

## Approximation methods for the actual trajectory of load carried by overhead crane to the required one – a comparative analysis

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### Abstract

In this article we present results of the comparative analysis of two different ways of approximation of actual spatial trajectory of load displacements carried by overhead crane. Sigmoidal functions and polynomials are used to achieve the required agreement with the real trajectories. It is also shown how to obtain maximum accuracy by changing the shape of the trajectory at a fixed displacement time.

**Keywords:** bridge crane, PID regulator, sigmoidal functions, accuracy of movement, acceleration of suspension, load, oscillation suppression.

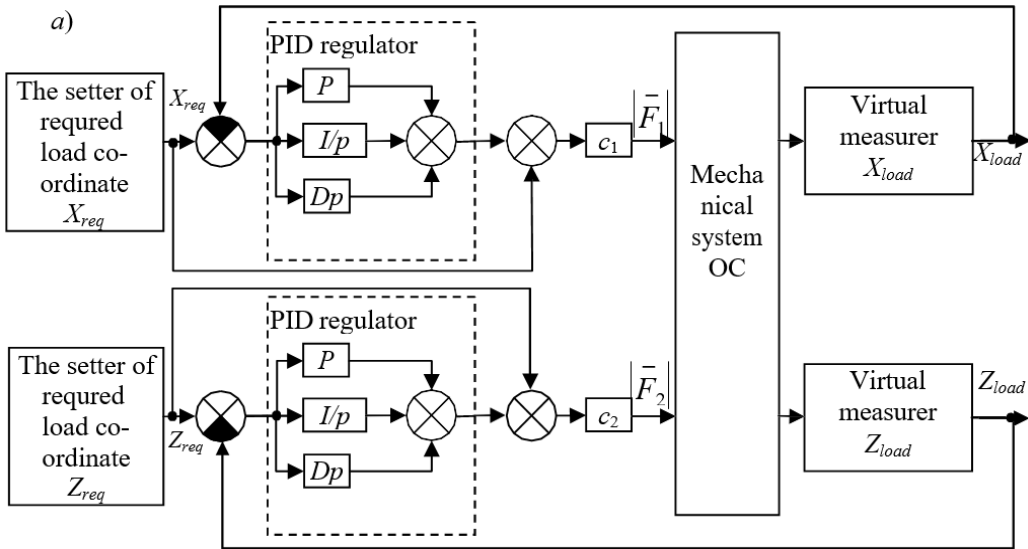
### 1. Introduction

In order to increase the productivity of overhead cranes (OC) it is necessary to damp the residual pendulum oscillation of the load (Shchedrin et al. 2007, Blackburn et al. 2010, Tolochko et al. 2010, Omar 2003, Abdel-Rahman et al. 2003, Fang et al. 2003). The method of providing the load movement along the required smooth trajectory has been developed, which reduces the load uncontrolled fluctuations (Shcherbakov et al. 2012-2014, Korytov et al. 2004-2015). This method has been developed for OC with non-rigid suspension load. For this purpose, a proportional-integral-derivative (PID) is used, which controls independently the two controlled axes of load horizontal flatness  $X_{load}$  of bridge and  $Z_{load}$  of trolley (Shcherbakov et al. 2014).

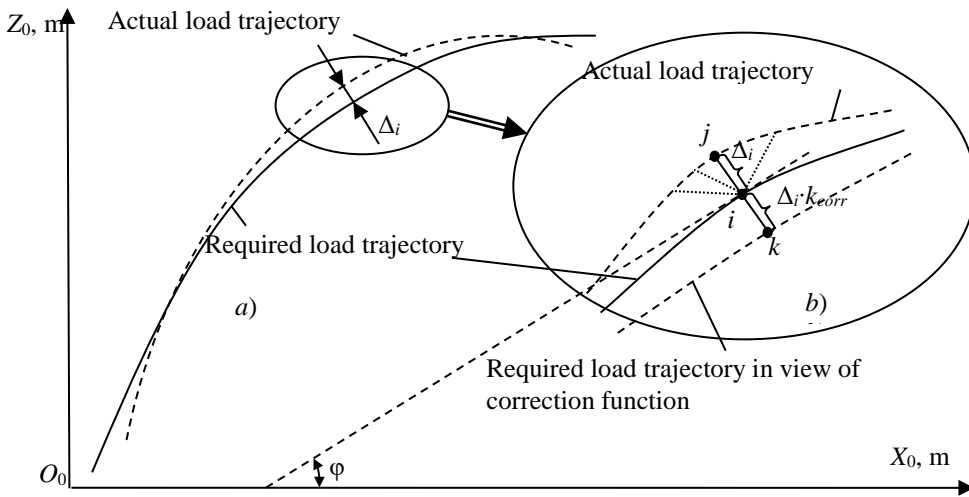
### 2. Problem Formulation

There is a simultaneous increase in the accuracy and the speed of movement at the required spatial trajectory of the cargo. This decrease is not accompanied by the deviation angles of the suspension in the gravitational vertical displacement. The angle control of OC load rope is carried out by means of the operational, in real-time, formation of the optimal law of motion of

suspension point in order to optimize approximation of the actual load trajectory (the required connections scheme is shown in Fig. 1 (Shcherbakov et al. 2015)).



**Fig. 1.** The scheme of connections for constructing a model of mechanical subsystem for the overhead crane using PID regulators (Shcherbakov et al. 2015)



**Fig. 2.** Diagram illustrating the determining principle the accuracy of setting the required load displacements trajectory (a), and principle of using the correction function by superimposing with the main function of setting for the required trajectory (b)

The proposed method makes it possible to disperse the load smoothly, gradually reduce the load handling speed when approaching the target point and extinguish the pendulum oscillation load. It uses the movement of the point of suspension. At the same time stabilizes the position

of the goods to the destination point. Special requirements for the form of the desired trajectory of the cargo and the methods of its task is not presented. In particular, it is not necessarily the analytical task trajectory. Conditions smoothness is desirable because this increases the accuracy of the implementation, but is not mandatory.

The disadvantage of this method is an absolute accuracy of displacement  $\Delta_i$  appearing in realization of any required curved trajectory. The value of the absolute accuracy of displacements in a point  $i$  on the required curve is defined as the minimum distance of this point  $i$  to the actual curve trajectory of load displacement (to the nearest point  $j$  on the actual cargo trajectory  $\{X_{load}(t); Z_{load}(t)\}$  Fig. 2).

The highest value of this displacement along the trajectory, measured in the direction perpendicular to the tangent to the desired path at the current point, gives the maximum absolute accuracy in the trajectory  $\Delta_{max}$  when implementing considered trajectory.

This accuracy is increased with the curvature of the required path of the cargo and with the cargo speed (less displacement time).

### 3. Problem Solution

For reduction of an error  $\Delta_{max}$  universal approach of adjustment is used: the corrected required trajectory is adjusted to the original required trajectory. These accuracies  $\{\Delta_i\}$  and  $\Delta_{max} = \max(|\Delta_i|)$  where calculated on original required path of the cargo displacement  $\{X_{req}(t); Z_{req}(t)\}_{init}$ , and the synthesis of the actual trajectory of the cargo displacement is carried out according to the diagram in Fig. 1, using the corrected required trajectory  $\{X_{req}(t); Z_{req}(t)\}_{corr}$  which differs from the original.

Corrected required trajectory  $\{X_{req}(t); Z_{req}(t)\}_{corr}$  can be formed in different ways and using different algorithms of «adapting». The differences in the methods and algorithms of «adapting» depend on the method of setting the original required path of the cargo displacement  $\{X_{req}(t); Z_{req}(t)\}_{init}$ .

We present here two ways of setting the smoothed trajectory  $\{X_{req}(t); Z_{req}(t)\}_{init}$ :

**Trajectory № 1.** Using two sigmoidal (logistic) time functions separately for each horizontal coordinate  $X_0, Z_0$  of space in a fixed Cartesian coordinate system  $O_0X_0Y_0Z_0$ , the overall trajectory is formed in an arc to bypass a single obstacle, not having in  $O_0X_0Y_0Z_0$  inflection point (Shcherbakov et al. 2015, Mitchell 1997):

$$X_{TR}(t, a, c) = l_x / (1 + e^{-a \cdot (t-c)}); \quad (1)$$

$$Z_{TR}(t, a_1, c_1, a_2, c_2) = (s_x \cdot k_{sx}) / ((1 + e^{-a_1 \cdot (t-c_1)}) \cdot (1 + e^{-a_2 \cdot (t-c_2)})); \quad (2)$$

$$Y_{TR} = \text{const}, \quad (3)$$

where  $t$  – time;  $X_{req}, Z_{req}$  – the required horizontal load coordinates at time  $t$ ;  $Y_{req}$  – vertical load coordinate;  $a, c, a_1, c_1, a_2, c_2$  – parameters of sigmoidal functions;  $l_x$  – set length of the load displacement along the axis  $X_0$  (start and end points have zero coordinate  $Z = 0$ );  $s_x$  – the maximum size of a set displacement of the arc of required load trajectory along the axis  $Z_0$  (to bypass obstacles);  $k_{sx}$  – correction coefficient of maximum value of load sideways displacement:

$$k_{sx} = \left(1 + e^{-a_1 \cdot (c-c_1)}\right) \cdot \left(1 + e^{-a_2 \cdot (c-c_2)}\right), \quad (4)$$

Function (2) is the product of two functions (1) – increasing and falling, with distinct points of time of inflection. The parameters  $c, c_1, c_2$  define the time points of inflection of sigmoidal functions. The parameters  $a, a_1, a_2$  determine the rate of functions change (growth or decline, depending on the sign).

As a result, in Cartesian coordinate system  $O_0X_0Y_0Z_0$ , the shaped trajectory in an arc is formed that has no point of inflection. This example is shown in Fig. 3.

The maximum sizes of the curve were specified by parameters  $l_x = 10$  m and  $s_x = 5$  m (see. Fig. 3).

**Trajectory № 2.** Using known algorithms of spline-interpolation (approximation by means of method of least squares) of the four reference points  $\{X_{rp}(j); Z_{rp}(j)\}, j \in [1,4]$ , set in the fixed Cartesian coordinate system  $O_0X_0Y_0Z_0$ , polynomial  $P$  of degree  $n$  is formed (Prasolov 2003):

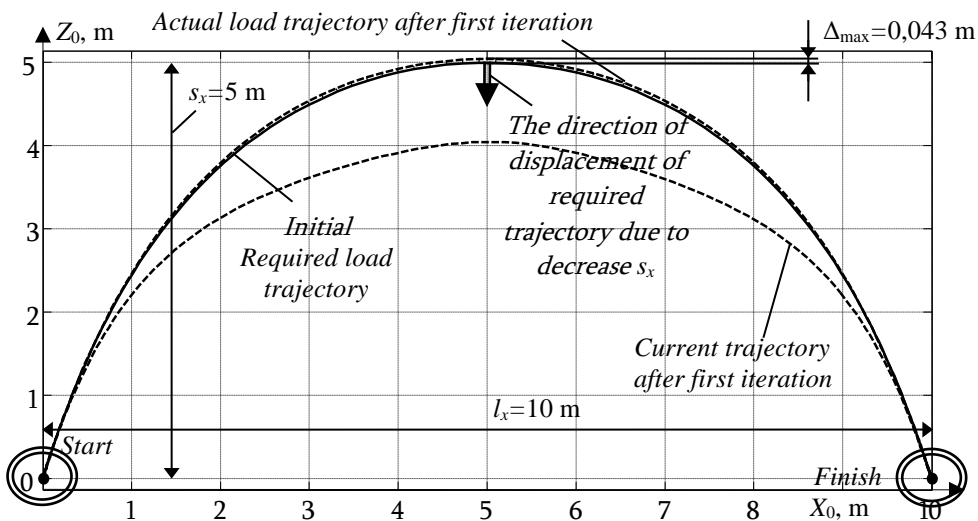
$$Z_{req} = p_1(X_{req})^n + p_2(X_{req})^{n-1} + p_3(X_{req})^{n-2} + \dots + p_n \cdot X_{req} + p_{n+1}, \tag{5}$$

where  $p_1 \dots p_{n+1}$  are coefficients of the polynomial  $P$  ordered by descending degree of argument.

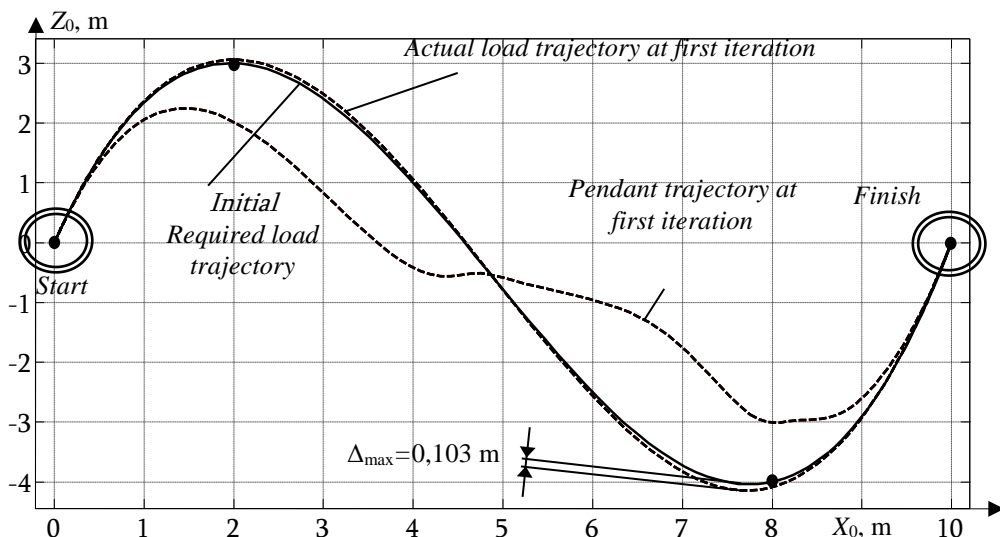
This polynomial with  $n = 3$  generates a trajectory as a function of  $Z_{req} = f(X_{req})$  of bypass of two obstacles having one point of inflection (Fig. 4).

Motion on the trajectory formed in this way  $Z_{req} = f(X_{req})$  is performed: along the axis  $O_0X_0$  – by the sigmoid function of the form (1), and along the axis  $O_0Z_0$  – from (1) using the expression (5) time dependence  $Z_{req} = f(t)$  is formed.

The reference points of this curve are shown as an example, with coordinates  $X_{rp}(j) = [0 \ 2 \ 8 \ 10]$  m;  $Z_{rp}(j) = [0 \ 0 \ 3 \ 4]$  m (see. Fig. 4).



**Fig. 3.** Initial required trajectory in the form of arc without an inflection point (-) and the actual load trajectory (---) (it is used the trajectory of example №1,  $l_x = 10$  m,  $s_x = 5$  m,  $\Delta_{max} = 0,043$  m,  $T_p = 25$  c)



**Fig. 4.** The initial required trajectory in the form of a spline- interpolation of four reference points (rp)(-) and the actual load trajectory (---) (The example of trajectory №2,  $X_{rp}(j) = [0, 2, 8, 10]$  m,  $Z_{rp}(j) = [0, 3, -4, 0]$  m,  $T_p = 30c$ )

The conditional time of suspension point displacement of load  $T_s$  which, according to the results in references, is included in the limits  $T_s = 1,33333 \cdot t_{load}$  where  $t_{load}$  – the conditional time of load displacement which has a fixed value of  $T_p = 25$  s in the first example of movement on the arc trajectory (see. Fig. 3), and  $T_p = 30$  s in the second example of movement on the trajectory of spline-interpolation of four reference points (see. Fig. 4).

As an example, we consider two ways of corrected required trajectory  $\{X_{req}(t); Z_{req}(t)\}_{corr}$  determination:

- 1) Determination by changing the coordinates (offset) reference points of the required initial trajectory;
- 2) Formation through the use of additional corrective functions. The latter is added to the basic function set the required trajectory.

The first method assumes local optimization by variable parameters. The second method assumes the execution of several recurring iterations. The absolute accuracy is always calculated by the initial required trajectory.

For trajectory 1, the first method is laid in reducing the value  $s_x$ , with respect to which (1) - (4) is formed as corrected required trajectory  $\{X_{req}(t); Z_{req}(t)\}_{corr}$ . For trajectory 2 it is necessary to correct coordinates  $Z_{rp}$  of intermediate reference points (2, 3) and  $X_{rp}(2,3)$ .

The second proposed method for any trajectory is calculated on the basis of the vector of accuracy  $\{\Delta_i\}$ , obtained by implementing the initial required trajectory, with two vectors of coordinatewise correction:

$$dX_i = \Delta_i \cdot \sin(\varphi_i) \cdot k_{korr}; dZ_i = \Delta_i \cdot \cos(\varphi_i) \cdot k_{korr}; \tag{6}$$

where

$$\varphi_i = \arctg((Z_{req\ i} - Z_{req\ i-1}) / (X_{req\ i} - X_{req\ i-1})); \tag{7}$$

$$\Delta_i = \frac{\left| \begin{aligned} & (Z_{load\ i} - Z_{load\ i-1}) \cdot X_{req\ i} + \\ & + (X_{load\ i-1} - X_{load\ i}) \cdot Z_{req\ i} + \\ & + (X_{load\ i} \cdot Z_{load\ i-1} - X_{load\ i-1} \cdot Z_{load\ i}) \end{aligned} \right|}{\sqrt{(Z_{load\ i} - Z_{load\ i-1})^2 + (X_{load\ i-1} - X_{load\ i})^2}}; \quad (8)$$

and  $k_{corr}$  is correction coefficient that determines the degree of displacement of the corrected trajectory from actual load trajectory which was obtained at the previous iteration of the algorithm. In this particular range of experiments this value is taken as  $k_{corr} = 1$ .

Taking into account that using mathematical modeling of OC with PID controls all virtually measured parameters that are stored in the discrete form with constant pitch sampling time ( $\Delta t = 0,1$  s, which corresponds to an increase of the index  $i$  by 1). Using expressions (6) - (8) allowed for a discrete time dependences obtained in the previous iteration (simulation of the displacement cycle is costly), we determine time dependences of the additional correction function for the next iteration of the algorithm in the form of  $dX(t)$ ,  $dZ(t)$ . The latter, in turn, were used to calculate the required corrected load displacement trajectory on a subsequent iteration by adding on axis:

$$\{X_{req}(t); Z_{req}(t)\}_{corr} = \{X_{req}(t); Z_{req}(t)\}_{init} + \{dX(t); dZ(t)\}. \quad (9)$$

By using expressions (6) - (9) at the current time step, by point  $i$  on the original required trajectory and the point  $j$  on the actual trajectory of load displacement in the previous iteration, we find point  $k$  as corrected by required trajectory of load displacement for the subsequent iteration (see. Fig. 2).

For a comparative analysis of the two methods of forming corrected desired trajectory we adduce computing experimental investigations on the simulation model of the dynamic system OC with PID regulators (see. Fig. 1) (Shcherbakov et al. 2015).

Proportional coefficient, integral and differential time constants of PID regulators of gear control of bridge and truck displacement in described range of computing experimental investigations have the following values:  $P = 20$ ;  $I = 5$ ;  $D = 5$ , respectively.

Other parameters of the OC and its workflow had the following fixed values: the weight of the overhead crane,  $m_1 = 3500$  kg; the weight of load truck,  $m_2 = 1250$  kg; the weight of load,  $m_3 = 100$  kg; the length of the rope on which there is suspended load = 12 m; coefficients of damping by the angular coordinates of deviations hoist rope from the vertical in two mutually perpendicular planes, 100 N·m·s / rad.

As the main criterion for comparison we use maximum absolute accuracy of the initial implementation of the required trajectory  $\Delta_{max}$ .

As an example, in Fig. 5 we give the results of the implementation of the method number 1 for selection of coordinates of reference point for the trajectory № 1. As a reference point, we adopt midpoint of the arc - the position which is specified by  $s_x$ . We adopt the initial value of this parameter  $(s_x)_{init} = 5$  m. When we have the reduction of corrected values  $(s_x)_{corr}$  from 5 to 4,92 m, the minimum value of the maximum absolute accuracy on the trajectory  $\Delta_{max} = 0,0085$  meters is achieved by  $(s_x)_{corr} = 4,96$  m (against  $\Delta_{max} = 0,043$  m with no correction). Thus we have the decline in  $\Delta_{max}$  by 80%.

Fig. 6 shows the results of the method 1 by selection of coordinates of two intermediate reference points for trajectory 2 given in Fig. 4. Selection is made on the coordinate  $Z_0$  for

reference point number 2 within the  $Z_{ref}(2) = 3... 2,8$  m, and for the reference point number 3 within the  $Z_{ref}(3) = -4... -3,8$  m.

When  $Z_{ref}(2) = 2,91$  m,  $Z_{ref}(3) = -3,925$  m we achieve local accuracy minimum  $\Delta_{max} = 0,0215$  m against  $\Delta_{max} = 0,103$  m without correction. Thus we have the decline  $\Delta_{max}$  by 80 %.

We note that for the first and for the second trajectories, the realization of method number 1 in selection of reference points coordinates, practically does not lead to changes in accelerations developed by the OC drives and the full operation of the drives (changes are less than 1%). The disadvantage of the method number 1 is the minimum reduction of accuracy of trajectory.

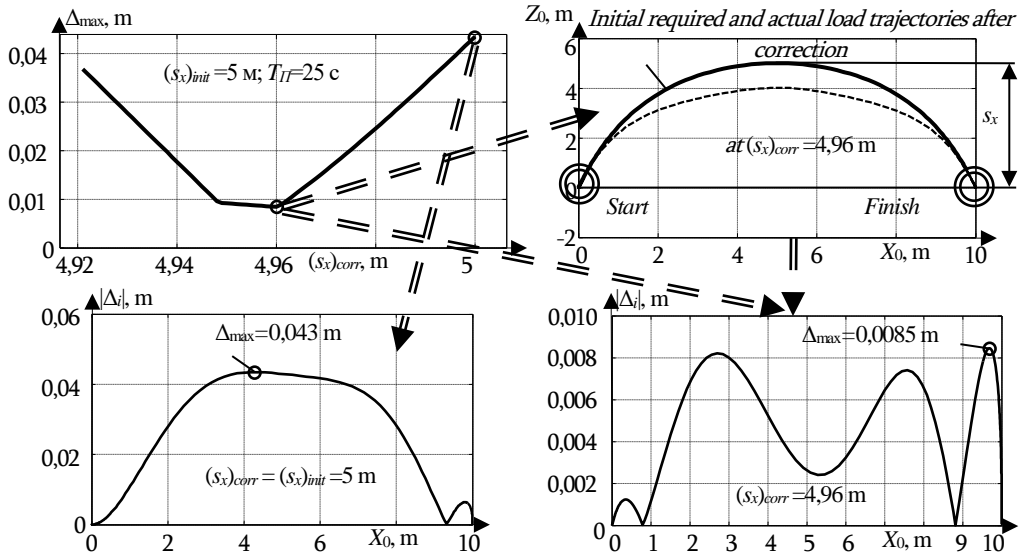
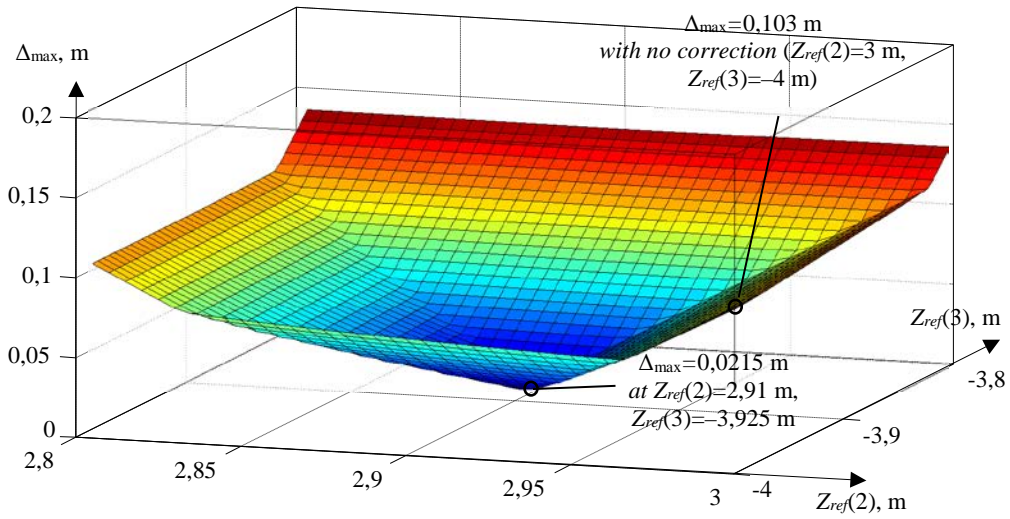
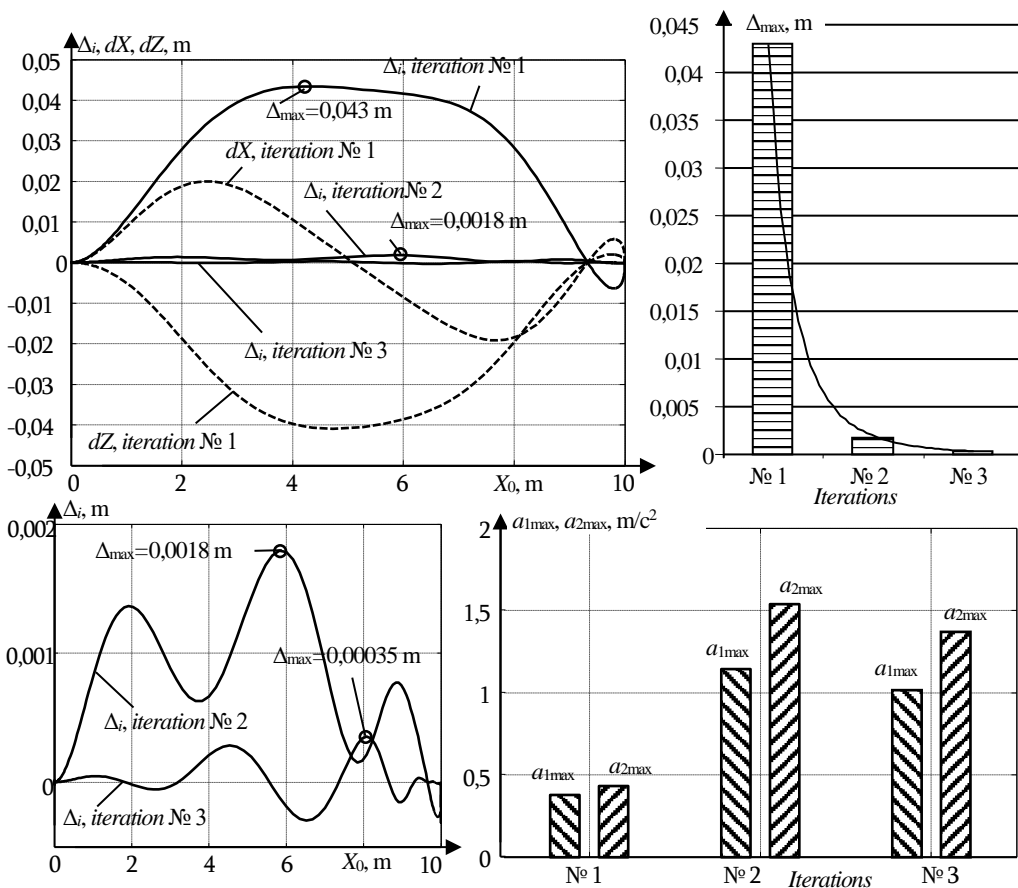


Fig. 5. The results of the method number 1 with selection of coordinates of reference point for trajectory 1 (an example)



**Fig. 6.** The results of the method 1 with selection of coordinates of reference points for trajectory 2 (an example)





**Fig. 7.** The results of the method number 2, with realization of additional corrective function superimposed with the main, for trajectory 1 (an example)

In Fig.7, we give the results of the method number 2 realization of additional corrective function superimposed with the main for trajectory 1. We consider the first three iterations of the algorithm.

On the second iteration for the same initial trajectory achieved accuracy an order of magnitude smaller than in the method of number 1 (less than 2 mm,  $\Delta_{max} = 0,0018$  m). However, significantly, by 200-250% increases the maximum of movement acceleration  $a_{1max}$ ,  $a_{2max}$  (bridge and trolley). In Fig. 8 we give the results of the method number 2 of additional correction function superimposed with the main, for trajectory 1.

The analysis of relations shown in Fig. 8 and 6 showed that the method number 2 regarding the trajectory 2 has more disadvantages in comparison with method № 1. The minimum accuracy  $\Delta_{max}$  which is achieved in the second iteration of the algorithm is larger than the same minimum accuracy for the same trajectory using the method of number 1 (0,0215 vs. 0,029 m). At the same time, at further iterations (iteration number 3) when using the method number 2, the error is not only reduced, but even slightly increases. There is a sharp increase (one order of magnitude or more) of maximum accelerations  $a_{1max}$   $a_{2max}$  developed by OC drives at the second or third iteration. Total work of OC drives at the iteration number 2 and 3 increases several times. Moreover, the following iterations cause even more substantial increase in both

acceleration and overall work. Practically, the obtained required peak values of accelerations which have value of ten  $\text{m/s}^2$  are not realizable by means of modern OC drives. That is, the method 2 for the trajectory for bypass of two obstacles, which has an inflection point, cannot be used in practice.

#### 4. Conclusion

It was proved that there is a possibility of significant (several times) increase of the accuracy of the smoothed implementation of the OC displacement with point of inflection, and also in case without inflection point due to slight changes in the shape of the trajectory at a fixed time movements.

Method number 2 has more possibilities to reduce maximum deviation for the trajectory in an arc without an inflection point for turning movement of a single obstacle (trajectory number 1).

Changing the path number 1 by a small change in the coordinates of the reference point (the value of lateral displacement) does not increase the maximum acceleration of the bridge and trolley and the full work at OC drives. However, this method can not reduce the error path of the implement load. Using the correction functions superimposed with the main function of required trajectory setting, allows to achieve maximum deviation decrease of trajectory 1. It does not increase maximum accelerations of bridge and load trolley and full operation of OC drives.

For the trajectory number 2 set by polynomial function and having four reference points and an inflection point, the way number 1 does not provide decrease in an error of realization of a trajectory of movement of freight. But it also does not cause increase in the maximum accelerations of the bridge and the cargo cart and full operation of OC drives. At the same time method number 2 for this trajectory gives the worst results in reducing deviation, and simultaneously it is not practically applicable because of excessively large maximum values of accelerations, developed by OC drives.

For trajectories in the form of an arc without a point of inflection method number 2 (the method of corrective functions realization superimposed with the main function of required trajectory setting) has advantages.

It is practically possible to use only method number 1 for the trajectory with four reference points (this trajectory has inflection point). Method number 1 is the method a small change in the coordinates of reference points.

Извод

### Методе апроксимације за стварну путању терета који носи мосни кран – компаративна анализа

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## Резиме

У овом раду представљени су резултати компаративне анализе два различита типа апроксимација стварне просторне путање премештања терета мосним краном. Сигмоидалне функције и полиноми су употребљени како би се постигло неопходно слагање са правим путањама. Такође је показана могућност добијања максималне тачности променом типа путање за фиксно време премештања.

**Кључне речи:** мосни кран, ПИД регулатор, сигмоидална функција, тачност покрета, убрзање вешања, терет, сузбијање осцилација

## References

- Abdel-Rahman EM, Nayfeh AH, Masoud ZN (2003). Dynamics and control of cranes: a review. *Journal of Vibration and Control*, 9, 863-908.
- Blackburn D, Singhose W, Kitchen J, Patrangenu V, Lawrence J (2010). Command Shaping for Nonlinear Crane Dynamics, *Journal of Vibration and Control*, 16, 477-501.
- Denisenko VV (2007). Varieties of PID-regulators, *Automation in the industry*, 6, 45-50.
- Fang Y, Dixon WE, Dawson DM, Zergeroglu E (2003). Nonlinear coupling control laws for an underactuated overhead crane system. *IEEE / ASME Trans. Mechatronics*, Vol. 8, No. 3, 418-423.
- Korytov MS, Glushets VA, Zyryanova SA (2004). Modelling of working crane movements using SimMechanics and Virtual Reality Toolbox, *Exponenta Pro. Mathematics in applications*, 3-4, 94-102.
- Korytov MS, Zyryanova SA (2004). Simulation of a dynamic system with the crane by means of block package «SIMMECHANICS» system MATLAB, *Omsk Scientific Bulletin*, 4, 88-90.
- Mitchell, Tom M (1997). *Machine Learning*, WCB/McGraw-Hill.
- Omar HM (2003). Control of gantry and tower cranes: PhD Dissertation, *Virginia Polytechnic Institute and State University*. Blacksburg, Virginia. 2003.
- Prasolov VV (2003). *Polynomials*, M.: MTSNMO.
- Shchedrin AV, Serikov SA, Kolmykov VV (2007). Automatic system of soothing vibrations for overhead crane, *Instruments and systems. Management, monitoring, diagnostics*, 8, 13-17.
- Shcherbakov V, Korytov M, Sukharev R, Volf E (2015). Mathematical modeling of process moving cargo by overhead crane, *Applied Mechanics and Materials*, 701-702, 715-720.
- Shcherbakov VS, Korytov MS, Kotkin SV (2012). *The system of autoimmunization of modeling of hoist arrow – shaped hoist cranes*: monograph, SibADI, Omsk.
- Shcherbakov VS, Korytov MS, Volf EO (2014). The increase of accuracy and speed of load displacement on required trajectory by means of overhead crane, *Systems. Methods. Technologies*, 4(24), 52-57.
- Shcherbakov VS, Korytov MS, Volf EO (2014). The system of damping of space load oscillations moved by overhead crane, *SibADI Bulletin*, 6(40), 56-61.
- Shcherbakov VS, Korytov MS, Wolf EO (2014). The method of improving of the accuracy of the object displacement by means of crane by compensating its uncontrolled spatial oscillations, *Mechanization of construction*, 2, 21-25.

Tolochko OI, Bazhutin DV (2010). A comparative analysis of methods of damping cargo vibrations suspended to the mechanism of translational motion of overhead crane, *Electrical engineering and electrical equipment*, 75, 22-28.