shear stresses obtained by the present theory (HBT) via integration of 2D elasticity equations $\left(\sigma_{ij,j} = 0, \quad i, j = x, z\right)$ satisfy zero shear stress conditions at the top and bottom surfaces of the sandwich beam and the continuity conditions at the interfaces between the layers. These stresses are in good agreement with the transverse stresses given in Kulkarni and Ghugal (2020). The through thickness variation of this stress under sinusoidal thermal load is shown in Fig. 8. The figure shows a realistic variation (parabolic in core and linear in face sheets) across the thickness of sandwich beam.

5. Conclusions

A higher order beam theory is used to study the thermal response of symmetric three-layer sandwich beam. The thermal flexural behaviour of a sandwich beam has been investigated under sinusoidal and uniformly distributed linear thermal loads. The theory shows parabolic variation of transverse shear stresses through the thickness of the sandwich beam. The effect of shear deformation is seen in the results obtained by the present theory. The present theory is applicable to thin as well as moderately thick sandwich beams. The present theory obviates the need of shear correction factor and satisfies shear stress free boundary conditions at the top and bottom surfaces of the sandwich beam. Numerical results for normalized displacements and stresses for various aspect ratios are presented in tabular form and their realistic variations across the thickness of very thick beam are presented graphically to demonstrate the validity of present theory. The results of the present theory agree closely with those of existing higher order theories which demonstrate the validity and efficacy of the present higher order beam theory under thermal environment.

Acknowledgements: The authors acknowledge the support and encouragement from Symbiosis Institute of Technology, Pune and Government Engineering College, Karad to complete this work.

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