Journal of the Serbian Society for Computational Mechanics / Vol. 12 / No. 1, 2018 / pp 52-79
(DOI: 10.24874/jsscm.2018.12.01.05)

Flexural Analysis of Composite Laminated Beams Subjected to Thermo-Mechanical Loads

S. K. Kulkarni[1*], Y. M. Ghugal[2]

[1] Department of Civil Engineering, D Y Patil School of Engineering and Technology, Pune - 412105, Maharashtra State, India
e-mail: sanjaykulkarni.sk@gmail.com
[2] Department of Applied Mechanics, Government Engineering College, Karad - 415124, Maharashtra State, India
e-mail: ghugal@rediffmail.com
corresponding author

Abstract

The present work is concerned with the flexural analysis of single layer orthotropic, two-layer antisymmetric and three-layer symmetric laminated beams subjected to uniformly distributed thermo-mechanical loads. The thermal load is varying linearly across the thickness of laminated beams. A combination of uniformly distributed thermal load with uniformly distributed transverse mechanical load is taken into consideration for the flexural analysis of laminated beams. A sinusoidal shear deformation theory is used. The displacement field of theory consists of sinusoidal function in terms of thickness co-ordinate to include the shear deformation effect. Governing equations and boundary conditions of theory are obtained by using the principal of virtual work. The theory obviates the need of shear correction factor and satisfies the shear stress free conditions at the top and bottom of the beam. Stresses and displacements in orthotropic, antisymmetric and symmetric cross ply laminated beams are obtained by using sinusoidal shear deformation theory. The results obtained by present theory are compared with Timoshenko beam theory, classical beam theory and also with the results which are available in the literature wherever possible.

Keywords: Cross ply laminated beams, sinusoidal shear deformation theory, thermal stress, thermo-mechanical loads, equivalent single layer theory

1. Introduction

Composite structures are widely used in industry, aerospace engineering, under water and automotive structures due their excellent properties such as high strength to weight ratio and high stiffness to weight ratio. This make them ideally suited for use in weight sensitive structures. Laminated beams are widely used in aircrafts and watercrafts. The effect of temperature on displacements and stresses in the laminated beam attracts more attentions. Laminates can be subjected to severe thermal conditions through heating. The inter-laminar stresses are the main cause of failure when the laminates are subjected to severe thermal loading. This is due to the fact that thermal expansion coefficients in the direction of fibers are usually smaller than in transverse direction resulting high inter-laminar stresses at the interfaces. Therefore, there is a need to predict the inter-laminar transverse shear stresses accurately.

Several theories exist in literature for the flexural analysis of composites structures. They have been extended to thermoelastic problems as well. Classical laminate beam theory (CBT) presented by (Tanigawa et al. 1989) is based on the Euler-Bernoulli hypothesis and leading to inaccurate results for moderately thick beams, because transverse shear strains are neglected. The first order shear deformation theory of (Mindlin, 1951) when applied to beams is known as Timoshenko beam theory (TBT). The transverse shear strains are assumed to be constant along the thickness of laminates; hence transverse shear stresses remain constant along the thickness of laminates and stress-free boundary conditions are not satisfied at the top and bottom of the laminates. Further, it requires a shear correction factor to obtain the correct stresses and displacements. In the present work, a shear correction factor of 5/6 has been used.

A higher order shear deformation theory takes into account transverse shear strains and obviates the need of shear correction factor. A non-constant polynomial expression for the out of plane displacement is considered by (Kant et al. 1997) with a higher order theory. An exponential function has also been used by (Soldatos et al. 1997) with higher order theory. All these studies have a displacement-based approach. On the basis of mixed formulations, other approaches are formulated and presented by (Carrera, 2000b). A finite element model is also applied to beams by using higher order shear deformation theory and presented by (Kant et al. 1988). A thermoelastic solution of linear uncoupled thermo-elasticity has been presented for certain problems of flexure of composite laminates by (Bhaskar et al. 1996). Both mechanical and thermo-mechanical tests for thin and thick beams are presented by (Philippe et al. 2009) in order to evaluate the capacity of newly developed three node beam finite elements including transverse and normal effects in the analysis of laminated beams. This work focuses on the necessity to take into account the transverse normal stress, especially for thick beams. Within the framework of thermal problems, different approaches have been developed; including mixed formulations presented by Tessler et al. (2001) and displacement based developed by Ali et al. (1999).

The higher order shear deformation theories with sinusoidal functions in terms of thickness coordinates to include thickness effect are termed as trigonometric or sinusoidal shear deformation theories. Trigonometric shear deformation theory has been developed by Touratier (1991). Various problems of plates have been investigated including bidirectional bending when acted upon by mechanical and thermal load. Thermal stresses and displacements for orthotropic, antisymmetric and symmetric laminates subjected to nonlinear thermal load using trigonometric shear deformation theory have been presented by Ghugal et al. (2013). This paper includes bidirectional bending of plates. A closed form solution to assess multilayered plate theories for various thermal stress problems is presented by (Carrera et al. 2004). This work is the further development of two-dimensional modelling in thermal stress analysis of multilayered composite plates. Various equivalent single layer and layerwise theories for laminated plates subjected to mechanical load are critically discussed by Ghugal et al. (2002). An assessment of mixed and classical theories for the thermal stress analysis of orthotropic multilayered plates has been presented by Carrera, (2000a).

However, it has been observed from the literature review that the trigonometric shear deformation theory is not fully explored for the one-dimensional analysis of laminated composite beams subjected to combined thermal and mechanical loadings. Hence, a sinusoidal shear deformation theory (SSDT) is applied for the flexural analysis of laminated composite beams subjected to combined uniformly distributed thermo-mechanical loadings and the results are presented for the first time in the literature.

The aim of this paper is to develop a mathematical model of sinusoidal shear deformation theory including shear deformation effect, to obtain accurate flexural response of laminated composite beams subjected to combined thermal and mechanical loadings. This is an equivalent single layer theory for the evaluation of displacements and inter-laminar stresses in composite laminated beams. The necessity to consider the transverse shear stress especially for thick beam in thermal conditions has been studied in this work.

The results of displacement and stresses obtained by sinusoidal shear deformation theory (SSDT) are compared with those of classical beam theory (CBT), Timoshenko beam theory (TBT), and other refined theories. The results of pure thermal load are compared with exact elasticity solutions.

2. Mathematical formulation

The mathematical formulation of present sinusoidal shear deformation theory for laminate composite beam is based on certain kinematical and physical assumptions. The governing equations and boundary conditions are obtained by using principal of virtual work. The Navier solution has been employed to develop the analytical solution for the simply supported boundary conditions.

2.1 A laminated beam under consideration: The geometry of beam is as shown below.

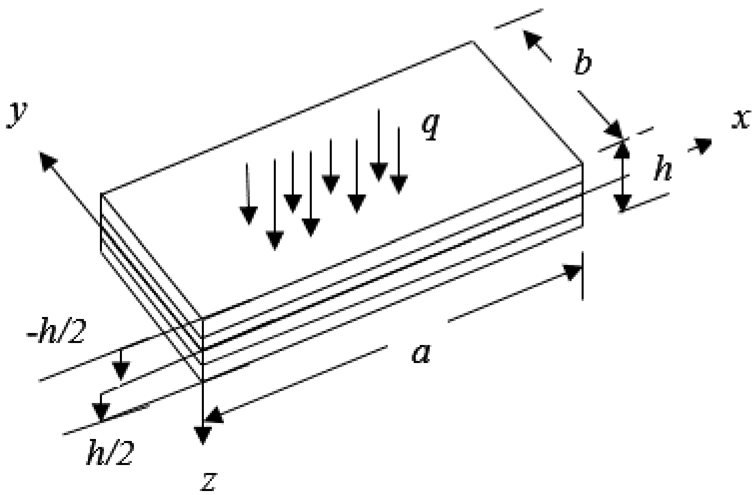

Fig. 1. Geometry of a laminated beam.

Let us consider a beam occupying the domain in Cartesian coordinate $\left(O-x,y,z\right)$.

$$0 \le x \le a,\ 0 \le y \le b,\ -\frac{h}{2} \le z \le \frac{h}{2} \tag{1}$$

The beam has a rectangular uniform cross section of height h and width b and is assumed to be straight. The beam is made up of N number of layers and each layer may be assumed to be made up of orthotropic material. The width b along y axis is very small as compared to length a along x axis. The z direction is assumed to be positive in the downward direction. The upper surface of laminated beam $(z=-h/2)$ is subjected to thermal load $T(x,z)$ and transverse mechanical load (q).

2.2 Assumptions made in mathematical formulation

The present sinusoidal shear deformation theory is based on following assumptions:

1. It is assumed that, the width of the beam b along y axis is very small as compared to length a along x axis while it is simply supported at its edges $x=0$ and $x=a$.
2. The displacement component u is the inplane displacement along x axis and w is the transverse displacement in z direction.
3. Since the theory has been applied to laminated beam, the displacement along y direction (v) is assumed to be zero.
4. Since, $a >> b$ we can state, $\left(\frac{\partial}{\partial y}=0\right)$ and $\left(\frac{\partial}{\partial x}=\frac{d}{\partial x}\right)$.
5. The axial displacement u in x direction has three components namely extension, bending and shear.

$$u=u_0+u_b+u_s \tag{2}$$

a) u_0 is the middle surface displacement in x direction known as extension component.

b) The bending component u_b is assumed to be analogous to the displacement given by classical laminate theory.

$$u_b=-z\frac{\partial w(x)}{\partial x} \tag{3}$$

c) The shear component is assumed to be sinusoidal in nature with respect to thickness coordinate, so that maximum shear stress occurs at neutral plane and zero at top and bottom surfaces of the beam.

$$u_s=\frac{h}{\pi}\sin\frac{\pi z}{h}\varphi(x) \tag{4}$$

6. The transverse displacement w in z direction is assumed to be a function of x coordinate only.

$$w=w(x) \tag{5}$$

7. The laminated beams are subjected to pure thermal load as well as a combination of uniformly distributed thermal and mechanical load.
8. The body forces are ignored in the thermo-mechanical analysis of laminated beam.

The assumptions made in this theory for 1D thermal flexural analysis of cross ply laminated beams are deduced from the 2D trigonometric shear deformation theory developed by Ghugal and Kulkarni (2013a, b) for thermoelastic analysis of laminated plates. The sinusoidal function introduced into the kinematics of theory is strongly based on theory of elasticity.

3. Kinematics of sinusoidal shear deformation theory

Based on the before mentioned assumptions the kinematics of sinusoidal shear deformation theory (SSDT) for beam is expressed as follows:

$$u(x,z) = u_0(x) - z\frac{\partial w}{\partial x} + \frac{h}{\pi}\sin\frac{\pi z}{h}\varphi(x) \tag{6}$$

$$w(x,z) = w(x) \tag{7}$$

Here, $u(x,z)$ is the axial displacement in x direction and $w(x)$ is the transverse displacement in the z direction. $u_0(x)$ is the centre line displacement and is the function of x only. The shear slope φ is also a function of x only. The trigonometric function in thickness coordinate considers the shear deformation effects and the displacement field of the theory represents a better kinematics. The kinematics of the theory enforces to satisfy shear stress free boundary conditions on top and bottom surfaces of the beam with realistic variation across the thickness.

3.1 Strain-displacement relationship

Within the framework of linear theory of elasticity, the normal and shear strains are obtained as follows.

$$\varepsilon_x = \frac{\partial u}{\partial x} \tag{8}$$

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \tag{9}$$

3.2 Constitutive relations

The thermo-elastic stress-strain relationships for laminated beam can be written as;

$$\begin{Bmatrix} \sigma_x \\ \tau_{zx} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & 0 \\ 0 & \bar{Q}_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x - \alpha_x T \\ \gamma_{zx} \end{Bmatrix}_k \tag{10}$$

where, $\bar{Q}_{ij}^{(k)}$ are the reduced stiffness coefficients as given below.

$$\bar{Q}_{11}^{(k)} = \frac{\bar{E}_1^{(k)}}{\left(1-\mu_{12}^{(k)}\mu_{21}^{(k)}\right)},\ \bar{Q}_{55}^{(k)} = \bar{G}_{13}^{(k)} \tag{11}$$

where, E is Young's modulus, G is shear modulus, μ_{ij} are Poisson's ratios and α_x is the coefficient of thermal expansion in x direction.

3.3 Temperature distribution

The temperature distribution across the thickness of laminated beam is assumed to be in the form as given below.

$$T(x,z) = \frac{2}{h} z T_0(x) \tag{12}$$

In the above equation, T is the temperature change from a reference state which is a function of x and z. The thermal load T_0 is linearly varying across the thickness of laminated beam and is a function of x. The temperature distribution through the thickness of laminated beam is as shown in figure 2.

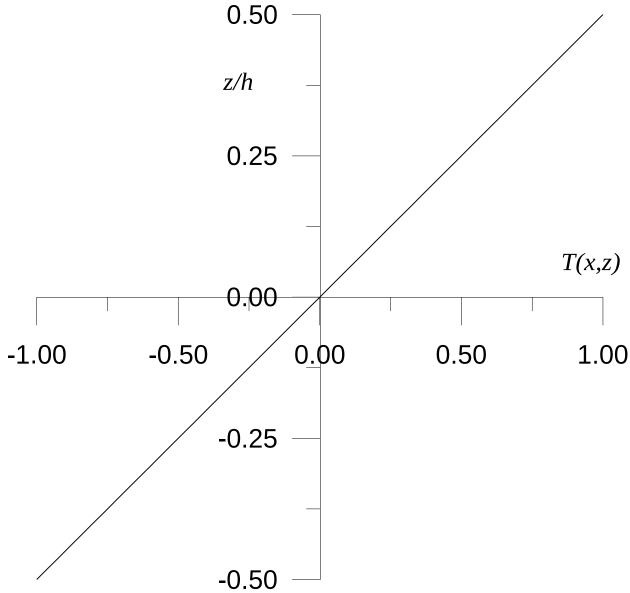

Fig. 2. Temperature distribution through the thickness of laminated beam.

4. Governing equations and boundary conditions

The governing equations and boundary conditions are derived by using principle of virtual work. The principal of virtual work states that of all possible displacements that satisfy the given conditions of constraints, that system which is associated with equilibrium makes the value of sum of potential energy of the prescribed external forces and the potential strain energy of the internal stresses maximum and in the case of stable equilibrium a minimum. The principle of virtual work when applied to the laminated beam leads to:

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^a \left(\sigma_x \delta\varepsilon_x + \tau_{zx} \delta\gamma_{zx} \right) dx\, dz - \int_0^a q \delta w dx = 0 \tag{13}$$

The governing equations of equilibrium can be derived from the above equation (13) by integrating the displacements gradients in ε_i by parts and setting the coefficients of $\delta u_0, \delta w$ and $\delta\varphi$ to zero separately. The symbol δ denotes variational operator and collecting the coefficients of $\delta u_0, \delta w$ and $\delta\varphi$ one can obtain the governing equations as follows.

$$\delta u_0 : -A_{11}\frac{\partial^2 u_0}{\partial x^2} + B_{11}\frac{\partial^3 w}{\partial x^3} - SB_{11}\frac{\partial^2 \varphi}{\partial x^2} + TB_{11}\frac{\partial T_0}{\partial x} = 0 \tag{14}$$

$$\delta w : D_{11}\frac{\partial^4 w}{\partial x^4} - B_{11}\frac{\partial^3 u_0}{\partial x^3} - S_{11}\frac{\partial^3 \varphi}{\partial x^3} + TD_{11}\frac{\partial^2 T_0}{\partial x^2} = q \tag{15}$$

$$\delta \varphi : S_{11}\frac{\partial^3 w}{\partial x^3} - SS_{11}\frac{\partial^2 \varphi}{\partial x^2} + C_{55}\varphi - SB_{11}\frac{\partial^2 u_0}{\partial x^2} + TS_{11}\frac{\partial T_0}{\partial x} = 0 \tag{16}$$

The associated boundary conditions are as follows:

Along edges $x = 0$ and $x = a$

$$B_{11}\frac{\partial^2 u_0}{\partial x^2} - D_{11}\frac{\partial^3 w}{\partial x^3} + S_{11}\frac{\partial^2 \varphi}{\partial x^2} = 0 \text{ or } w \text{ is prescribed.} \tag{17}$$

$$-B_{11}\frac{\partial u_0}{\partial x} + D_{11}\frac{\partial^2 w}{\partial x^2} - S_{11}\frac{\partial \varphi}{\partial x} + TD_{11}T_0 = 0 \text{ or } \frac{dw}{dx} \text{ is prescribed.} \tag{18}$$

$$SB_{11}\frac{\partial u_0}{\partial x} - S_{11}\frac{\partial^2 w}{\partial x^2} + SS_{11}\frac{\partial \varphi}{\partial x} - TS_{11}T_0 = 0 \text{ or } \varphi \text{ is prescribed.} \tag{19}$$

$$A_{11}\frac{\partial u_0}{\partial x} - B_{11}\frac{\partial^2 w}{\partial x^2} + SB_{11}\frac{\partial \varphi}{\partial x} - TB_{11}T_0 = 0 \text{ or } u_0 \text{ is prescribed.} \tag{20}$$

where the stiffness coefficients $A_{ij}, B_{ij}, ...$ etc are defined as follows:

$$\left(A_{11}, B_{11}, D_{11}\right) = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)}\left(1, z, z^2\right)dz \tag{21}$$

$$\left(SB_{11}, S_{11}, SS_{11}\right) = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)}\frac{h}{\pi}\sin\frac{\pi z}{h}\left(1, z, \frac{h}{\pi}\sin\frac{\pi z}{h}\right)dz \tag{22}$$

$$\left(TB_{11}, TD_{11}, TS_{11}\right) = \sum_{k=1}^{N}\left(\alpha_x\right)\int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)}\left(z, z^2, \frac{h}{\pi}\sin\frac{\pi z}{h}\right)dz \tag{23}$$

$$C_{55} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{55}^{(k)}\cos^2\frac{\pi z}{h}dz \tag{24}$$

For orthotropic and three-layer symmetric laminated beam $B_{11}, TB_{11}, SB_{11} = 0$ and $u_0 = 0$.

5. Application of theory

To assess the performance of the sinusoidal shear deformation theory when applied to laminated beams subjected to thermal load and combined thermo-mechanical load, a simply supported orthotropic (0^0), antisymmetric ($0^0/90^0$) and symmetric ($0^0/90^0/0^0$) cross ply laminated beams are considered. The material properties of high modulus Graphite-Epoxy orthotropic layer are taken from Philippe et al. (2009) and Bhaskar et al. (1996). The material properties are as follows:

$$\begin{aligned} &E_L = 172.4\,\text{GPa},\ E_T = 6.895\ \text{GPa},\ G_{LT} = 3.448\ \text{GPa}, \\ &G_{TT} = 1.379\ \text{GPa},\ \mu_{12} = 0.25,\ \alpha_T = 1125\alpha_L \end{aligned} \tag{25}$$

where L refers to the fiber direction, and T refers to the transverse direction. α_L and α_T are the thermal expansion coefficients in the fiber and normal direction.

To assess the effectiveness of the present theory, numerical investigations have been carried out for following examples with different configurations and the material properties as mentioned above.

Example 1: A single layer orthotropic beam (0^0) subjected to sinusoidal and uniformly distributed pure thermal load is considered for thermal bending analysis (Table 1).

Example 2: A two-layer antisymmetric laminated beam ($0^0/90^0$) subjected to sinusoidal and uniformly distributed pure thermal load is taken in to consideration for thermal analysis (Table 2).

Example 3: A three-layer symmetric laminated beam ($0^0/90^0/0^0$) subjected to sinusoidal and uniformly distributed pure thermal load is considered for thermal flexural analysis (Table 3).

Example 4: An orthotropic beam (0^0) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load is taken in to consideration for thermo-mechanical bending analysis (Table 4).

Example 5: A two-layer antisymmetric laminated beam ($0^0/90^0$) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load is taken into consideration for thermo-mechanical flexural analysis (Table 5).

Example 6: A three-layer symmetric laminated beam ($0^0/90^0/0^0$) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load is taken into consideration for thermo-mechanical flexural analysis (Table 6).

6. Navier solution

Following are the boundary conditions used for simply supported laminated composite beam along the edges $x = 0$ and $x = a$.

$$w = 0,\ M_x = 0,\ N_x = 0,\ M_x^S = 0$$

Navier's solution procedure is adopted to compute displacement variables. The following is the solution forms for $u_0(x), w(x)$ and $\varphi(x)$ that satisfies the boundary conditions exactly.

$$u_0(x) = \sum_{m=1.3.5}^{\infty} u_m \cos\left(\frac{m\pi x}{a}\right) \tag{26}$$

$$w(x) = \sum_{m=1.3.5}^{\infty} w_m \sin\left(\frac{m\pi x}{a}\right) \tag{27}$$

$$\varphi(x) = \sum_{m=1,3,5}^{\infty} \varphi_m \cos\left(\frac{m\pi x}{a}\right) \tag{28}$$

where, u_m, w_m and φ_m are the unknown coefficients to be determined. The thermal and transverse mechanical loads are expanded in single Fourier sine series as given below.

$$\begin{aligned} T_0(x) &= \sum_{m=1}^{\infty} T_{0m} \sin\left(\frac{m\pi x}{a}\right) \\ q(x) &= \sum_{m=1}^{\infty} q_m \sin\left(\frac{m\pi x}{a}\right) \end{aligned} \tag{29}$$

where m is the positive integer and T_{0m} and q_m are the coefficients of Fourier series expansions, respectively for thermal and transverse mechanical loads as follows:

$$\begin{aligned} T_{0m} &= \begin{cases} T_0 & \text{for sinusoidal thermal load, } m = 1 \\ \dfrac{4T_0}{m\pi} & \text{for uniform thermal load, } m = 1,3,5.... \end{cases} \\ q_m &= \left\{ \dfrac{4q_0}{m\pi} \quad \text{for uniformly distributed load, } m = 1,3,5... \right. \end{aligned} \tag{30}$$

In which T_0 and q_0 are the intensities of thermal and mechanical load respectively. Substitution of equations (26), (27), (28) and (29) in to governing equations (14), (15) and (16) leads to the set of algebraic equations which can be written in matrix form as follows.

$$\begin{bmatrix} A_{11}\dfrac{m^2\pi^2}{a^2} & -B_{11}\dfrac{m^3\pi^3}{a^3} & SB_{11}\dfrac{m^2\pi^2}{a^2} \\ -B_{11}\dfrac{m^3\pi^3}{a^3} & D_{11}\dfrac{m^4\pi^4}{a^4} & -S_{11}\dfrac{m^3\pi^3}{a^3} \\ SB_{11}\dfrac{m^2\pi^2}{a^2} & -S_{11}\dfrac{m^3\pi^3}{a^3} & SS_{11}\dfrac{m^2\pi^2}{a^2} + C_{55} \end{bmatrix} \begin{Bmatrix} u_{0m} \\ w_m \\ \varphi_m \end{Bmatrix} = \begin{Bmatrix} -T_{0m} TB_{11} \dfrac{2m\pi}{ha} \\ T_{0m} TD_{11} \dfrac{2m^2\pi^2}{ha^2} + q_m \\ -T_{0m} TS_{11} \dfrac{2m\pi}{ha} \end{Bmatrix} \tag{31}$$

Solving the above set of algebraic equations, the values of u_{0m}, w_m and φ_m can be obtained. Having obtained the values of u_{0m}, w_m and φ_m one can then calculate all the thermal displacements and stresses within the beam by using equations (6), (7), (8), (9) and (10). Transverse shear stresses are obtained by integrating equilibrium equations $\left(\tau_{zx}^{EE}\right)$ of theory of elasticity with respect to the thickness coordinate, satisfying shear stress free conditions at the top and bottom surface of the laminated beam and which ascertains the continuity of transverse shear stress at the layer interface. This relation can be expressed as given below.

$$\tau_{zx}^{EE} = \int_{-\frac{h}{2}}^{z_k} \frac{\partial \sigma_x}{\partial x} dz + C_1$$

The constant of integration can be obtained from appropriate boundary conditions. It is expected that this relation will produce accurate transverse shear stresses.

6.1 Numerical results

In this paper, displacements and stresses are obtained for single layer orthotropic beam (0^0), two-layer antisymmetric beam ($0^0/90^0$) and three-layer symmetric laminated beam ($0^0/90^0/0^0$) with simply supported boundary conditions and subjected to pure thermal load as well as combined thermo-mechanical loads for various aspect ratios (S). The numerical results for single layer orthotropic beam (0^0) subjected to pure thermal load are presented in following normalized forms for the purpose of discussion (Table 1).

$$\bar{u}\left(0,\frac{h}{2}\right)=\frac{u}{\alpha_L T_0 a},\ \bar{w}\left(\frac{a}{2},0\right)=\frac{hw}{\alpha_L T_0 a^2},\ \bar{\sigma}_x\left(\frac{a}{2},-\frac{h}{2}\right)=\frac{\sigma_x}{\alpha_L E_T T_0},\ \bar{\tau}_{zx}\left(0,0\right)=\frac{\tau_{zx}}{E_T \alpha_L T_0}$$

Source	Model	S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{EE}$
Sinusoidal Thermal load (0^0)						
Present	SSDT	4	0.3183	0.2026	0.0000	4.2951
Present	TBT	4	0.3183	0.2026	0.0000	4.2951
Present	CBT	4	0.3183	0.2026	0.0000	4.2951
Present	SSDT	10	0.3183	0.2026	0.0000	1.8653
Present	TBT	10	0.3183	0.2026	0.0000	1.8653
Present	CBT	10	0.3183	0.2026	0.0000	1.8653
Present	SSDT	100	0.3183	0.2026	0.0000	0.1954
Present	TBT	100	0.3183	0.2026	0.0002	0.1954
Present	CBT	100	0.3183	0.2026	0.0000	0.1954
Uniformly Distributed Thermal load (0^0)						
Present	SSDT	4	0.4899	0.2500	0.0000	54.6875
Present	TBT	4	0.4899	0.2500	0.0000	54.6875
Present	CBT	4	0.4899	0.2500	0.0000	54.6875
Present	SSDT	10	0.4899	0.2500	0.0000	23.7500
Present	TBT	10	0.4899	0.2500	0.0000	23.7500
Present	CBT	10	0.4899	0.2500	0.0000	23.7500
Present	SSDT	100	0.4899	0.2500	0.0000	2.4875
Present	TBT	100	0.4899	0.2500	0.0002	2.4875
Present	CBT	100	0.4899	0.2500	0.0000	2.4875

Table 1. Normalized displacements and stresses in single layer orthotropic beam (0^0) subjected sinusoidal and uniformly distributed thermal load (Example 1)

The numerical results for two-layer antisymmetric laminated beam ($0^0/90^0$) subjected to pure thermal load are presented in following normalized forms for the purpose of comparison and discussion (Table 2).

$$\bar{u}\left(0,\frac{h}{2}\right)=\frac{u}{\alpha_L T_0 a},\ \bar{w}\left(\frac{a}{2},0\right)=\frac{hw}{\alpha_L T_0 a^2},\ \bar{\sigma}_x\left(\frac{a}{2},-\frac{h}{2}\right)=\frac{\sigma_x}{\alpha_L E_T T_0},\ \bar{\tau}_{zx}\left(0,0\right)=\frac{\tau_{zx}}{E_T\alpha_L T_0}$$

Source	Model	S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{EE}$
Sinusoidal Thermal load (0/90)						
Present	SSDT	4	101.3046	38.1550	2123.2790	134.8439
Present	TBT	4	101.4938	41.2759	2188.1140	135.6736
Present	CBT	4	101.4844	41.2721	2188.1140	135.6737
Philippe et al. (2009)	SinRef-7p	4	155.1000	42.6520	2044.1000	-
Philippe et al. (2009)	SinRef-6p	4	156.6000	41.3020	2075.4000	-
Philippe et al. (2009)	SinRef-c	4	153.3900	45.8820	1774.5000	-
Philippe et al. (2009)	exact	4	155.3600	42.8890	1994.7000	-
Present	SSDT	10	101.4551	40.7663	2177.5280	54.2153
Present	TBT	10	101.4939	41.2759	2188.1160	54.2695
Present	CBT	10	101.4845	41.2721	2188.1150	54.2695
Philippe et al. (2009)	SinRef-7p	10	114.2200	43.2850	2142.4000	-
Philippe et al. (2009)	SinRef-6p	10	114.3500	43.0230	2150.1000	-
Philippe et al. (2009)	SinRef-c	10	111.8800	42.1910	2115.3000	-
Philippe et al. (2009)	exact	10	114.1800	43.2930	2129.0000	-
Present	SSDT	100	101.4841	41.2708	2188.0100	5.4269
Present	TBT	100	101.4928	41.2754	2188.0830	5.4270
Present	CBT	100	101.4845	41.2721	2188.1150	5.4270
(Philippe et al. 2009)	SinRef-7p	100	104.1800	43.1520	2181.1000	-
(Philippe et al. 2009)	SinRef-6p	100	104.1800	43.1490	2181.2000	-
(Philippe et al. 2009)	SinRef-c	100	101.6000	41.2860	2199.8000	-
(Philippe et al. 2009)	exact	100	104.1900	43.1550	2171.7000	-
Uniformly Distributed Thermal load (0/90)						
Present	SSDT	4	155.2392	47.7309	2196.4270	1464.1830
Present	TBT	4	156.1982	50.9189	2118.6360	1727.4510
Present	CBT	4	156.1837	50.9142	2118.6370	1727.4510
Present	SSDT	10	155.9150	50.4136	2171.3000	640.9235

Present	TBT	10	156.1983	50.9189	2118.6390	690.9801
Present	CBT	10	156.1838	50.9142	2118.6370	690.9805
Present	SSDT	100	156.1800	50.9140	2119.9720	69.0068
Present	TBT	100	156.1968	50.9183	2118.6020	69.0981
Present	CBT	100	156.1838	50.9142	2118.6390	69.0981

Table 2. Normalized displacements and stresses in antisymmetric laminated cross ply beam subjected sinusoidal and uniformly distributed thermal load (Example 2)

The numerical results for three-layer symmetric laminated beam ($0^0/90^0/0^0$) subjected to pure thermal load are presented in following normalized forms for the purpose of discussion (Table 3).

$$\bar{u}\left(0,\frac{h}{2}\right)=\frac{u}{\alpha_L T_0 a},\ \bar{w}\left(\frac{a}{2},0\right)=\frac{hw}{\alpha_L T_0 a^2},\ \bar{\sigma}_x\left(\frac{a}{2},-\frac{h}{2}\right)=\frac{\sigma_x}{\alpha_L E_T T_0},\ \bar{\tau}_{zx}\left(0,-\frac{h}{6}\right)=\frac{\tau_{zx}}{E_T \alpha_L T_0}$$

Source	Model	S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{EE}$
Sinusoidal Thermal load (0/90/0)						
Present	SSDT	4	0.7606	0.1499	34.7579	7.8886
Present	TBT	4	0.8679	0.5525	43.1644	7.6074
Present	CBT	4	0.8679	0.5525	43.1644	7.6074
(Philippe et al. 2009)	SinRef-c	4	0.2608	0.2332	4.46270	-
(Philippe et al. 2009)	exact	4	7.4696	3.6156	281.110	-
Present	SSDT	10	0.8499	0.4850	41.7541	3.0618
Present	TBT	10	0.8679	0.5525	43.1644	3.0430
Present	CBT	10	0.8679	0.5525	43.1644	3.0430
(Philippe et al. 2009)	SinRef-c	10	0.7346	0.3762	32.9600	-
(Philippe et al. 2009)	exact	10	5.0095	1.8934	87.4100	-
Present	SSDT	100	0.8677	0.5518	43.1501	0.3043
Present	TBT	100	0.8679	0.5525	43.1640	0.3043
Present	CBT	100	0.8679	0.5525	43.1644	0.3043
(Philippe et al. 2009)	SinRef-c	100	0.8665	0.5506	43.3780	-
(Philippe et al. 2009)	exact	100	4.4494	2.8197	43.3100	-
Uniformly Distributed Thermal load (0/90/0)						
Present	SSDT	4	0.9268	0.2572	45.9882	143.4607
Present	TBT	4	1.3357	0.6816	41.7938	96.8608
Present	CBT	4	1.3357	0.6816	41.7938	96.8608
Present	SSDT	10	1.2048	0.6139	45.6244	50.0257
Present	TBT	10	1.3357	0.6816	41.7938	38.7443
Present	CBT	10	1.3357	0.6816	41.7938	38.7443
Present	SSDT	100	1.3334	0.6809	41.9686	3.9060
Present	TBT	100	1.3357	0.6816	41.7933	3.8744
Present	CBT	100	1.3357	0.6816	41.7938	3.8744

Table 3. Normalized displacements and stresses in symmetric laminated cross ply beam (0/90/0) subjected sinusoidal and uniformly distributed thermal load (Example 3)

The numerical results for single layer orthotropic beam (0^0) subjected to uniformly distributed combined thermo-mechanical load are presented in following normalized form for the purpose of discussion (Table 4).

$$\bar{u}\left(0,\frac{h}{2}\right)=\frac{u}{\left(\dfrac{q_0 h S^3}{E_T}+\dfrac{\alpha_L T_0 a}{1}\right)},\ \bar{w}\left(\frac{a}{2},0\right)=\frac{w\times 100}{\left(\dfrac{q_0 a^4}{E_T h^3}+\dfrac{\alpha_L T_0 a}{10h}\right)}$$

$$\bar{\sigma}_x\left(\frac{a}{2},-\frac{h}{2}\right)=\frac{\sigma_x}{\left(\dfrac{q_0 a^2}{h^2}+\dfrac{E_T\alpha_L T_0}{1}\right)},\bar{\tau}_{zx}(0,0)=\frac{\tau_{zx}}{\left(\dfrac{q_0 a}{h}+\dfrac{E_T\alpha_L T_0}{1}\right)}$$

Combined uniformly distributed thermo-mechanical load (0^0)						
Source	Model	S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{EE}$
Present	SSDT	4	0.0382	2.1801	0.7071	11.5881
Present	TBT	4	0.0382	4.0378	0.7058	11.5884
Present	CBT	4	0.0382	2.1738	0.7058	11.5884
Present	SSDT	10	0.0148	0.8804	0.7438	2.8984
Present	TBT	10	0.0148	1.1739	0.7425	2.8988
Present	CBT	10	0.0148	0.8741	0.7425	2.8988
Present	SSDT	100	0.0101	0.6296	0.7503	0.8301
Present	TBT	100	0.0100	0.6305	0.7499	0.8302
Present	CBT	100	0.0100	0.6275	0.7499	0.8302

Table 4. Normalized displacements and stresses in orthotropic beam (0^0) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load (Example 4)

The numerical results for two-layer antisymmetric laminated beam ($0^0/90^0$) subjected to uniformly distributed combined thermo-mechanical load are presented in following normalized form for the purpose of discussion (Table 5).

$$\bar{u}\left(0,\frac{h}{2}\right)=\frac{u}{\left(\dfrac{q_0 h S^3}{E_T}+\dfrac{\alpha_L T_0 a}{1}\right)},\ \bar{w}\left(\frac{a}{2},0\right)=\frac{w\times 100}{\left(\dfrac{q_0 a^4}{E_T h^3}+\dfrac{\alpha_L T_0 a}{10h}\right)}$$

$$\bar{\sigma}_x\left(\frac{a}{2},-\frac{h}{2}\right)=\frac{\sigma_x}{\left(\dfrac{q_0 a^2}{h^2}+\dfrac{E_T\alpha_L T_0}{1}\right)},\ \bar{\tau}_{zx}(0,0)=\frac{\tau_{zx}}{\left(\dfrac{q_0 a}{h}+\dfrac{E_T\alpha_L T_0}{1}\right)}$$

Combined uniformly distributed thermo-mechanical load ($0^0/90^0$)						
Source	Model	S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{EE}$
Present	SSDT	4	46.8069	301.9622	131.5816	292.6084

Present	TBT	4	47.0903	322.2385	126.6506	345.2234
Present	CBT	4	47.0903	319.5756	126.6507	345.2236
Present	SSDT	10	10.1300	54.0477	23.6874	57.9713
Present	TBT	10	10.1471	54.6229	23.1068	62.5132
Present	CBT	10	10.1471	54.1946	23.1068	62.5133
Present	SSDT	100	0.1854	3.8426	2.3639	0.3532
Present	TBT	100	0.1854	3.8433	2.3631	0.3540
Present	CBT	100	0.1854	3.8391	2.3631	0.3540

Table 5. Normalized displacements and stresses in antisymmetric laminated beam ($0^0/90^0$) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load (Example 5)

The numerical results for three-layer symmetric laminated beam ($0^0/90^0/0^0$) subjected to uniformly distributed combined thermo-mechanical load are presented in following normalized form for the purpose of discussion (Table 6).

$$\bar{u}\left(0,\frac{h}{2}\right)=\frac{u}{\left(\dfrac{q_0 hS^3}{E_T}+\dfrac{\alpha_L T_0 a}{1}\right)},\ \bar{w}\left(\frac{a}{2},0\right)=\frac{w\times 100}{\left(\dfrac{q_0 a^4}{E_T h^3}+\dfrac{\alpha_L T_0 a}{10h}\right)}$$

$$\bar{\sigma}_x\left(\frac{a}{2},-\frac{h}{2}\right)=\frac{\sigma_x}{\left(\dfrac{q_0 a^2}{h^2}+\dfrac{E_T\alpha_L T_0}{1}\right)},\ \bar{\tau}_{zx}\left(0,-\frac{h}{6}\right)=\frac{\tau_{zx}}{\left(\dfrac{q_0 a}{h}+\dfrac{E_T\alpha_L T_0}{1}\right)}$$

Combined uniformly distributed thermo-mechanical load ($0^0/90^0/0^0$)						
Source	Model	S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{EE}$
Present	SSDT	4	0.0721	4.9820	3.8974	29.1846
Present	TBT	4	0.0883	7.2160	3.1903	19.9780
Present	CBT	4	0.0883	4.8776	3.1903	19.9780
Present	SSDT	10	0.0236	1.7184	1.2989	5.2092
Present	TBT	10	0.0235	1.7044	1.1837	4.2106
Present	CBT	10	0.0235	1.3283	1.1837	4.2106
Present	SSDT	100	0.0105	0.6579	0.7825	0.7881
Present	TBT	100	0.0105	0.6586	0.7817	0.7881
Present	CBT	100	0.0105	0.6549	0.7817	0.7881

Table 6. Normalized displacements and stresses in symmetric laminated beam ($0^0/90^0/0^0$) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load (Example 6)

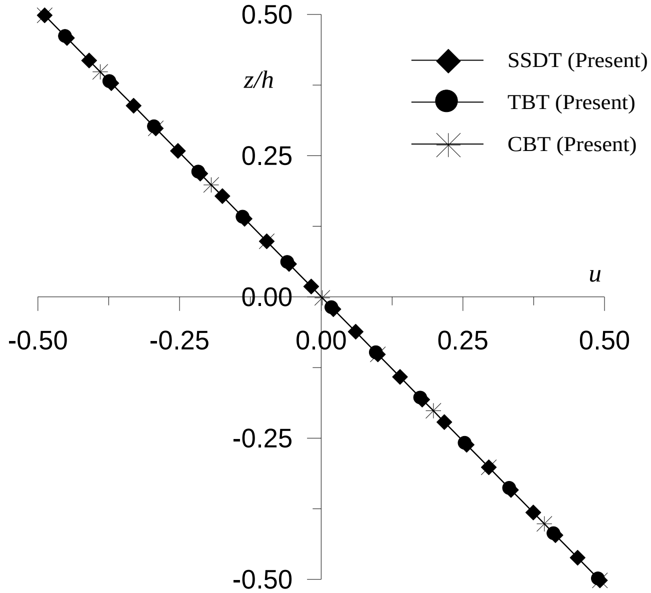

Fig. 3. Normalized axial displacement $(\bar{u})$ through the thickness of orthotropic beam (0^0) subjected to uniformly distributed thermal load aspect ratio 4

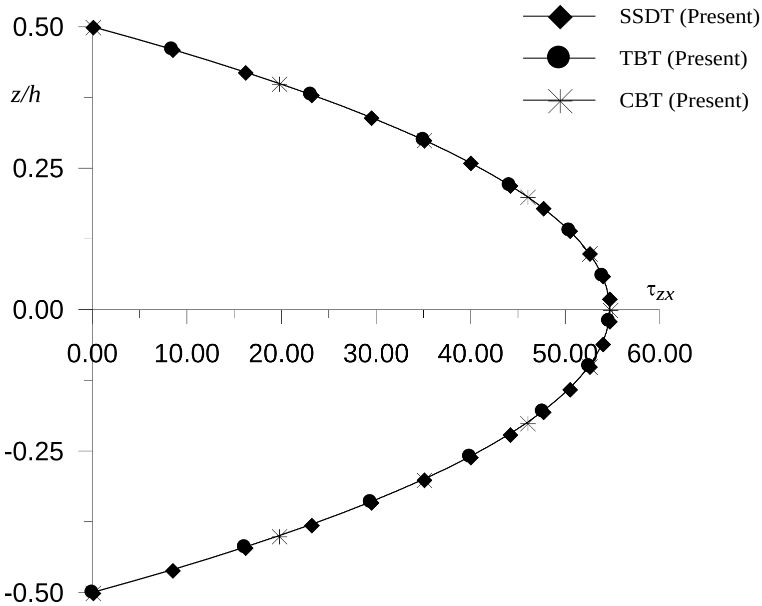

Fig. 4. Normalized transverse shear stress $(\bar{\tau}_{zx})$ through the thickness of orthotropic beam (0^0) subjected to uniformly distributed thermal load and obtained by equilibrium equation for aspect ratio 4

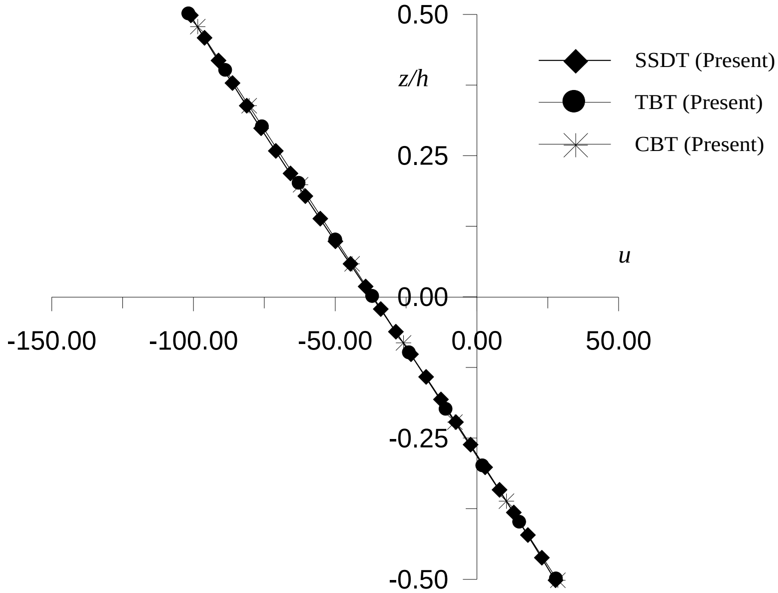

Fig. 5. Normalized axial displacement $(\bar{u})$ through the thickness of two layer laminated beam for aspect ratio 4 under sinusoidal thermal load.

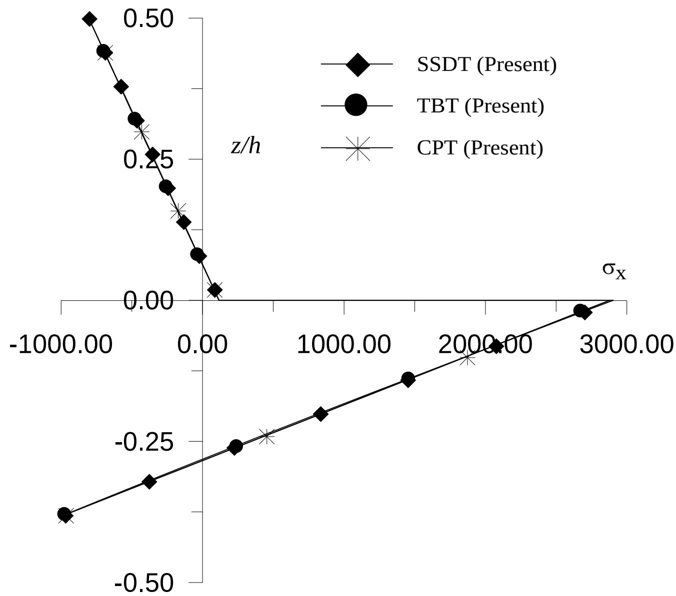

Fig. 6. Normalized normal stress $(\bar{\sigma}_x)$ through the thickness of two-layer laminated beam for aspect ratio 4 under sinusoidal thermal load

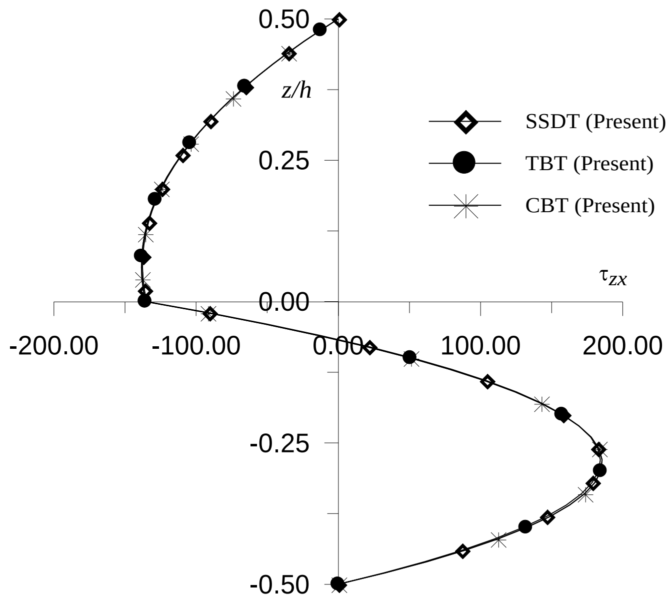

Fig. 7. Normalized transverse shear stress $\left(\bar{\tau}_{zx}\right)$ through the thickness of two-layer laminated beam subjected to sinusoidal thermal load and obtained by equilibrium equation for aspect ratio 4 under sinusoidal thermal load.

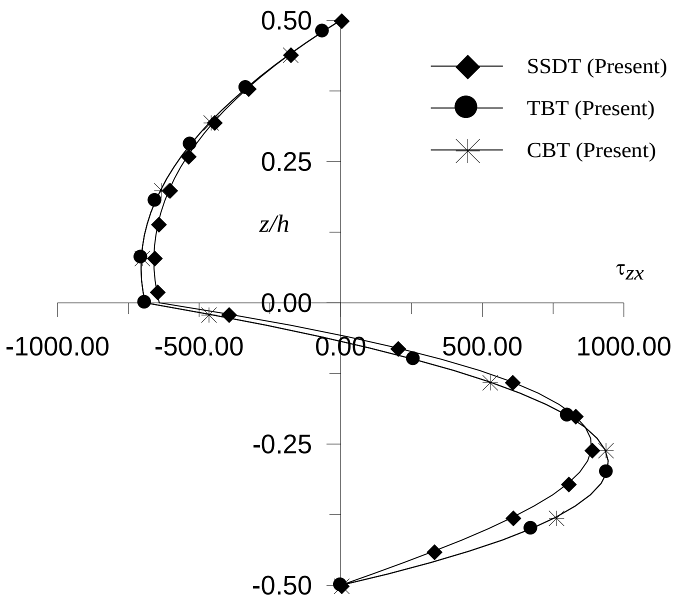

Fig. 8. Normalized transverse shear stress $\left(\bar{\tau}_{zx}\right)$ through the thickness of two-layer laminated beam subjected to uniformly distributed thermal load and obtained by equilibrium equation for aspect ratio 10

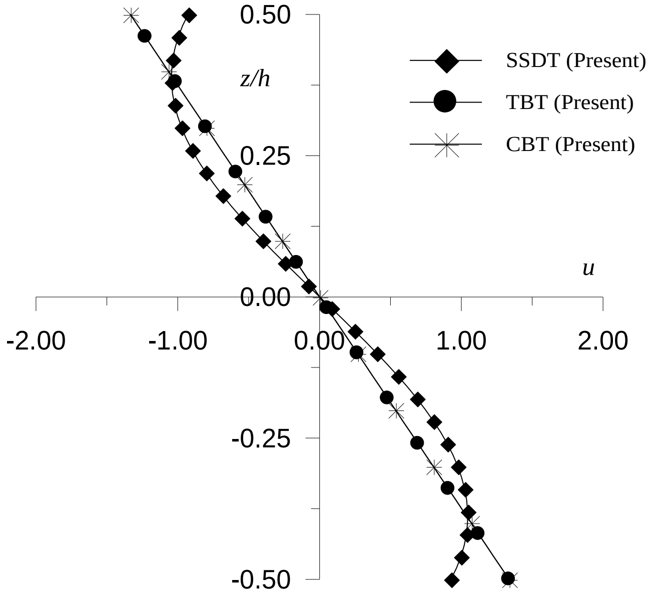

Fig. 9. Normalized axial displacement $(\bar{u})$ through the thickness of three-layer laminated beam (0/90/0) subjected to uniformly distributed thermal load for aspect ratio 4

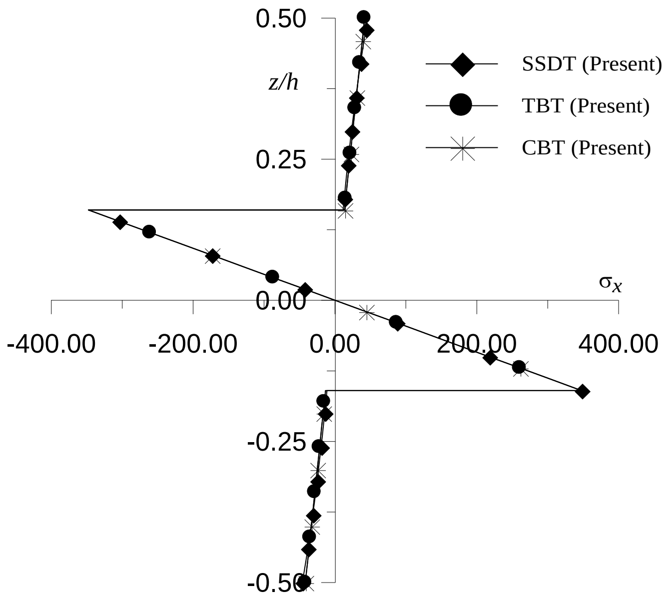

Fig. 10. Normalized normal stress $(\bar{\sigma}_x)$ through the thickness of three-layer laminated beam (0/90/0) subjected to uniformly distributed thermal load aspect ratio 4

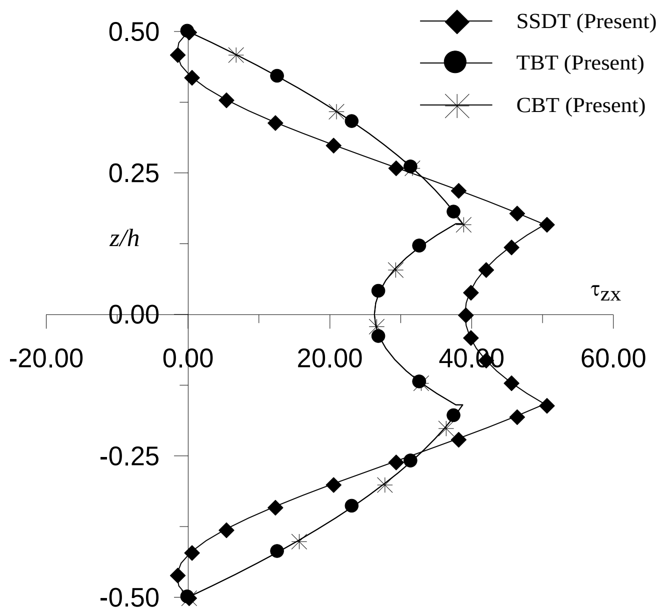

Fig. 11. Normalized transverse shear stress $\left(\bar{\tau}_{zx}\right)$ through the thickness of three-layer laminated beam (0/90/0) subjected to uniformly distributed thermal load and obtained by equilibrium equation for aspect ratio 10

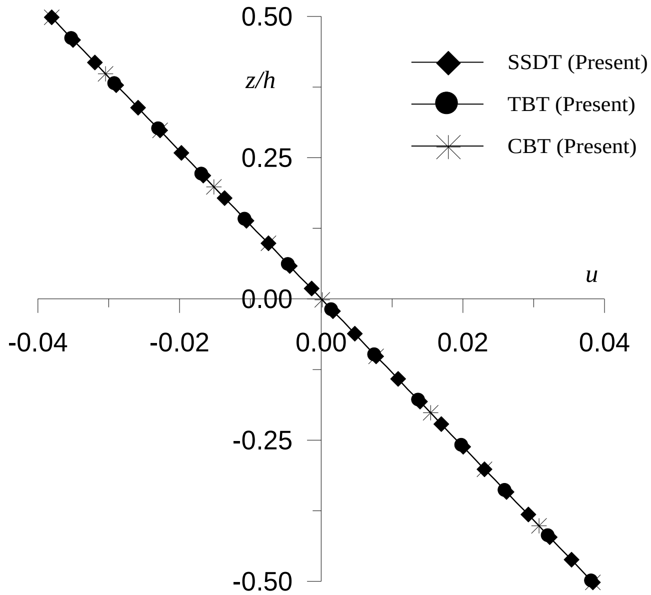

Fig. 12. Normalized axial displacement $\left(\bar{u}\right)$ through the thickness of orthotropic beam (0^0) subjected to combined uniformly distributed thermo-mechanical load for aspect ratio 4

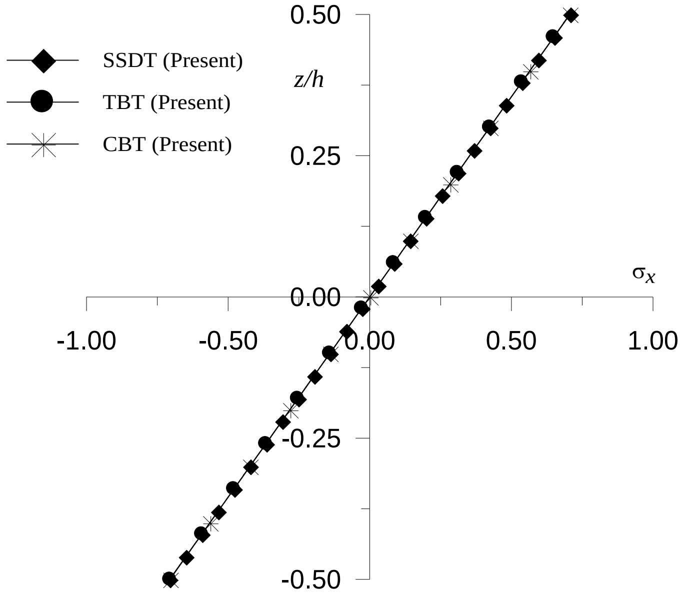

Fig. 13. Normalized normal stress $\left(\bar{\sigma}_x\right)$ through the thickness of orthotropic beam (0^0) subjected to combined uniformly distributed thermo-mechanical load for aspect ratio 4

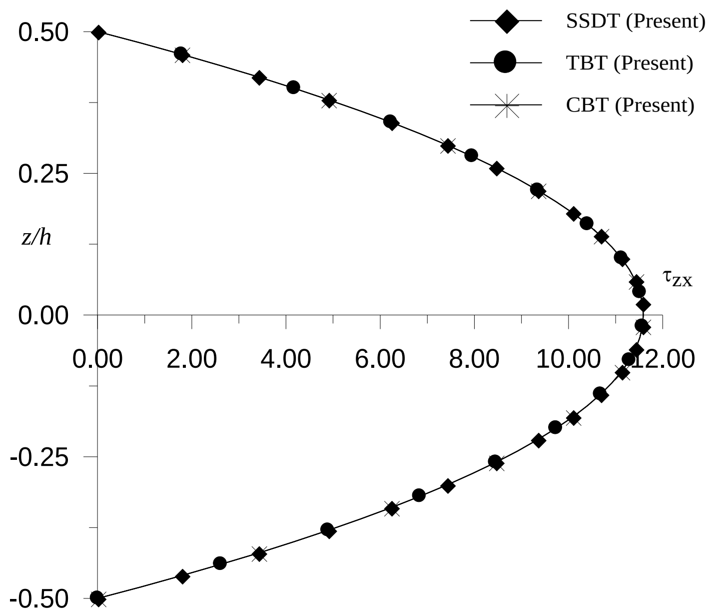

Fig. 14. Normalized transverse shear stress $\left(\bar{\tau}_{zx}\right)$ through the thickness of orthotropic beam (0^0) subjected to combined uniformly distributed thermo-mechanical load for aspect ratio 4

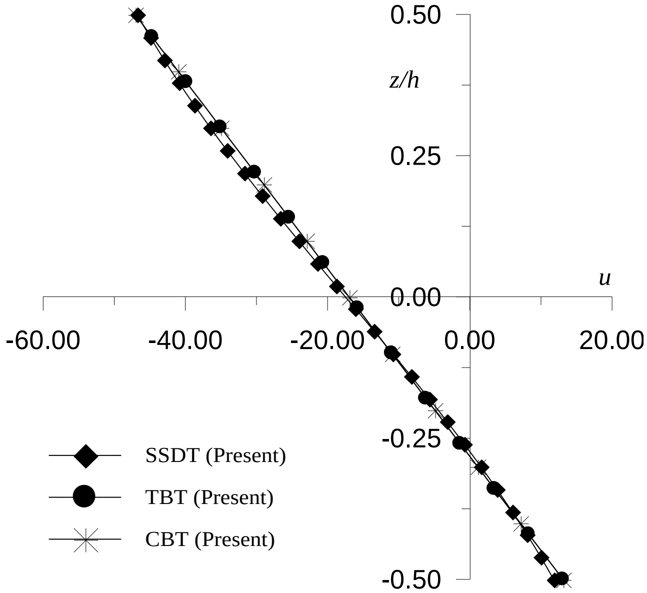

Fig. 15. Normalized axial displacement $(\bar{u})$ through the thickness of two-layer laminated beam (0/90) subjected to combined uniformly distributed thermo-mechanical load for aspect ratio 4

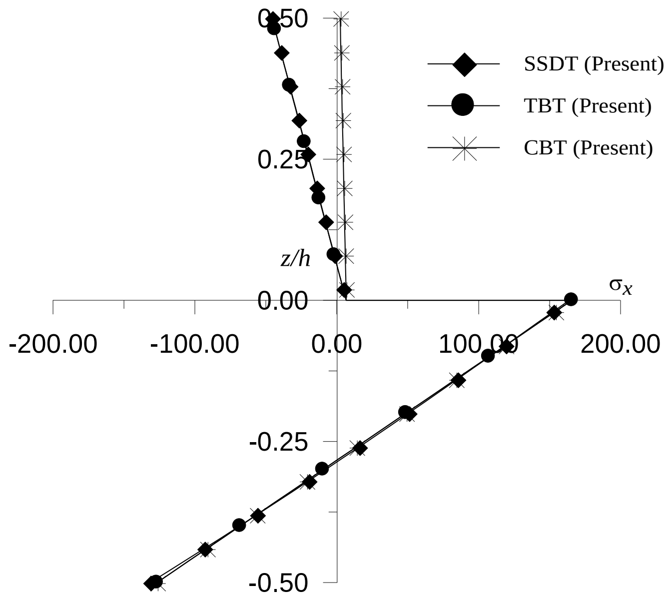

Fig. 16. Normalized axial stress $(\bar{\sigma}_x)$ through the thickness of two-layer laminated beam (0/90) subjected to combined uniformly distributed thermo-mechanical load for aspect ratio 4

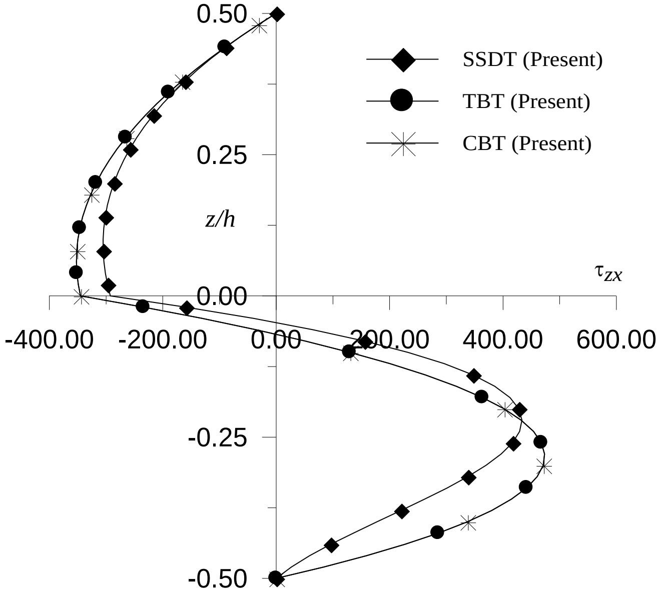

Fig. 17. Normalized transverse shear stress $\left(\bar{\tau}_{zx}\right)$ through the thickness of two-layer laminated beam (0/90) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load for aspect ratio 4

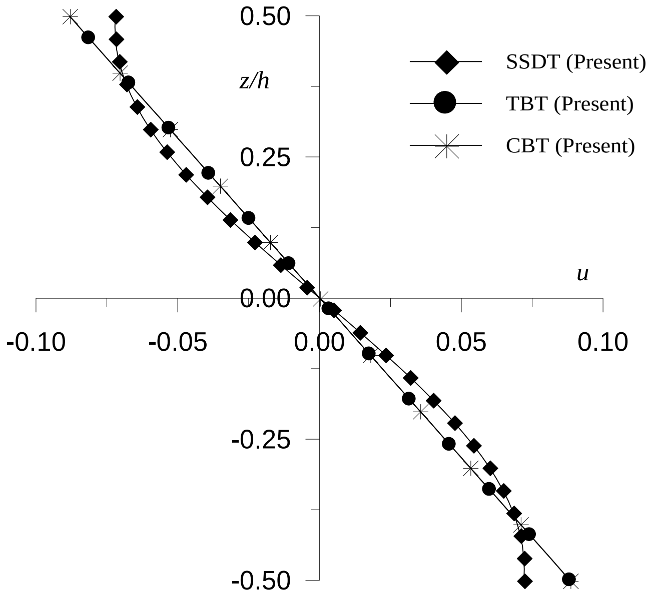

Fig. 18. Dimensionless axial displacement $\left(\bar{u}\right)$ through the thickness of three-layer laminated beam (0/90/0) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load for aspect ratio 4.

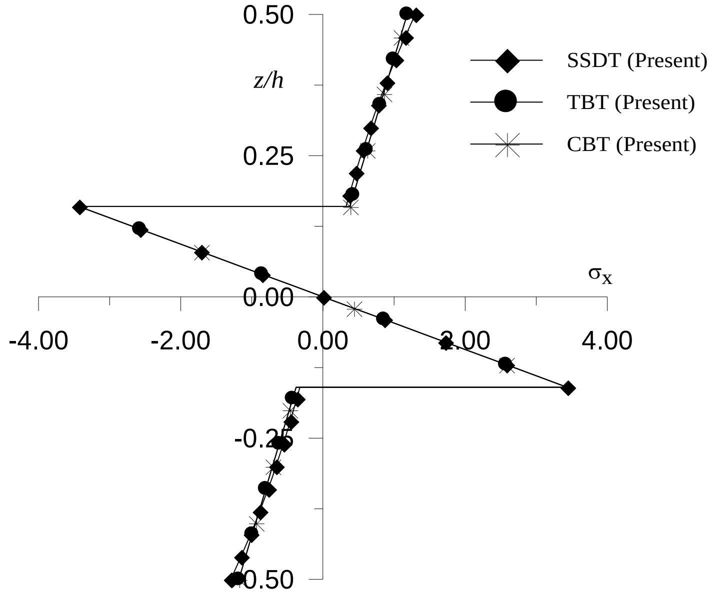

Fig. 19. Normalized normal stress $\left(\bar{\sigma}_x\right)$ through the thickness of three-layer laminated beam (0/90/0) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load for aspect ratio 10

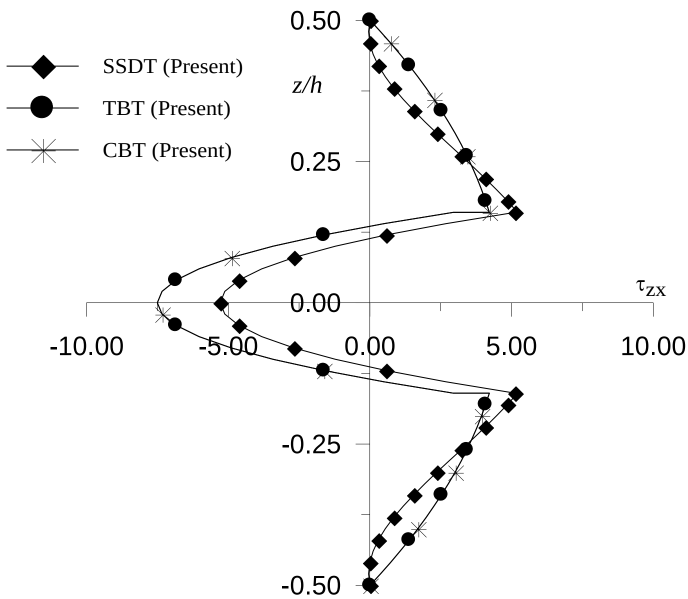

Fig. 20. Normalized transverse shear stress $\left(\bar{\tau}_{zx}\right)$ through the thickness of three-layer laminated beam (0/90/0) subjected to uniformly distributed thermal load in combination with uniformly distributed transverse mechanical load and obtained by equilibrium equation for aspect ratio 10

7. Discussion of results

Axial displacement $(\bar{u})$:

Single layer orthotropic beam, 0^0:

It has been observed that, the axial displacements obtained by sinusoidal shear deformation theory, Timoshenko beam theory and classical beam theory are identical irrespective of the aspect ratios in case of pure thermal load. The through thickness variation of axial displacement for single layer orthotropic beam under pure thermal load is shown in Fig. 3 for aspect ratio 4. The combined thermo-mechanical loads found to yield different values of this displacement with different aspect ratios as shown in Table 4. The through thickness variation of axial displacements under combined thermo-mechanical loads is shown in Fig. 12 for aspect ratio 4.

Two-layer $0^0/90^0$ beam:

The results of axial displacement along x axis obtained by sinusoidal shear deformation theory (SSDT) are in good agreement with the results obtained by Timoshenko beam theory (TBT) and classical beam theory (CBT) for aspect ratios 4, 10 and 100 (Table 2). The percentage error when compared with exact elasticity solution reduces as aspect ratio changes from 4 to 10 and found to be 11.14 % for aspect ratio 10. Further, this percentage error reduces to 2.59 % for aspect ratio 100. The through thickness variation of axial displacement under sinusoidal thermal load is as shown in Fig. 5 for aspect ratio 4. The results of axial displacement obtained by sinusoidal shear deformation theory (SSDT) for aspect ratio 4 under uniformly distributed thermal load increases slightly when aspect ratio changes from 4 to 10, whereas the results obtained by TBT and CBT are independent of aspect ratio. The shear deformation effect has been observed in the result shown by SSDT. The results of axial displacement obtained by SSDT, TBT and CBT for aspect ratio 100 are in good agreement with each other. The effect of shear deformation has been observed when acted upon by combined thermo-mechanical loads as shown in Table 5. The through thickness variation of axial displacement under combined thermo-mechanical loads is shown in Fig. 15 for aspect ratio 4 which shows slight deviation of SSDT as compared to TBT and CBT.

Three-layer $0^0/90^0/0^0$ beam:

The results of axial displacement in three-layer laminated beam subjected to sinusoidal, uniformly distributed pure thermal load and combined thermo-mechanical loads are shown in Table 3 and Table 6, respectively. It has been observed that axial displacements obtained by Timoshenko beam theory and classical beam theory have identical values and are independent of aspect ratios, whereas the results of axial displacements obtained by sinusoidal shear deformation theory (SSDT) changes as aspect ratio changes from 4 to 100. The through thickness variation of axial displacements under pure thermal load and combined thermo-mechanical load is shown Figs. 9 and 18 respectively. These figures show the realistic variation obtained by sinusoidal shear deformation theory (SSDT) indicating the shear deformation effect under pure thermal load and combined thermo-mechanical loads.

Transverse displacement $(\bar{w})$:

Single layer orthotropic beam, 0^0:

The results of transverse displacement for orthotropic beam under pure thermal load and combined thermo-mechanical loads are shown in Tables 1 and 4. In case of pure thermal load identical values are obtained by present theory, Timoshenko beam theory and classical beam theory for all aspect ratio. When orthotropic beam is acted upon by combined thermo-mechanical load, the considerable difference has been observed in the results obtained by sinusoidal shear deformation theory and Timoshenko beam theory for aspect ratio 4 and 10.

Two-layer 0/90 beam:

The results of transverse displacement for two-layer antisymmetric laminated beam under pure thermal load are shown in Table 2. Transverse displacement along z direction obtained by sinusoidal shear deformation theory (SSDT) for aspect ratio 4 shows the effect of shear deformation, whereas the results of transverse displacement obtained by Timoshenko beam theory (TBT) and classical beam theory (CBT) do not show any change in the result even though aspect ratio changes from 4 to 10. The results of transverse displacement obtained by sinusoidal shear deformation theory (SSDT) are found to increase with increase in aspect ratio from 4 to 10 and 10 to 100. The percentage error when the results obtained by SSDT are compared with exact solution of elasticity reduces from 11.03 % to 5.8 % when aspect ratio changes from 4 to 10. Further, for aspect ratio 100, transverse displacement obtained by SSDT, TBT and CBT are in good agreement with each other and percentage error reduces to 4.3 % when results obtained by SSDT are compared with exact solution of elasticity. The results of transverse displacement under combined thermo-mechanical loads are presented in the Table 5. The effect of transverse mechanical load on this displacement has been observed for aspect ratio 4 when obtained by sinusoidal shear deformation theory.

Three-layer symmetric laminated beam 0/90/0:

The results of transverse displacements for three-layer symmetric laminated beam under pure thermal load are shown in Table 3. It is observed that the results obtained by Timoshenko beam theory (TBT) and classical beam theory (CBT) have identical values for all aspect ratios whereas the transverse displacement obtained by present theory increases as aspect ratio increases from 4 to 100, this shows the effect of shear deformation. The results of transverse displacement under combined thermo-mechanical loads are presented in Table 6. A considerable difference has been observed in the results of SSDT and TBT for aspect ratio 4 under combined thermo-mechanical load. This effect is due to additional mechanical load.

Axial normal stress $\left(\bar{\sigma}_x\right)$:

Single layer orthotropic beam, 0^0:

The results of axial normal stress for orthotropic beam when subjected to pure thermal load are presented in Table 1. It has been noted that the results obtained by present theory, Timoshenko beam theory and classical beam theory under pure thermal load are zero. That is no bending stresses are developed due to pure thermal load. It means the effect of pure thermal load on bending of orthotropic beam is negligible. The effect of mechanical load has been observed when an orthotropic beam is acted upon by combined thermo-mechanical load as shown in Table 4. The through thickness variation of these stresses under combined thermo-mechanical load is shown in Fig. 13 for aspect ratio 4 indicating the effect of mechanical load alone.

Two-layer 0/90 beam:

The results of axial normal stress for two-layer laminated beam when subjected to pure thermal load are presented in Table 2. The results of axial normal stress for two-layer laminated beam obtained by sinusoidal shear deformation theory (SSDT) show 6.4 % error when compared with exact elasticity solution and the effect of shear deformation has been observed for aspect ratio 4. A notable change has been observed in the results obtained by SSDT when aspect ratio changes from 4 to 10 and 10 to 100, whereas the results obtained by TBT and CBT remains same even though aspect ratio changes from 4 to 10 and 10 to 100. It is noted that the results of axial normal stress evaluated by SSDT increase with increase in aspect ratio from 4 to 100 under sinusoidal thermal load and decrease in this stress is observed in case of uniformly distributed thermal load with increase in aspect ratio from 4 to 100. The percentage error between sinusoidal shear deformation theory (SSDT) and exact elasticity solution for aspect ratio 10 is found to be 2.2 % which further decreases to 0.75 % for aspect ratio 100. The through thickness variation of normal stress across the thickness of laminated beam is as shown in Fig. 6 for aspect ratio 4 under sinusoidal thermal load with change in direction. These stresses under combined thermo-mechanical load are presented in Table 5. The results of axial normal stress obtained by present theory show considerable difference when compared with the results of Timoshenko beam theory and classical beam theory for aspect ratio 4 under uniformly distributed combined thermo-mechanical load. The through thickness variation of these stresses under combined thermo-mechanical load is shown in Fig. 16 for aspect ratio 4. The variation of this stress shows the notable deviation from classical beam theory.

Three-layer symmetric laminated 0/90/0 beam:

The results of axial normal stress for three-layer symmetric laminated beams subjected to pure thermal load and combined thermo-mechanical load are presented in Tables 3 and 6 respectively. It is noted that the results obtained by Timoshenko beam theory and classical beam theory have identical values for all aspect ratios whereas axial normal stress obtained by sinusoidal shear deformation theory (SSDT) decreases as aspect ratio increases. The through thickness variation of theses stresses under pure thermal load and combined thermo-mechanical load are shown in Figs. 10 and 19 respectively. The effect of additional mechanical load has been observed in figure 19.

Transverse shear stress $\left(\bar{\tau}_{zx}^{EE}\right)$:

Single layer orthotropic beam, 0^0:

The results of transverse shear stress for orthotropic beam subjected to pure thermal load and combined thermo-mechanical loads are shown in Tables 1 and 4 respectively. Transverse shear stresses obtained by sinusoidal shear deformation theory, Timoshenko beam theory and classical beam theory have identical values for aspect ratios 4, 10 and 100. Through thickness variations of these stresses under pure thermal load and combined thermo-mechanical load are shown in Figs. 4 and 14, respectively.

Two-layer 0/90 beam:

The results of transverse shear stress for two-layer laminated beam subjected to pure thermal load are shown in Table 2 for various aspect ratios. Transverse shear stresses are obtained by integrating the equations of equilibrium from theory of elasticity. This satisfies the shear stress free boundary conditions on the top and bottom surfaces of laminated beam. The results of transverse shear stress obtained by sinusoidal shear deformation theory (SSDT) show shear deformation effect for aspect ratio 4 under sinusoidal and uniform thermal load. The through thickness variation of transverse shear stress is as shown in Figs. 7 and 8 under sinusoidal and uniform thermal loads, respectively. The continuity of stress has been observed at the interface with reversal of sign under sinusoidal and uniform thermal load. It is noted Timoshenko beam theory (TBT) and classical beam theory (CBT) underpredicts these stresses under uniform thermal load as shown in Fig. 8 for aspect ratio 10. The results of transverse shear stress for two-layer laminated beam when subjected to combined thermo-mechanical loads are shown in Table 5. It has been noted that transverse shear stresses obtained by TBT and CBT have identical values for aspect ratios 4 and 10, whereas the results of transverse shear stress obtained by SSDT show the effect of additional mechanical load. The through thickness variation of this stress is shown in Fig. 17 for aspect ratio 4 under combined thermo-mechanical loads.

Three-layer symmetric laminated 0/90/0 beam:

The results of transverse shear stress for three-layer symmetric laminated beam subjected to pure thermal load are shown in Table 3. A considerable difference has been noted in the results obtained by SSDT, TBT and CBT for aspect ratios 4 and 10. The effect of shear deformation has been observed in the result obtained by SSDT for thick beam under pure thermal load. The shear stress free boundary conditions are observed. In case of combined thermo-mechanical loads, the effect of additional mechanical load has been observed and shown in Table 6 for various aspect ratios. The through thickness variation of theses stresses under pure thermal load and combined thermo-mechanical loads are shown in Figs. 11 and 20, respectively. In case of thermo-mechanical load shear stress distribution deviates considerably with change in sign as compared to the distribution in case of pure thermal load.

8. Conclusions

In this article, a numerical model of sinusoidal shear deformation theory has been presented to investigate the thermal and thermo-mechanical response of single layer orthotropic, two-layer antisymmetric and three-layer symmetric cross ply laminated beams subjected to pure thermal load and combined thermo-mechanical loads for different aspect ratios. Special attention is pointed towards the transverse shear stresses, which plays an important role in thick beams. This study shows the necessity of taking in to account the transverse shear effect which cannot be neglected, particularly in thermal problems of laminated beams. It has been observed that the Timoshenko beam theory and classical beam theory underpredicts the transverse shear stress and overpredicts the transverse displacement for thick beam. The effect of shear deformation has been observed in the results of axial stress obtained by sinusoidal shear deformation theory for thick beam. Timoshenko beam theory and classical beam theory are observed to be suitable for thin laminated beams whereas sinusoidal shear deformation theory is suitable for thin as well as moderately thick laminated beam under pure thermal load and combined thermo-mechanical loads. Further, sinusoidal shear deformation theory obviates the need of shear correction factor and yields realistic displacements and stresses. Hence, the equivalent single layer sinusoidal shear deformation theory is strongly suitable for thick laminated beams under thermal environment.

References

Ali JSM, Bhaskar K, Vardan TK (1999). A new theory for accurate thermal/mechanical flexural analysis of symmetric laminated plates, Composite Structures, 45, 227-232.

Bhaskar K, Vardan TK, Ali JSM (1996). Thermoelastic solutions for orthotropic and anisotropic composite laminate, Composites: Part B, 27B, 415-420.

Carrera E (2000a). An assessment of mixed and classical theories for the thermal stress analysis of orthotropic multilayered plates, Journal of Thermal Stresses, 23, 797-831.

Carrera E (2000b). A priori vs. a posteriori evaluation of transverse stresses in multilayered orthotropic plates, Composite Structures, 48, 245-260.

Carrera E, Ciuffreda A (2004). Closed form solutions to assess multilayered plate theories for various thermal for various thermal stress problems, Journal of Thermal Stresses, 27, 1001-1031.

Ghugal YM, Kulkarni SK (2013). Flexural analysis of cross-ply laminated plates subjected to nonlinear thermal and mechanical loadings, Acta Mechanica, 224, 675-690.

Ghugal YM, Kulkarni SK (2013). Thermal flexural analysis of cross ply laminated plates using trigonometric shear deformation theory, Latin American Journal of Solids and Structures, 10, 1001-1023.

Ghugal YM, Shimpi RP (2002). A review of refined shear deformation theories of isotropic and anisotropic laminated plates, Journal of Reinforced Plastics and Composites, 21, 775-813.

Kant T, Gupta A (1988). A finite element model for a higher order shear-deformable beam theory, Journal of Sound and Vibration, 125, 193-202.

Kant T, Marur S, Rao G (1997). Analytical solution to the dynamic analysis of laminated beams using higher order refined theory, Composite Structures, 40, 1-9.

Mindlin R (1951). Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates, Journal of Applied Mechanics, 18, 31-38.

Philippe V, Olivier P, (2009). A refined sine-based finite element with transverse normal deformation for the analysis of laminated beams under thermo-mechanical loads, Journal of Mechanics of Materials and Structures, 4, 1127-1155.

Soldatos KP, Watson (1997) A general theory for the accurate stress analysis of homogeneous and laminated composite beams, International journal of solids and structures, 34, 2857-2885.

Tanigawa Y, Murakami H, Ootao Y (1989). Transient thermal stress analysis of a laminated composite beam, Journal of Thermal Stresses, 12, 25-39.

Tessler A, Annett M, Gendron G (2001). A {1,2}-order plate theory accounting for three-dimensional thermoelastic deformations in thick composite and sandwich laminates, Composite Structures, 52, 67-84.

Touratier M (1991). An efficient standard plate theory, International Journal of Engineering Sciences, 29, 901-916.