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ON THE EQUILIBRIUM OF STRATIFIED VISCOELASTIC PLASMA WITH QUANTUM PRESSURE AND SUSPENDED PARTICLES SATURATING POROUS MEDIUM

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Abstract

Stability of stratified incompressible viscoelastic plasma arranged in horizontal strata with quantum pressure and dust particles saturated by a porous medium is investigated. The rheology of the plasma is described by the Walters'(model B').The set of non-linear partial differential equations defining the physical system are reduced to linear ordinary differential equations by using the perturbation method, linear theory and normal mode technique. The density, viscosity, viscoelasticity and quantum pressure are assumed to stratify exponentially along the vertical, to obtain exact solutions satisfying the physical boundary conditions and the dispersion relation. The values of growth rate of the unstable perturbed modes are computed numerically to investigate roles that the various variables play on the stability on the considered physical system and are shown graphically. It is observed that the suspended dust particles density and relaxation time factor have a destabilizing effect on the system; whereas viscoelasticity in the presence of suspended dust particles lead to more damping in the frequency of perturbed waves. This work finds applications in diverse fields viz. modern technology, industries, astrophysics, petroleum oil additives, equipment of aero planes etc.

Keywords: Quantum pressure, stratified plasma, Walters' (model B'), porous medium, dust (suspended) particles.

1. Introduction

Rayleigh-Taylor instability (RTI) arises from the equilibrium of an incompressible in which density of a layer is higher than its adjacent layer and is continuously varying along the vertical chosen direction under the action of vertical gravity field Rayleigh (1900) and Taylor (1950). The experimental demonstration of the development of the Rayleigh–Taylor instability (in case of heavier fluid overlaying a lighter one, is accelerated towards it) was described by Lewis (1950). This instability is of significance in the extraction of oil from the earth to eliminate water drops, in analyzing the frequency of gravity waves formed in deep oceans, lazer and inertial confinement fusion (ICF) etc.

Quantum physics is the branch of science that deals with discrete, individual units of energy is called quanta as described in quantum theory. The pressure term in the equations of motion is divided into two terms, $p = p^C + p^Q$. The classical pressure, p^C and quantum pressure, p^Q have been investigated by Gardner and Haas (1994, 2005), by using Wingen principle and Schrödinger wave equation. In the momentum equations, the classical pressure and the quantum pressure (Bohm vector potential) are defined by $(-\nabla p)$ and $Q = \frac{\hat{h}}{2m_e m_i}\rho\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$, where $\hat{h}, \rho, m_e$ and m_i are the Plancks constant, density of fluid, masses of electron and ion, respectively.

Plasma is comprised of an electrically neutral medium of positive and negative particles. The mobility of charges influences each other's fields due to the generated electrical currents due to magnetic fields. As far as the electron plasma frequency is larger than the electron-neutral collision frequency, dominance of electrostatic interactions takes place over the ordinary gas kinetics processes. Plasma is an ionized gas that is also called the fourth state of matter. To form plasma, the gas may be heated or an excess of free electrons is needed to displace electrons in the atoms and molecules of the gas. The degree of ionization of a plasma is defined as proportion of charged particles to the total number of particles including neutral and ions. It is mainly controlled by temperature. It is surprising that a partially ionized gas with hardly 1% ionized particles, may act as plasma. Lightning is an example of plasma present at Earth's surface.

Partially ionized plasma is presented by a condition that often exists everywhere. The interaction between the ionized and neutral gas components is one of the situations of great importance in cosmic physics. Ionized hydrogen has been reported to be limited to certain sharply bounded regimes of space by Strömgren (1939). O-type stars and their clusters are essentially non-ionized. Gardner (1994) has introduced the quantum hydro-dynamic model (QHDM) for semiconductor physics to describe the mobility of charge, energy and momentum in plasma. The impact of quantum pressure with inclusion of magnetic field on RTI has been investigated by Hoshoudy (2009) and the vertical magnetic field is found to bring more stability on the growth rates of unstable configuration along with quantum effect. External magnetic field effect on RTI in non-homogeneous rotating plasma/fluids with an angular velocity has been demonstrated by Hoshoudy (2012). The plasma or fluids have been taken to be Newtonian in the aforesaid studies.

With the growing potential of the non-Newtonian fluids saturating a porous medium in industrial processes, petroleum engineering and astrophysical situations (Larson (1992), Bird et al. (1987)), the researchers are attracted and show interest to investigate RTI of such fluids worldwide. There is a variety of models of such fluids whose behavior is described by constitutive relations. We are interested therein the Walters' (model $B^{'}$) proposed theoretically by Walter (1960). RTI for both cases of superposed and exponentially stratified non-Newtonian fluids have been investigated theoretically and analytically by many authors (Sharma and Sharma (1977), Sharma and Kumar (2004), Sunil et al. (2004), Kumar and Lal (2007), Kumar and Singh (2011)). Stability of stratified viscoelastic fluid through porous medium with magnetic field has been established by Sharma and Urvashi (2006) and magnetic field is found to stabilize the system substantially.

In general, comets may comprise of a dusty snowball, being a mixture of frozen gases and transform from gas to solid and vice versa. Dust (suspended) particles play a vital role in industries, space, astrophysical plasmas and laboratory problems. Interstellar media has been found to contain small particles in the outer atmosphere termed as grains, galaxies and get ejected into the medium. This problem was first demonstrated by Alfvén and Carlqvis (1978) to analyze the formation of stars through Jeans instability. Sharma (1975) studied the impact of dust particles on the gravitational instability of an infinite homogeneous gas–particle medium, while that of a finitely conducting, rotating with uniform vertical magnetic field has been investigated by Sharma and Sharma (1980). Prajapati et al. (2009) have also discussed the influence of magnetic field of two streaming superimposed fluids. Later, Prajapati and Chhajlani (2010) found the impact of suspended dust particles on streaming superposed fluids in porous media and observed substantial stabilizing influence of suspended dust particles density and medium porosity on the unstable growth rates. A time dependent flow problem of dusty fluid flow in a rotating horizontal channel has been studied by Singh et al. (2016). Dolai and Prajapati (2018) have investigated the stability of two rotating superposed dusty plasmas and found a stabilizing influence in the presence of magnetic field, dust cloud and rotation. Spacecraft observations have emphasized upon a vital role of dust particles in dynamics of atmosphere and diurnal with variations in the surface temperature of Martin weather.

The empirical formula of flow of a fluid saturating a homogeneous and isotropic porous medium has been postulated by Darcy (1856). It replaces the usual viscous and viscoelastic terms in the equations of motion in an incompressible Walters' (model $B^{'}$) fluid by the resistance terms $\left[-\frac{1}{k_1}\left(\mu-\mu^{'}\frac{\partial}{\partial t}\right)\boldsymbol{q}\right]$ where $\boldsymbol{q}, \mu, \mu^{'} and\, k_1$ are filter velocity of pure fluid, kinematic viscosity, kinematic viscoelasticity and medium permeability.

Yadav and Ray (1991) have analyzed the unsteady flow of n-immiscible Walters'(model $B^{'}$) fluid in a porous medium within two parallel plates to include a vertical magnetic field. Sharma and Rana (1999) analyzed the stability of viscoelastic fluid with horizontal magnetic field and rotation with a uniform vertical angular velocity in a porous medium and the system is found to be stabilized substantially due to magnetic field which was otherwise unstable. Numerical investigations on the stability of viscoelastic Walters'(model $B^{'}$) fluid/plasma with exponential variations in density ,viscosity, viscoelasticity and quantum pressure in porous medium has been established by Sharma et al. (2014) and quantum pressure is found to bring about more stability for a certain wave number band, on the growth rate of unstable configuration. Prajapati (2016) has analyzed the RTI of strongly coupled viscoelastic fluid with non-uniform magnetic field and rotation and magnetic field, viscoelasticity and rotation are found to suppress the RTI substantially. Hoshoudy and Awasthi (2020) have analytically demonstrated the compressibility effects in Kelvin Helmholtz instability, KHI and RTI of two immiscible fluids in a porous medium and found that compressibility suppresses the Kelvin Helomholtz instability.

Motivated by diverse applications of various parameters mentioned above, the present work aims at to examine the stability of stratified viscoelastic plasma embedded with dust particles and quantum pressure saturating a porous layer, which is primarily devoted to the research work of Sharma et al. (2014). The viscoelastic behavior of the plasma is described by Walter'(model $B^{'}$) and Darcy model (1856) is deployed to explore the characteristics of porous medium.

2. Physical problem and mathematical analysis

An infinitely non-compressible, infinitely extending viscoelastic Walters' (model $B^{'}$) heterogeneous (heavy) plasma bounded by the planes and is arranged in horizontal strata of electrons and immobile ions saturating porous medium. The density, coefficient of viscosity, viscoelasticity and hydrodynamic pressure are assumed to vary with respect to vertical co-ordinate of inertial frame of reference i.e. z-axis.

Due to the dust particles, the fluid exert a force on particles which is equal and opposite to that of particles, an extra force term given by $\frac{KN}{\varepsilon}(\boldsymbol{V}-\boldsymbol{q})$ is added in the equations of motion where $\boldsymbol{V}, N, \varepsilon$ are suspended particle velocity, the particle number density, medium permeability and K is the Stokes' coefficient of resistance, given by $K = 6\pi a\mu$ for spherical particles, a is the particle radius. The inter-particle reactions are ignored due to large enough distance among the particles.

The modified conservation equations of the problem (Chandrasekhar (1961), Hoshoudy (2011, 2016)) are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial}{\partial t}+\frac{1}{\varepsilon}(\nabla.\boldsymbol{q})\right]\boldsymbol{q} = -\nabla p + \rho\boldsymbol{g} - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\boldsymbol{q} + \boldsymbol{Q} + \frac{KN}{\varepsilon}(\boldsymbol{V}-\boldsymbol{q}), \tag{1}$$

$$\nabla.\boldsymbol{q} = 0, \tag{2}$$

$$\frac{\rho}{\varepsilon}\frac{\partial\rho}{\partial t} + (\boldsymbol{q}.\nabla)\rho = 0. \tag{3}$$

The equations of motion and continuity of the dust particles are given by

$$mN\left[\frac{\partial\boldsymbol{V}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{V}.\nabla)\boldsymbol{V}\right] = KN(\boldsymbol{q}-\boldsymbol{V}), \tag{4}$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla.(N\boldsymbol{V}) = 0, \tag{5}$$

where m is the mass of dust particles and mN represents mass of particles contained in unit volume.

The buoyancy force on the particles is neglected.

The basic state solutions for which the stability is to be examined, is characterized by

$$\boldsymbol{q} = (0,0,0), \boldsymbol{V} = (0,0,0), \rho = \rho(z) \text{ and } \boldsymbol{Q} = \boldsymbol{Q}(z). \tag{6}$$

The stability of the system of flow of motion is examined by superimposing perturbations with infinite amplitude on the basic state solutions (6).

Let $\boldsymbol{q}(u,v,w), \boldsymbol{V}(l,r,s), \delta\rho, \delta p, \delta\boldsymbol{Q}(Q_{x1}, Q_{y1}, Q_{z1})$ represent the respective perturbations in fluid velocity, particle velocity, density, pressure and quantum pressure, which are assumed to be functions of space as well as time variables. Thus the disturbed flow is represented by

$$\boldsymbol{q} = (0,0,0)+(u,v,w), \boldsymbol{V} = (0,0,0)+(l,r,s), p = p(z)+\delta p, \rho = \rho(z)+\delta\rho$$

$$\text{and } \boldsymbol{Q} = \boldsymbol{Q}(z)+\delta\boldsymbol{Q}. \tag{7}$$

Using perturbations given by (7) and linear theory (neglecting the products of second and higher order perturbations because their contributions are infinitesimally very small), equations (1) – (4) in the linear perturbed form become

$$\frac{\rho}{\varepsilon}\frac{\partial\boldsymbol{q}}{\partial t} = -\delta p + \delta\rho\boldsymbol{g} - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\boldsymbol{q} + \delta\boldsymbol{Q} + \frac{KN}{\varepsilon}(\boldsymbol{V}-\boldsymbol{q}), \tag{8}$$

$$\nabla.\boldsymbol{q} = 0, \tag{9}$$

$$\frac{\rho}{\varepsilon}\frac{\partial \delta\rho}{\partial t} + (\boldsymbol{q}.\nabla)\,\rho = 0, \tag{10}$$

where

$$\delta \boldsymbol{Q} = \frac{\hat{h}^2}{2m_e m_i}\left[\begin{array}{l}\frac{1}{2}\nabla\left(\nabla^2\delta\rho\right) - \frac{1}{2\rho}\nabla\delta\rho\nabla^2\rho - \frac{1}{2\rho}\nabla\rho\nabla^2\delta\rho + \frac{\delta\rho}{2\rho^2}\nabla\rho\nabla^2\rho - \frac{1}{2\rho}\nabla\left(\nabla\rho\nabla\delta\rho\right) + \frac{\delta\rho}{4\rho^2}\nabla\left(\nabla\rho\right)^2 + \\ \frac{1}{2\rho^2}\left(\nabla\rho\right)^2\nabla\delta\rho + \frac{1}{\rho^2}\left(\nabla\rho\nabla\delta\rho\right)\nabla\rho - \frac{\delta\rho}{\rho^3}\left(\nabla\rho\right)^3\end{array}\right]$$

Both the bounding surfaces are supposed to be rigid. Thus, appropriate boundary conditions to be satisfied by the problem are

$$w = 0\ ,\ Dw = 0 \text{ at } z = 0 \text{ and } z = d. \tag{11}$$

To examine the stability of the system, perturbations are analyzed in terms of modes by ascribing a wave number, whose dependence on the space (x, y, z) and time, t is of the form

$$f^{'}(x, y, z, t) = f(z)\exp i\left(k_x x + k_y y + nt\right), \tag{12}$$

where $k = k_x{}^2 + k_y{}^2$ is the resultant real wave number and n is the complex growth rate.

Now, using the expression (12) and equation (4), equations (7) - (9) in the Cartesian form are

$$\frac{\rho}{\varepsilon}N^{'}u = -ik_x\delta p - \frac{1}{k_1}\left(\mu + \mu^{'}in\right)u + Q_{x1}, \tag{13}$$

$$\frac{\rho}{\varepsilon}N^{'}v = -ik_y\delta p - \frac{1}{k_1}\left(\mu + \mu^{'}in\right)v + Q_{y1}, \tag{14}$$

$$\frac{\rho}{\varepsilon}N^{'}w = -D\delta p - g\delta\rho - \frac{1}{k_1}\left(\mu + \mu^{'}in\right)w + Q_{z1}, \tag{15}$$

$$ik_x u + ik_y v + Dw = 0, \tag{16}$$

$$\varepsilon in\delta\rho = wD\rho, \tag{17}$$

where $N^{'} = in + \dfrac{imnN}{\rho\left(1 + \dfrac{imn}{k}\right)}$,

$$Q_{x1} = \frac{\hat{h}^2}{2\varepsilon n m_e m_i}\left[\begin{array}{l}\frac{1}{2}D\rho D^2 w + \left\{D^2\rho - \frac{1}{2\rho}\left(D^2\rho\right)^2\right\}Dw \\ + \left\{\frac{1}{2}D^3\rho - \frac{1}{\rho}D\rho D^2\rho - \frac{k^2}{2}D\rho + \frac{1}{2\rho^2}\left(D\rho\right)^3\right\}w\end{array}\right],$$

$$Q_{y1} = \frac{k_y}{k_x}Q_{x1},$$

$$Q_{z1} = \frac{\hat{h}^2}{2\varepsilon n m_e m_i}\begin{bmatrix} \frac{1}{2}D\rho D^3 w + \left\{\frac{3}{2}D^2\rho - \frac{1}{\rho}\left(D^2\rho\right)^2\right\}D^2 w \\ +\left\{\frac{1}{2}D^3\rho - \frac{1}{\rho}D\rho D^2\rho - \frac{k^2}{2}D\rho + \frac{3}{2\rho^2}\left(D\rho\right)^3\right\}Dwk^2 \\ +\frac{1}{2}D^4\rho - \frac{1}{\rho}D\rho D^3\rho - \frac{k^2}{2}D^2\rho - \frac{1}{\rho}\left(D^2\rho\right)^2 \\ +\frac{5}{2\rho^2}\left(D\rho\right)^2 D^2\rho + \frac{k^2}{2\rho}\left(D\rho\right)^2 - \frac{1}{\rho}\left(D\rho\right)^4 \end{bmatrix}.$$

Eliminating variables $u, v, \delta p$ and using equations (13) - (17), we get characteristic equation in w as

$$\rho k^2 \frac{n^*}{in} w - \frac{\rho n^*}{in} D^2 w - \frac{n^*}{in} D\rho Dw + \frac{gk^2}{in}\left(D\rho\right)w + \frac{k^2}{in}\left(\frac{\hat{h}}{4m_e m_i}\right)\begin{bmatrix} \frac{1}{\rho}\left(D\rho\right)^2 Dw - \frac{1}{\rho^2}\left(D\rho\right)\left\{\left(D\rho\right)^2 - 2\rho D^2\rho\right\} \\ Dw - \frac{k^2}{\rho}\left(D\rho\right)^2 w \end{bmatrix} = 0, \tag{18}$$

where $n^* = in\left\{n' + \frac{\varepsilon}{k_1}\left(\nu + \nu' in\right)\right\}$

3. Solution of the problem

Now the fluid density, coefficient of viscosity, viscoelasticity, pressure of quantum plasma are assumed to stratify continuously of the form

$$\begin{gathered} \rho_0(z) = \rho_0(0)\exp\left(\frac{z}{L_D}\right), \mu_0(z) = \mu_0(0)\exp\left(\frac{z}{L_D}\right), \\ \mu_0'(z) = \mu_0'(0)\exp\left(\frac{z}{L_D}\right), n_{q0}(z) = n_{q0}(0)\exp\left(\frac{z}{L_D}\right), \end{gathered} \tag{19}$$

where $\rho_0(0), \mu_0(0), \mu_0'(0) n_{q0}(0)$ and L_D are constants.

Using the stratifications of the form given by (19), the characteristic equation (18) transforms to

$$\left(n^* - {n_q}^2\right)D^2 w + \frac{1}{L_D}\left(n^* - {n_q}^2\right)Dw - k^2\left[\left(n^* - {n_q}^2\right) + \frac{g}{L_D}\right]w = 0, \tag{20}$$

where ${n_q}^2 = \frac{\hat{h}^2 k^2}{2\varepsilon n m_e m_i}$ represents the parameter accounting for quantum pressure.

Using the boundary conditions (10), the equation (20) implies that

$$D^2 w = 0 \text{ at } z = 0 \text{ and } z = d. \quad (21)$$

Therefore, the exact analytical base functions of equation (20) satisfying the boundary conditions (10) and (21), are taken as

$$w = \sin\left(\frac{m\pi}{d} z\right)\exp(\lambda z), \quad (22)$$

where m is a positive integer.

Substituting the solution given by (22) in equation (20), we obtain

$$\left(n^* - n_q^{\ 2}\right)\left[-\left(\frac{m\pi}{d} z\right) + 2\lambda\left(\frac{m\pi}{d}\right)\cos\left(\frac{m\pi}{d} z\right) + \lambda^2 \sin\left(\frac{m\pi}{d} z\right)\right] +$$
$$\frac{1}{L_D}\left(n^* - n_q^{\ 2}\right)\begin{bmatrix}\left(\frac{m\pi}{d}\right)\cos\left(\frac{m\pi}{d} z\right) + \lambda\sin\left(\frac{m\pi}{d} z\right) \\ -k^2\left(n^* - n_q^{\ 2}\right) + \frac{g}{L_D}\lambda\sin\left(\frac{m\pi}{d} z\right)\end{bmatrix} = 0. \quad (23)$$

Equating the coefficients of $\sin\left(\frac{m\pi}{d} z\right)$ and $\cos\left(\frac{n'\pi}{d} z\right)$ of equation (23) yield that

$$\left(n^* - n_q^{\ 2}\right)\left[\lambda^2 - \left(\frac{m\pi}{d}\right)^2 + \frac{\lambda}{L_D}\right] - k^2\left[\left(n^* - n_q^{\ 2}\right) + \frac{g}{L_D}\right] = 0 \quad (24)$$

and

$$\left(n^* - n_q^{\ 2}\right)\left[2\lambda\left(\frac{m\pi}{d}\right) + \frac{1}{L_D}\left(\frac{m\pi}{d}\right)\right] = 0. \quad (25)$$

As $n^* \neq n_q^{\ 2}$, therefore, equation (25) implies that $\lambda = -\frac{1}{2L_D}$.

Putting this value of λ in equation (24), the dispersion relation so obtained is

$$n^* = n_q^{\ 2} - \frac{4gk^2 d^2 L_D}{d^2 + 4k^2 d^2 L_D^{\ 2} + 4m^2\pi^2 L_D^{\ 2}},$$

which on substituting the value of n^* implies to

$$in\left\{in\left(\frac{1 + \tau^2 n^2 + \alpha_0 - \alpha_0 \tau in}{1 + \tau^2 n^2}\right) + \frac{\varepsilon}{k_1}\left(\nu + \nu' in\right)\right\} = n_q^{\ 2} + \frac{4gk^2 d^2 L_D}{d^2 + 4k^2 d^2 L_D^{\ 2} + 4m^2\pi^2 L_D^{\ 2}}, \quad (26)$$

where $\tau = \frac{m}{6\pi a\mu}$ and $\alpha_0 = \frac{mN}{\rho}$ represent the relaxation time and mass concentration of dust particles, respectively.

Now introducing the non-dimensional quantities

$$n^* = \frac{n_1^2}{n_{pe}^2}, n^*{}_q{}^2 = \frac{\hat{h}^2}{4m_e m_i L_D{}^4 n_{pe}{}^2}, \varepsilon^* = \frac{\varepsilon}{n_{pe}}, \nu^* = \frac{\nu}{n_{pe}}, \nu^{'*} = \frac{\nu^{'}}{n_{pe}},$$

$$k_1{}^* = \frac{k_1}{n_{pe}}, d^{*2} = \frac{d^2}{L_D{}^2}, k^{*2} = k^2 L_D{}^2, g^* = \frac{g}{n_{pe}{}^2 L_D}, \tau^* = \frac{\tau}{n_{pe}}$$

in equation (26) (the asterisks are omitted for the sake of convenience) yields

$$a_1 (in)^4 + a_2 (in)^3 + a_3 (in)^2 + a_4 (in) + a_5 = 0, \tag{27}$$

where $n_{pe} = \left(\frac{pe^2}{m_e{}^2 \varepsilon_0} \right)^{\frac{1}{2}}$ is the frequency of plasma and

$$a_1 = \tau^2 \left(k_1 + \varepsilon \nu^{'} \right), a_2 = \tau \left(\alpha_0 k_1 + \tau \nu \varepsilon \right),$$

$$a_3 = k_1 + \alpha_0 k_1 + \varepsilon \nu^{'} + n_q{}^2 k^2 \tau^2 k_1 - \frac{4gk^2 d^2 k_1}{d^2 + 4k^2 d^2 + 4m^2 \pi^2} \tag{28}$$

$$a_4 = \varepsilon \nu, a_5 = n_q{}^2 k^2 k_1 - \frac{4gk^2 d^2 k_1}{d^2 + 4k^2 d^2 + 4m^2 \pi^2}.$$

Since $n = n_r + in_i$ and pure oscillations occurs for $n_r = 0$ and $n_i \neq 0$, therefore, the equation (27) implies that

$$a_1 n_i^4 + a_2 n_i^3 + a_3 n_i^2 + a_4 n_i + a_5 = 0. \tag{29}$$

Equation (29) is the required dispersion relation between growth rate, n and the wave number, k to examine the stability of the system.

Now special cases arise:

Case I: In the absence of suspended particles i.e. $\tau = 0$ and $\alpha_0 = 0,$ equation (29) reduces to

$$\left\{ \left(1 + \frac{\varepsilon \nu^{'}}{k_1} \right) n^2 - \left(\frac{\varepsilon \nu}{k_1} \right) n \right\} \left(d^2 + 4gk^2 d^2 + 4m^2 \pi^2 \right) + \left(n_q{}^2 - 4gk^2 d^2 L_D \right) = 0, \tag{30}$$

which is in good agreement with the earlier result by Sharma et al. (2014).

Case II: In the absence of quantum pressure i.e. $n_q = 0$ equation (30) shrinks further to

$$\left\{ \left(1 + \frac{\varepsilon \nu^{'}}{k_1} \right) n^2 - \left(\frac{\varepsilon \nu}{k_1} \right) n \right\} \left(d^2 + 4gk^2 d^2 + 4m^2 \pi^2 \right) - 4gk^2 d^2 L_D = 0, \tag{31}$$

which coincides well with the earlier results of Sunil et al. (2004).

4. Results and discussion

The numerical values of the growth rate of unstable mode are computed numerically from the dispersion relation encapsulated in equation (31) with the help of software Mathematica version-12. The fixed permissible values of the involved pertinent parameters are taken as $\varepsilon = 0.6, \nu = 0.4, n_q = 0.6, k_1 = 0.4, m = 1, d = 1, \tau = 2, \alpha_0 = 0.4$ and $g = 10$ respectively, (Hoshoudy 2011, 2016).

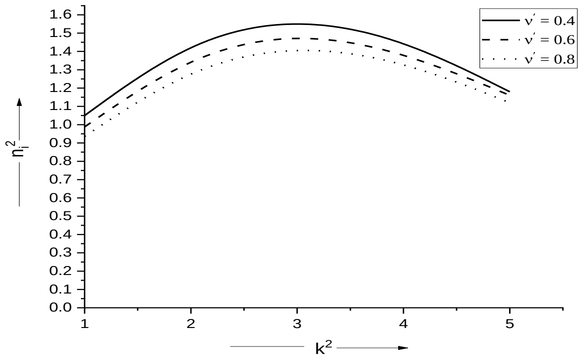

Fig. 1. The square of normalized growth rate, n_i^2 versus the square of normalized wave number, k^2 with respect to kinematic viscosity, ν'.

Figure 1 illustrates the variation of the square of normalized growth rate, n_i^2 versus the square of normalized wave number, k^2 for distinct values of kinematic viscoelasticity, $\nu' = 0.4, 0.6, 0.8.$ It has been found from the figure that the viscoelasticity decreases the maximum point, $k_{\max}$ for the instability and the magnitude of n_i^2 decreases with increase in ν' implying thereby the stabilizing effect of kinematic viscoelasticity on the system.

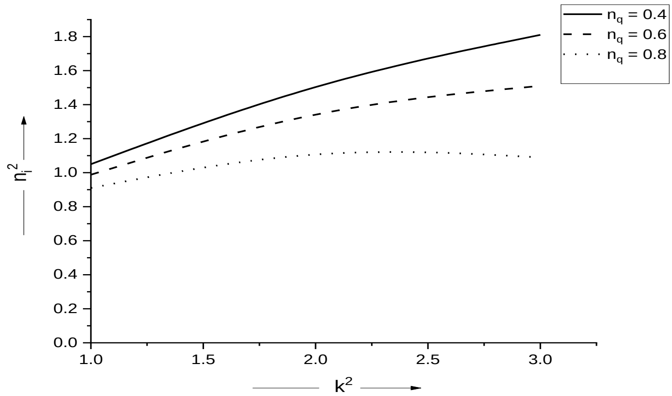

Fig. 2. The square of normalized growth rate, n_i^2 versus the square of normalized wave number, k^2 with respect to quantum pressure, n_q .

In Fig. 2, the influence of the quantum pressure of plasma, is visualized on the growth rates. The graphs depict a large enough stabilizing role on the growth rate of RTI of stratified plasma as the amplitude of growth rate educes with the increase in the value of quantum pressure parameter, n_q . Thus the instability region is shrunk.

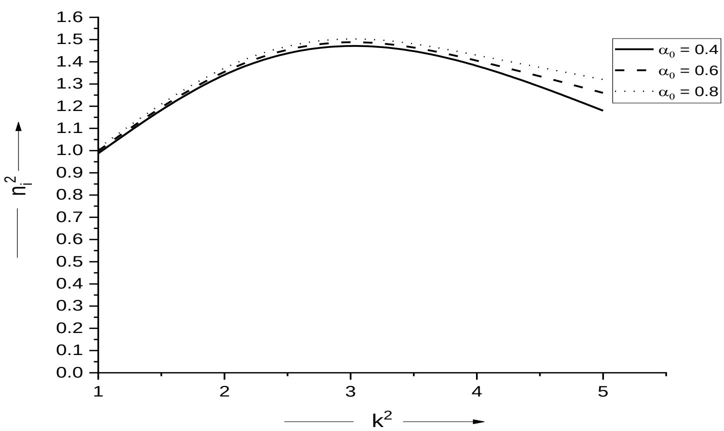

Fig. 3. The square of normalized growth rate, n_i^2 versus the square of normalized wave number, k^2 with respect to mass concentration, τ .

The variation of the square of the normalized growth rate, n_i^2 versus the square of normalized wave number, k^2 for distinct values of mass concentration parameter accounting for suspended particles, $\alpha_0 = 0.4, 0.6, 0.8$ has been displayed in Fig. 3. It is depicted from the graph that there is an increment in the amplitude of growth rate with increase in the value of mass concentration parameter. Consequently, the mass concentration of the dust particles has a destabilizing role on a system.

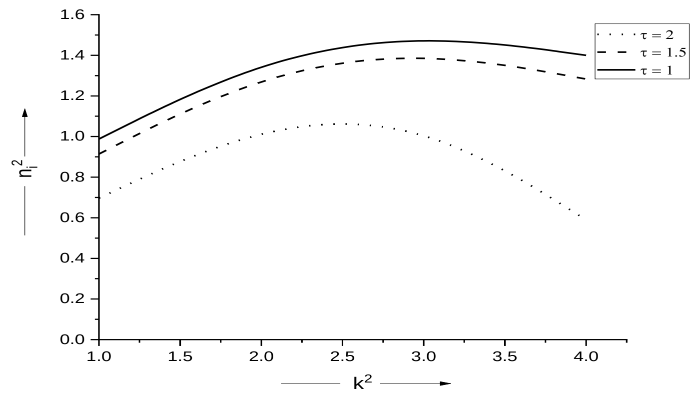

Fig. 4. The square of normalized growth rate, n_i^2 versus the square of normalized wave number, k^2 with respect to relaxation time factor, τ

The growth rate, n_i^2 is plotted against the wave number, k^2 in Fig. 4 for three distinct values of relaxation time factor, $\tau = 0.6, 1, 2$. It is assessed from the graph that relaxation time factor has stabilized the RTI, as the amplitude of growth rate falls with rise in the relaxation time factor.

It is noteworthy from Fig. 1, 3, 4 that the presence of the viscoelasticity, quantum pressure and suspended particles reduce the values of cut-off wave number.

5. Conclusion

Influence of quantum pressure and dust particles on the stability of stratified Walters' (model B') fluid / plasma in a porous medium is investigated. The relevant quantum hydrodynamic equations with dust particles and porous medium are formed. These equations are solved analytically using the normal mode technique to derive a dispersion relation of the RTI to assess impact of pertinent parameters on this instability. The effect of elasticity is revealed through the quantum pressure in the presence of dusty particles. The relaxation time of dust particles, the quantum pressure and the kinematic viscoelasticity are found to stabilize the RTI by lowering the amplitude of growth rate of perturbation substantially; whereas the effect of the mass concentration of the dust particles is to destabilize the RTI. Thus the simultaneous presence of quantum pressure and suspended particles play a major role in stabilizing quantum hydrodynamic RTI. This research work can be further extended for the presence of uniform as well as variable magnetic field to explore the suppression of RTI in laser plasma interaction, crab nebula, MTF (Modulation Transfer Function) device, white dwarfs in which the growth rate of RTI will be stabilized substantially due to magnetic field.

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