

## The effect of slip velocity on the ferrofluid based squeeze film in longitudinally rough conical plates

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### Abstract

This article makes an attempt to study and analyze the effect of slip velocity on the performance of a magnetic fluid based squeeze film in conical plates considering the longitudinal roughness pattern. The slip model of Beavers and Joseph has been deployed to calculate the effect of slip velocity. The stochastic averaging model of Christensen and Tonder has been used to evaluate the longitudinal roughness effect. The concerned stochastically averaged Reynolds type equation is solved to get the pressure distribution which results in the calculation of load carrying capacity. The findings indicate that the combined adverse effect of slip velocity and roughness can be overcome to a large extent by the positive effect of magnetization and the standard deviation in the case of negatively skewed roughness. This effect further enhances when variance(-ve) is in place. A significant aspect of our study is that in spite of the adverse effect of slip velocity the rough bearing system sustains certain amount of load, even in the absence of the flow which is rarely seen in the case of traditional lubricant based conical bearing system.

**Keywords:** conical plate bearing, magnetic fluid, longitudinal roughness, slip velocity

### 1. Introduction

Magnetic fluid is referred to as a stable colloid dispersion of magnetic nanoparticles in a carrier liquid which mainly consists of three parts: ferromagnetic nanoparticles, coating of these magnetic nanoparticles and a carrier fluid. A simple flow model to explain the steady flow of magnetic fluids in the presence of slowly changing external magnetic fields was proposed by (Neuringer and Rosensweig 1964). A good number of research papers are available in the literature for the study of different types of bearing using Neuringer and Rosensweig flow model. For example, (Tipei 1982) in short bearing, (Agrawal 1986), (Shah and Bhat 2003) and (Deheri and Patel 2011) in slider bearing, journal bearing by (Nada and Osman 2007) and (Patel et al. 2012), (Andharia and Deheri 2010) in conical plates and circular plates by (Shah and Bhat 2000) and (Deheri and Abhangi 2011).

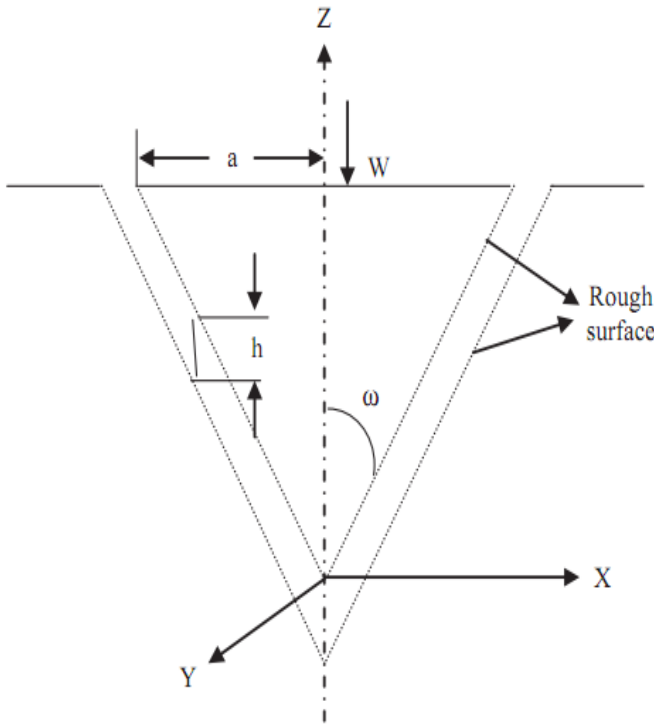
Most of the bearing surfaces tend to be rough after some run in and wear. During the last few years, it was established that the roughness of the surfaces significantly affects the bearing performance. Various Methods have been proposed to study the effect of surface roughness; (Christensen and Tonder 1969a, 1969b, 1970) employed a stochastic concept and found an averaging film model of lubricated surfaces with transverse roughness and longitudinal roughness. Using the Christensen and Tonder's stochastic model of roughness, (Ting 1972), (Praksh and Tiwari 1983), (Guha 1993), (Gupta and Deheri 1996), (Turaga et al. 1997), (Gururajan and Prakash 2000), (Gadelmawla et al. 2002), (Sinha and Adamu 2009), (Adamu and Sinha 2012), (Patel and Deheri 2013) dealt with the effect of surface roughness on the performance of various bearing systems with several geometries. It was found that the load carrying capacity of the bearing system increased with increasing magnetization of the magnetic fluid. (Patel and Deheri 2014) analyzed the effect of different porous structures on the performance of a Shliomis model based magnetic squeeze film in rotating rough porous curved circular plates. It was established that the adverse effect of transverse roughness could be compensated by the positive effect of magnetization in the case of negatively skewed roughness, suitably choosing the rotation ratio and the curvature parameters. (Patel and Deheri 2015) dealt with the combined effect of slip velocity and surface roughness on the performance of Jenkins model based magnetic squeeze film in curved rough annular plates. It was noticed that the effect of transverse surface roughness was adverse in general, Jenkins model based ferrofluid lubrication provided some measures in mitigating the adverse effect and this became more manifest when the slip parameter was reduced and negatively skewed roughness occurred.

In Tribology, the reduction of friction is quite crucial for the effective performance of the bearing system. It is found that slip velocity supports to reduce the friction. (Beavers and Joseph 1967) investigated the interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary condition at the interface. Flow with slip velocity becomes very useful for problems in chemical engineering for example, flow through pipes in which chemical reactions occur at the walls. (Patel 1980) discussed the performance of hydro-magnetic squeeze film between porous circular disks with velocity slip. Many investigations have studied, both theoretically and experimentally, the effects of slip on various types of bearings (Thompson and Troian 1997, Zhu and Granick 2001, Salant and Fortier 2004, Wu et al. 2006, Ahmed and Singh 2007, Patel and Deheri 2011, Wang et al. 2012). In all the above studies, it was obtained that the slip effect significantly affected the bearing system. (Rao et al. 2013) dealt with the effects of velocity slip and viscosity variation on squeeze film lubrication of two circular plates. So far no study has discussed the effect of velocity slip on the combined influences of longitudinal roughness and magnetism. However, there is a study (Patel and Deheri 2014) on infinitely long bearings with transverse surface roughness under the presence of a magnetic fluid. It is known that the longitudinal roughness induces an enhanced performance in some suitable situation.

Therefore, it was deemed proper to investigate the slip effect on the performance of a longitudinally rough ferrofluid squeeze film in conical plates. In fact, efforts have been made to analyze the extent to which the longitudinal roughness and magnetism combine compensated the adverse effect of velocity slip.

## 2. Analysis

The physical configuration of the bearing system, which is infinite in the Y- direction is shown in Figure 1. Here, squeeze film velocity  $\dot{h}_0 = dh_0/dt$  is in the z-direction. The magnetic field  $M$  is oblique to the lower plate.



**Fig. 1.** Configuration of the bearing system

The bearing surfaces are assumed to be transversely rough. According to discussions of (Christensen and Tonder 1969a, 1969b, 1970), the stochastic film thickness  $h$  of the lubricant film is considered as

$$h = \bar{h} + h_s \tag{1}$$

where  $\bar{h}$  denotes the mean film thickness and  $h_s$  is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces.  $h_s$  is assumed to be stochastic in nature and governed by the probability density function:

$$f(h_s) = \begin{cases} \frac{35}{32c} \left(1 - \frac{h_s^2}{c^2}\right)^3, & -c \leq h_s \leq c \\ 0, & \text{elsewhere} \end{cases}$$

where  $c$  denotes the maximum deviation from the mean film thickness. The mean  $\alpha$ , the standard deviation  $\sigma$  and the parameter  $\varepsilon$ , which is the measure of symmetry of the random variable  $h_s$ , are found and explained in (Christensen and Tonder 1969a, 1969b, 1970).

The flow is laminar and lubricant film is considered to be isoviscous and incompressible. Assuming the usual assumptions of the hydromagnetic lubrication, the concerned Reynold's equation (Prakash and Vij 1973), (Deheri et al. 2005), (Deheri and Patel 2011) and (Patel and Deheri 2013) governing the film pressure in this case is derived as

$$\frac{1}{x} \frac{d}{dx} \left[ x h^3 \frac{d}{dx} \left( p - \frac{1}{2} \mu_0 \bar{\mu} M^2 \right) \right] = \frac{12 \mu \dot{h}_0}{\sin^2 \omega} \tag{2}$$

where

$$M^2 = a^2 - \sin^2 \omega$$

$\alpha$  is dimension of the bearing,  $h$  denotes film thickness,  $\omega$  stands for semi-vertical angle of cone,  $\mu$  being fluid viscosity,  $\bar{\mu}$  represents the magnetic susceptibility and  $\mu_0$  denotes the permeability of the free space.

Following the averaging process of (Christensen and Tonder 1969a, 1969b, 1970) discussed by (Andharia and Deheri 2010), equation (2) takes the form:

$$\frac{1}{x} \frac{d}{dx} \left[ x m(\bar{h})^{-1} \frac{d}{dx} \left( \bar{p} - \frac{1}{2} \mu_0 \bar{\mu} M^2 \right) \right] = \frac{12 \mu h_0}{\sin^2 \omega} \quad (3)$$

where  $\bar{p}$  is the expected value of the lubricant pressure  $p$  and

$$m(\bar{h}) = (\bar{h})^{-3} \left[ 1 - 3(\bar{h})^{-1} \alpha + 6(\bar{h})^{-2} (\sigma^2 + \alpha^2) - 20(\bar{h})^{-3} (3\sigma^2 \alpha + \alpha^3 + \varepsilon) \right] \left( \frac{2 + s\bar{h}}{4 + s\bar{h}} \right)$$

The concerned boundary conditions are:

$$\bar{p}(a \operatorname{cosec} \omega) = 0, \left( \frac{d\bar{p}}{dx} \right)_{x=0} = 0 \quad (4)$$

Introducing the non dimensional quantities:

$$H = \frac{\bar{h}}{h_0}, X = \frac{x}{a \operatorname{cosec} \omega}, M(H) = h_0^3 m(\bar{h}), \bar{\sigma} = \frac{\sigma}{h_0}, \bar{\alpha} = \frac{\alpha}{h_0}, \bar{\varepsilon} = \frac{\varepsilon}{h_0^3},$$

$$P = -\frac{h_0^3 \bar{p}}{\mu h_0 A}, \mu^* = -\frac{\mu_0 \bar{\mu} h_0^3}{\mu h_0} \quad (5)$$

wherein

$$A = \frac{a^2 \pi}{\sin \omega}.$$

Using equation (5) in equation (3), the dimensionless pressure is obtained in the form of:

$$P = \frac{\mu^* \sin \omega}{2\pi} (1 - X^2) + \frac{3M(H)}{\pi \sin^2 \omega} (1 - X^2) \quad (6)$$

where

$$M(H) = H^{-3} \left[ 1 - 3H^{-1} \bar{\alpha} + 6H^{-2} (\bar{\sigma}^2 + \bar{\alpha}^2) - 20H^{-3} (3\bar{\sigma}^2 \bar{\alpha} + \bar{\alpha}^3 + \bar{\varepsilon}) \right] \left( \frac{2 + sH}{4 + sH} \right)$$

The non-dimensional load carrying capacity then, is derived as:

$$W = -\frac{w h_0^3}{\mu h_0 A^2} = \int_0^1 P dX = \frac{\mu^* \sin \omega}{3\pi} + \frac{2M(H)}{\pi \sin^2 \omega} \quad (7)$$

where

$$w = 2\pi \int_0^{a \operatorname{cosec} \omega} x p(x) dx$$

### 3. Results and Discussions

It is noticed that the non dimensional pressure is obtained from equation (6) while equation (7) presents the dimensionless load carrying capacity. It is observed that the dimensionless pressure increased by

$$\frac{\mu^* \sin \omega}{2\pi} (1 - X^2)$$

while the non dimensional load carrying capacity enhances by

$$\frac{\mu^* \sin \omega}{3 \pi}$$

as compared to the case of traditional lubricant based bearing system. Probably, this may be due to the fact that the viscosity of the lubricant gets increase because of magnetization. This leads to the increased pressure and hence enhanced load carrying capacity. Besides, it can be seen that the expression involved in the equation (7) is linear with respect to the magnetization parameter  $\mu^*$  and hence an increase in  $\mu^*$  would lead to increased load carrying capacity. In the absence of slip velocity this study reduces to the investigation of (Andharia and Deheri 2010).

The fact that the magnetization causes increased load carrying capacity can be seen from Figures 2-3. Here the increase is nominal. It is interesting to note that the standard deviation associated with roughness increases the load carrying capacity which does not happen in the case of transverse pattern of the roughness.

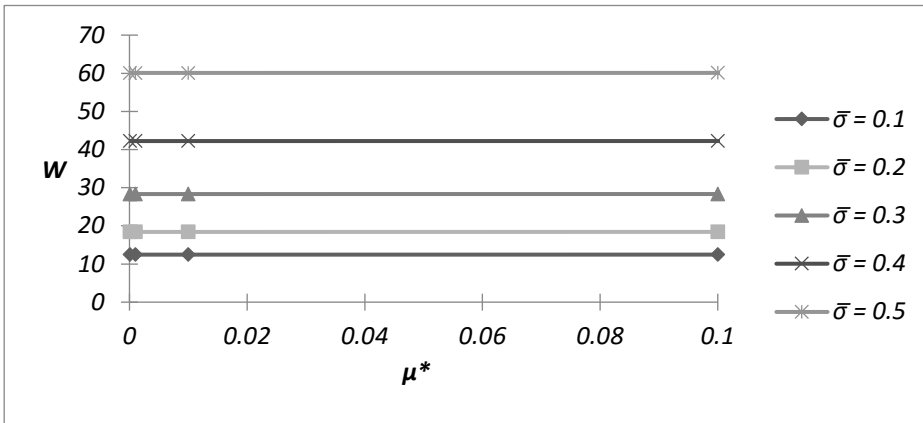


Fig. 2. Variation of load carrying capacity with respect to  $\mu^*$  and  $\bar{\sigma}$

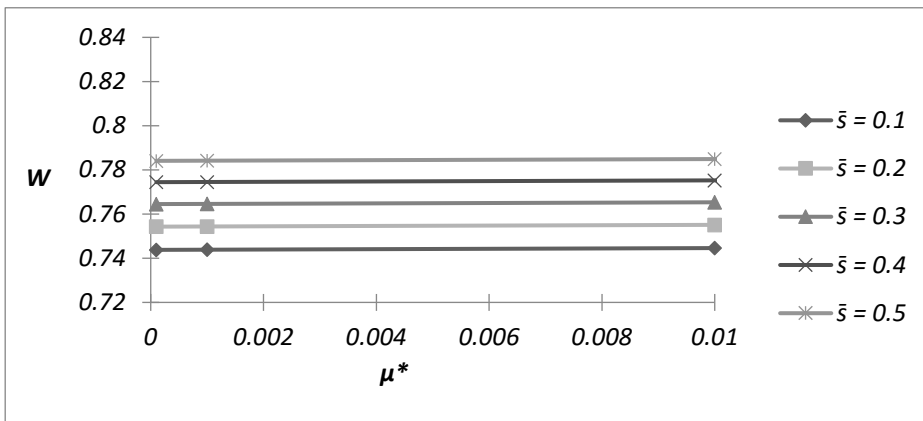
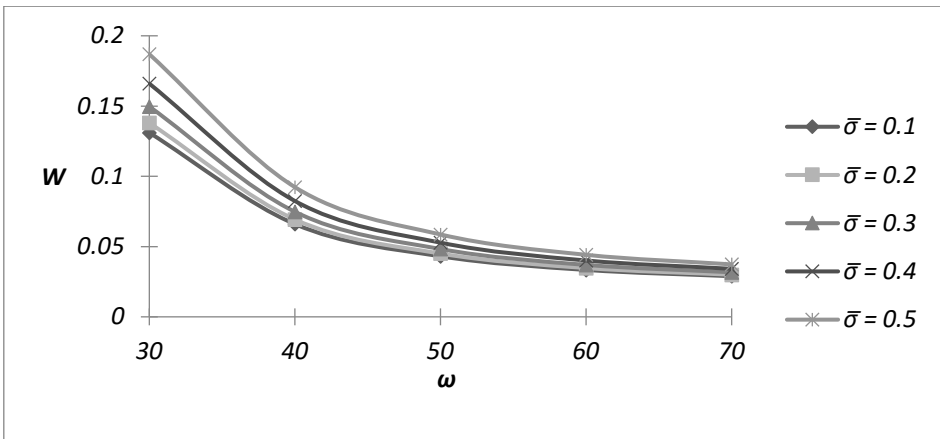
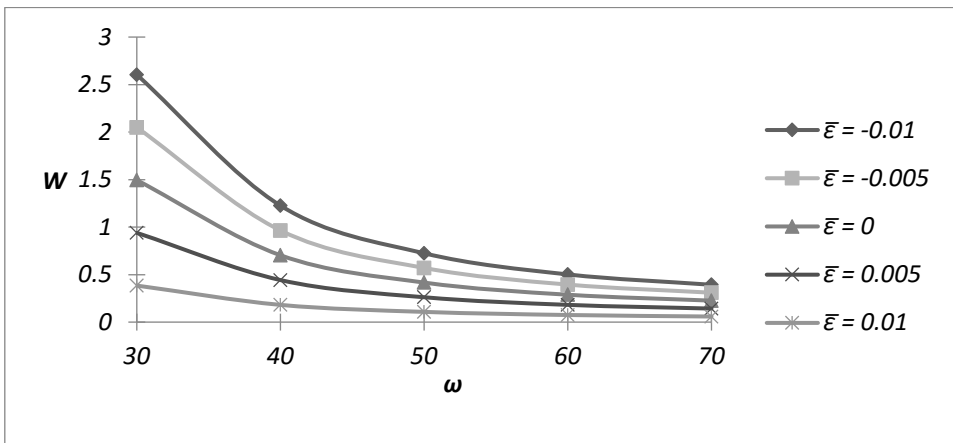


Fig. 3. Variation of load carrying capacity with respect to  $\mu^*$  and  $\bar{\sigma}$

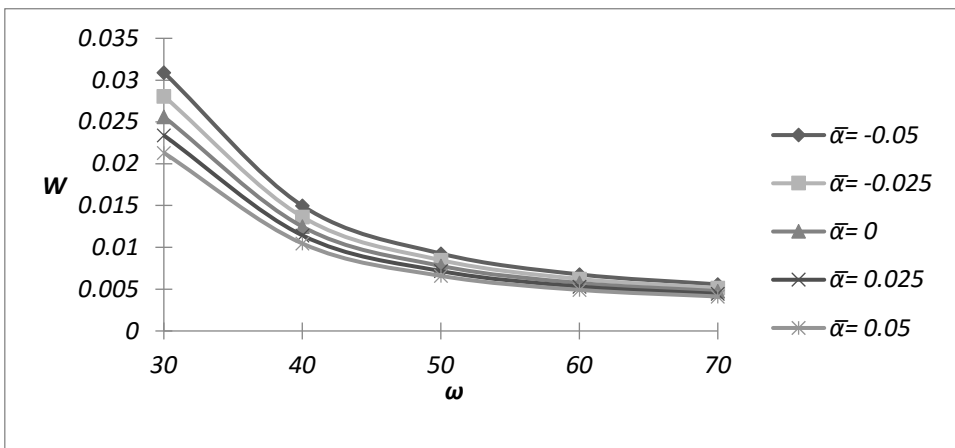
The effect of semi-vertical angle of the cone is presented in Figures 4-7. It is seen that there is heavy load reduction in between 30-50 degree. However, the effect of slip velocity on the load carrying capacity with respect to the semi vertical angle is almost negligible (Figure 7).



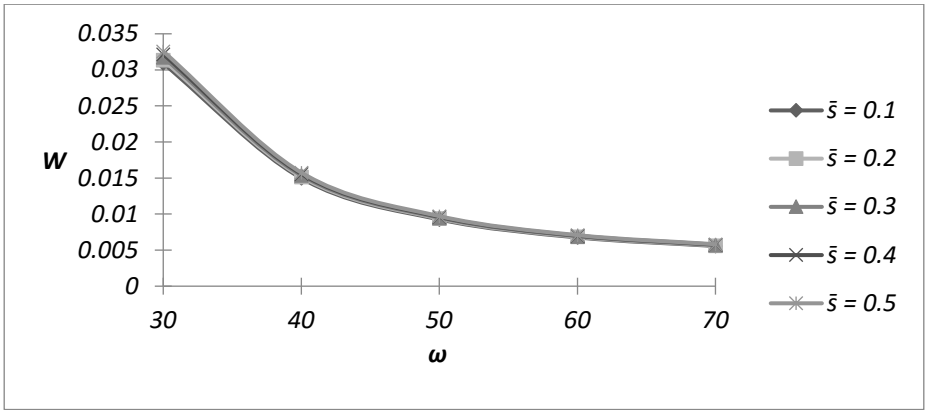
**Fig. 4.** Variation of load carrying capacity with respect to  $\omega$  and  $\bar{\sigma}$



**Fig. 5.** Variation of load carrying capacity with respect to  $\omega$  and  $\bar{\epsilon}$

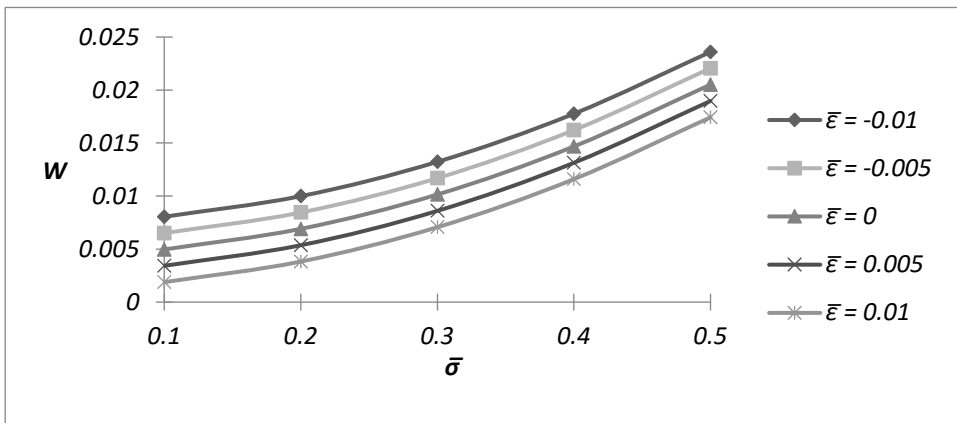


**Fig. 6.** Variation of load carrying capacity with respect to  $\omega$  and  $\bar{\alpha}$

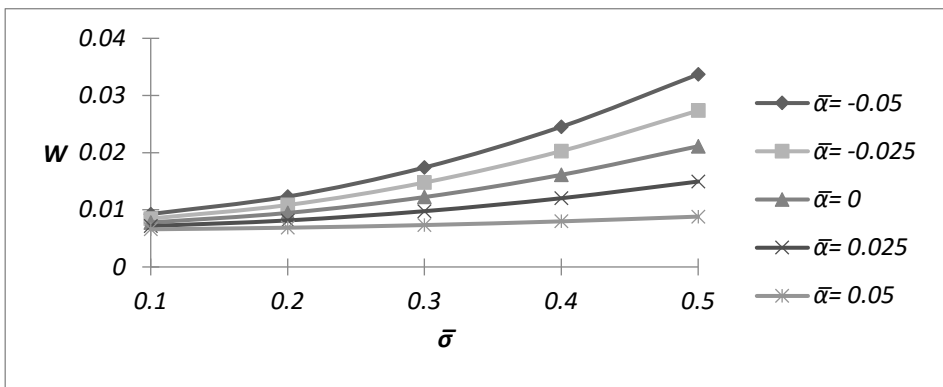


**Fig. 7.** Variation of load carrying capacity with respect to  $\omega$  and  $\bar{s}$

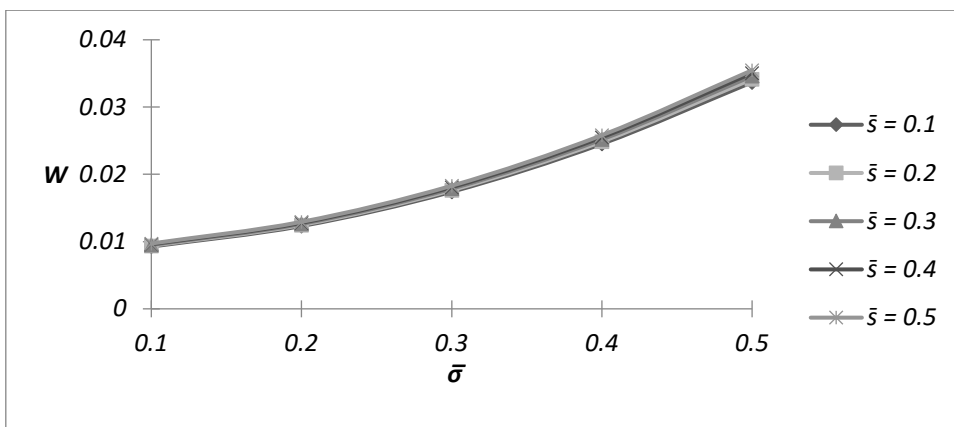
From Figures 8-10, it can be noticed that although the standard deviation causes an increase in load carrying capacity, the effect of slip on the variation of load carrying capacity with respect to standard deviation is negligible (Figure 10).



**Fig. 8.** Variation of load carrying capacity with respect to  $\bar{\sigma}$  and  $\bar{\epsilon}$

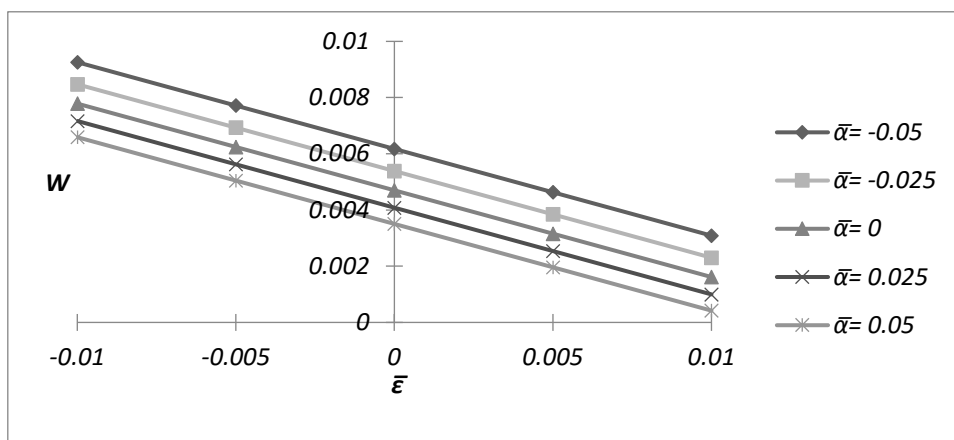


**Fig. 9.** Variation of load carrying capacity with respect to  $\bar{\sigma}$  and  $\bar{\alpha}$



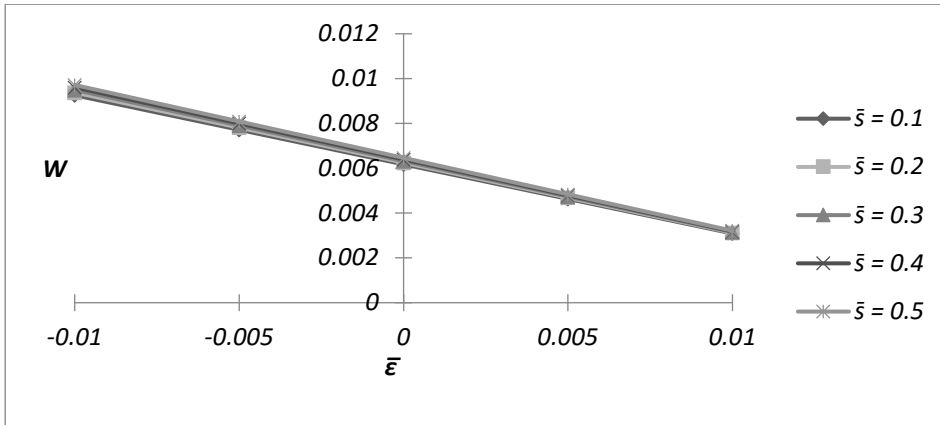
**Fig. 10.** Variation of load carrying capacity with respect to  $\bar{\sigma}$  and  $\bar{s}$

The positively skewed roughness decreases the load carrying capacity while the load carrying capacity gets increase owing to negatively skewed roughness. Besides, the effect of slip on the distribution of load carrying capacity with respect to skewness is negligible (Figure 12). Further, it is seen that the variance follows the path of skewness so far as the trends of load carrying capacity is concerned (Figures 11-13).

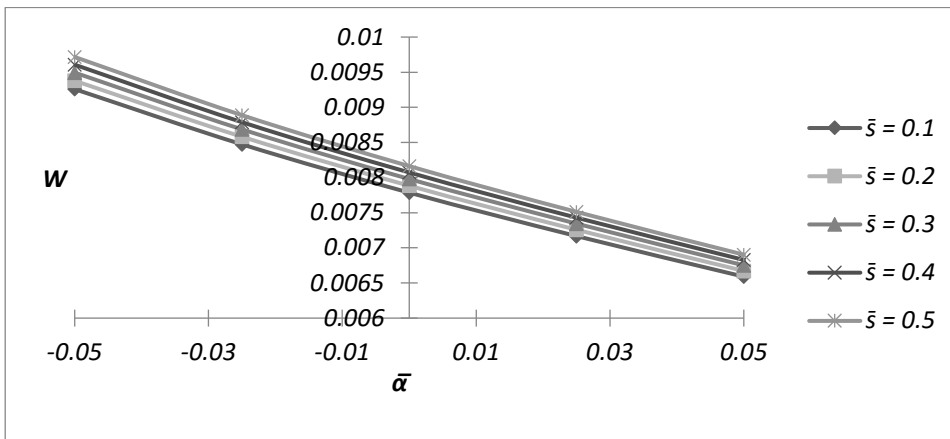


**Fig. 11.** Variation of load carrying capacity with respect to  $\bar{\sigma}$  and  $\bar{\alpha}$





**Fig. 12.** Variation of load carrying capacity with respect to  $\bar{\epsilon}$  and  $\bar{s}$



**Fig.13.** Variation of load carrying capacity with respect to  $\bar{\alpha}$  and  $\bar{s}$

The figures establish that the combined positive effect of negatively skewed roughness and variance (-ve) may lead to an improved performance of the bearing system as the slip effect can be contained by the positive effect of magnetization and standard deviation.

**4. Conclusions**

It is appealing to note that in the most of the situations the slip effect is at the most nominal which can be overcome by the positive effect of magnetization and negatively skewed roughness as the standard deviation also increases the load carrying capacity. However, the slip parameter may be taken at the reduced level to derive an improved performance in the general situation. Therefore, the roughness aspects must be addressed carefully while designing the bearing system.

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## Извод

**Ефекат брзине клизања на ферофлуидну превлаку код уздужних храпавих коничних плоча****J. R. Patel<sup>1\*</sup>, G. M. Deheri<sup>2</sup>**

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**Резиме**

У овом раду дат је покушај испитавања и анализе ефекта брзине клизања на перформансе превлаке са магнетном течношћу код коничних плоча уз узимање у обзир уздужног обрасца храпавости. Клизни модел Биверса и Џозефа (Beavers and Joseph) коришћен је како би се израчунао ефекат брзине клизања. Узет је стохастички усредњен модел Кристенсена и Тондера (Christensen and Tonder) како би се проценио ефекат уздужне храпавости. Стохастички усредњене једначине Рејнолдсовог типа решене су како би се добила расподела притиска која резултира израчунавањем капацитета ношења терета. Резултати показују да се комбиновани негативни ефекат брзине клизања и храпавости може превазићи у великој мери позитивним ефектом магнетизације и стандардном девијацијом у случају негативно искошене храпавости. Овај ефекат се даље појачава када је варијација (-ve) на месту. Значајан аспект нашег истраживања јесте да упркос негативном ефекту брзине клизања, храпави систем задржава одређену количину терета, чак и у одсутности тока који се ретко виђа у случају традиционалних коничних система са лубрикантом.

**Кључне речи:** коничне плоче, магнетна течност, уздужна храпавост, брзина клизања

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